TIME-DEPENDENT RELIABILITY ANALYSIS OF REINFORCED-CONCRETE BRIDGES UNDER THE COMBINED EFFECT OF CORROSION, CREEP AND SHRINKAGE

FATIMA EL HAJJ CHEHADE[1,2], RAFIC YOUNES[3], HUSSEIN MROUEH[1] & FADI HAGE CHEHADE[2]
[1]Laboratory of Civil Engineering and geo-Environment (LGCgE), Lille 1 University, France
[2]Modeling Center, Doctoral School of Science and Technology, Lebanon
[3]Faculty of Engineering, Lebanese University, Lebanon

ABSTRACT

This paper proposes a methodology for the time-dependent reliability assessment of reinforced-concrete bridges, taking into account the combined effect of steel corrosion due to the aggressive environment, creep and shrinkage under realistic traffic. For this purpose, traffic is simulated over time according to weight in motion, data recorded in Europe, then time-dependent deflection is calculated by considering the cyclic creep effect due to traffic load in addition to shrinkage and tension stiffening. So, the time-dependent bending moment can be deduced using the finite element method; this moment is then introduced into the limit state function with the nominal resistance (flexure) which decreases over time due to the reduction of steel section under corrosion, the reliability index and the probability of failure are then calculated using the first-order reliability method. This procedure is illustrated through an example for a simply supported reinforced-concrete bridge.

Keywords: corrosion, creep, shrinkage, realistic traffic, reliability, reinforced-concrete bridge.

1 INTRODUCTION

During their service life, civil infrastructures are exposed to operating and environmental conditions that can be the main causes for structural deficiency and deterioration. In this work, we are interested in the assessment of road bridges, especially short- to medium-span reinforced concrete (RC) bridges that constitute an essential part of civil infrastructures.

One of the major forms of environmental attack for this kind of structure is chloride diffusion in concrete, leading to reinforcement corrosion. Another challenge related to varying cyclic loading caused by traffic is posed for these structures and may also affect durability and public safety. Therefore, the determination of the need or not for maintenance or replacement becomes a major issue, especially because of the high replacement cost and limited budgets allocated for these purposes.

The performance evaluation of any structure requires the estimation of the resistance and load effect. In order to evaluate the high variability of the parameters involved in the assessment, the reliability theory is used to determine the probabilistic measure of safe performance.

Reliability engineering is relatively recent; it appeared as an independent discipline in the 1950s and concerns several fields of engineering. At the beginning, it was focused on the reliability of machines and electronic equipment for estimating failure rate, then gradually it began to have applications in wider fields. Thus, software-reliability assessment is now the main source of reliability problems as a result of the increasing use of software programs [1]. The concept of reliability analysis is also transmitted to the field of structural design and assessment in civil engineering because it suits the uncertainties existing in most areas of this field. This theory becomes more likely to be applied by the end of 1970s thanks to the

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pioneering work of Cornell et al. [2]. Structural reliability analysis is now the most sophisticated level of assessment of highway structures [3].

In recent years, there has been a considerable amount of publications concerning the time-dependent reliability assessment of highway bridges. The time-dependent behavior of structural resistance for RC bridges is evaluated in most cases by considering the influence of corrosion with time on the reinforcement steel which causes the loss of flexural strength [4]–[8]; the stochastic nature of live load is also involved in some studies trying to predict a reasonable loading model but assuming a stationary load process without taking into account the time-variant nature of load intensity and frequency [9], [10]. In this study, the reliability analysis will take into account the reduction of the resistance caused by steel corrosion due to chloride ingress in addition to creep, shrinkage and tension stiffening included in the side of load effect. For this purpose, a time-dependent traffic simulation based on European weight in motion (WIM) data is performed. The approach will be clarified by an illustrative example of a simply supported RC bridge.

2 RELIABILITY GENERAL FORMULATION

Structural reliability theory is concerned with the assessment methods of safety and serviceability for civil engineering structures while giving a rational treatment of uncertainties; in other words, structural reliability is the probability that a structure will not attain a specified limit state (ultimate or serviceability) at a given period of time – each limit state can be defined by a particular form of a function, called the limit state function or failure function, and the general form of a limit state function can be divided into a resistance term R and a load effect term S. The time-dependent limit state function can be given by the following general form [11]:

$$G(t) = R(t) - S(t). \tag{1}$$

The most suitable form of the limit state function is by expressing it in terms of the set of n basic variables $\bar{X}$, which affect the structural performance because these variables may not be statistically independent, so we obtain the following expression of the limit state function:

$$G(X_i(t)) = R\big(X_1(t), X_2(t), \dots, X_n(t)\big) - S(DD, LL(t), \dots), \tag{2}$$

where $R(t)$ is the resistance function of the system of random variables which influence the limit state – for example, for a concrete section these variables are related to material properties and section dimensions – and $S(t)$ is the random function of load effects resulting from dead loads DD and live loads LL.

This definition implies the splitting of the space into two zones: the reliable zone and the failure zone, the boundary between these two subspaces is a hyper surface of equation $G(X)$ which is called the failure surface (i.e. $R = S$); thus, the probability that $G(X) < 0$ corresponds to the probability of failure of the section p_f, to calculate this probability the integration is to be performed over the failure domain D where the strength is smaller than or equal to the load effect. In the case of independent variables, the volume integral can be transformed into a plane line integral as follows:

$$p_f = P(Z = R - S \leq 0) = \iint_{R<S} f_R(r).f_S(s)\, dr\, ds = \int_{-\infty}^{+\infty} F_R(s).f_S(s)\, ds, \tag{3}$$

where $f_R$and f_S are the probability density function for the time-variant resistance and load effect respectively, and F_R is the cumulative distribution function of the time-variant resistance.

Due to the high number of variables in the limit state function, the analytical solution of this integral is very complex. Monte Carlo simulation is sometimes used to calculate this type of integral by generating a very large number of variables with known distributions and then calculating the difference between R and S at each time; the probability of failure can be deduced from the rate of the cases where R was less than S. In this study, the first-order reliability method (FORM) is used to calculate the failure probability because it leads to more refined results [12]. By using the FORM method, the structural safety can also be estimated in terms of the reliability index β. Thus, the following steps are required:

1. Transform the random variable vector {X} from the physical variable space to standardized space with standard normal variables {U}, $G(\{X\}) \rightarrow \mathcal{G}(\{U\})$.
2. In the standardized space, the reliability index corresponds to the minimum distance between the origin of the space and the limit state surface; this optimization problem can be formulated as $\beta = min(\sum_{i=1}^{n} u_i{}^2)^{1/2}$.
3. The failure probability is deduced from the reliability index using first-order approximation by FORM as follows $p_f = \Phi(-\beta)$, where $\Phi(.)$ denotes the cumulative distribution function of the standard normal distribution.

3 STRENGTH LIMIT STATE

In this study, the strength or bending limit state is considered and the limit state function can be written in the form:

$$G(X_i(t)) = M_R(f_c', f_y, d, b, c, \dots) - M_a(M_{ss}, M_{traffic}, \dots), \tag{4}$$

where M_R is the resistance bending moment and M_a is the applied moment including traffic $M_{traffic}$, superstructure M_{ss} and bridge self-weight.

An overall description of the proposed method is illustrated through a flowchart in Fig. 1.

3.1 Calculation of time-variant flexural capacity

The nominal bending moment is used to calculate the flexural resistance for a RC beam cross-section, and is given by [12]:

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = A_s f_y \left(d - \frac{1}{2} \times \frac{A_s f_y}{0.85 f_c' b}\right), \tag{5}$$

where A_s is the reinforcement area, f_y is the steel yielding stress, d the effective height of the beam, f_c' the concrete compressive strength and b the width of the girder flange in case of the T section.

3.1.1 Reduction of steel cross-section

For RC bridges, the loss of strength is mainly caused by the corrosion of steel [4], [5], [13]. The time-variant resistance is then calculated by considering the reduction of the steel section with reference to the corrosion initiation time. The reduction of the bar diameter is given by [4]:

$$D(t) = \begin{cases} Di \; for \; t < T_i \\ D_i - 0.0232 \, i_{corr}(t - T_i) \; for \; T_i < t < T_i + \dfrac{Di}{0.0232 \, i_{corr}}, \\ 0 \; for \; t > T_i + \dfrac{Di}{0.0232 \, i_{corr}} \end{cases} \tag{6}$$

where $D(t)$ is the diameter of a reinforcement bar at time t (years), Di is the initial bar diameter, i_{corr} is the corrosion rate parameter in $\mu A/cm^2$.

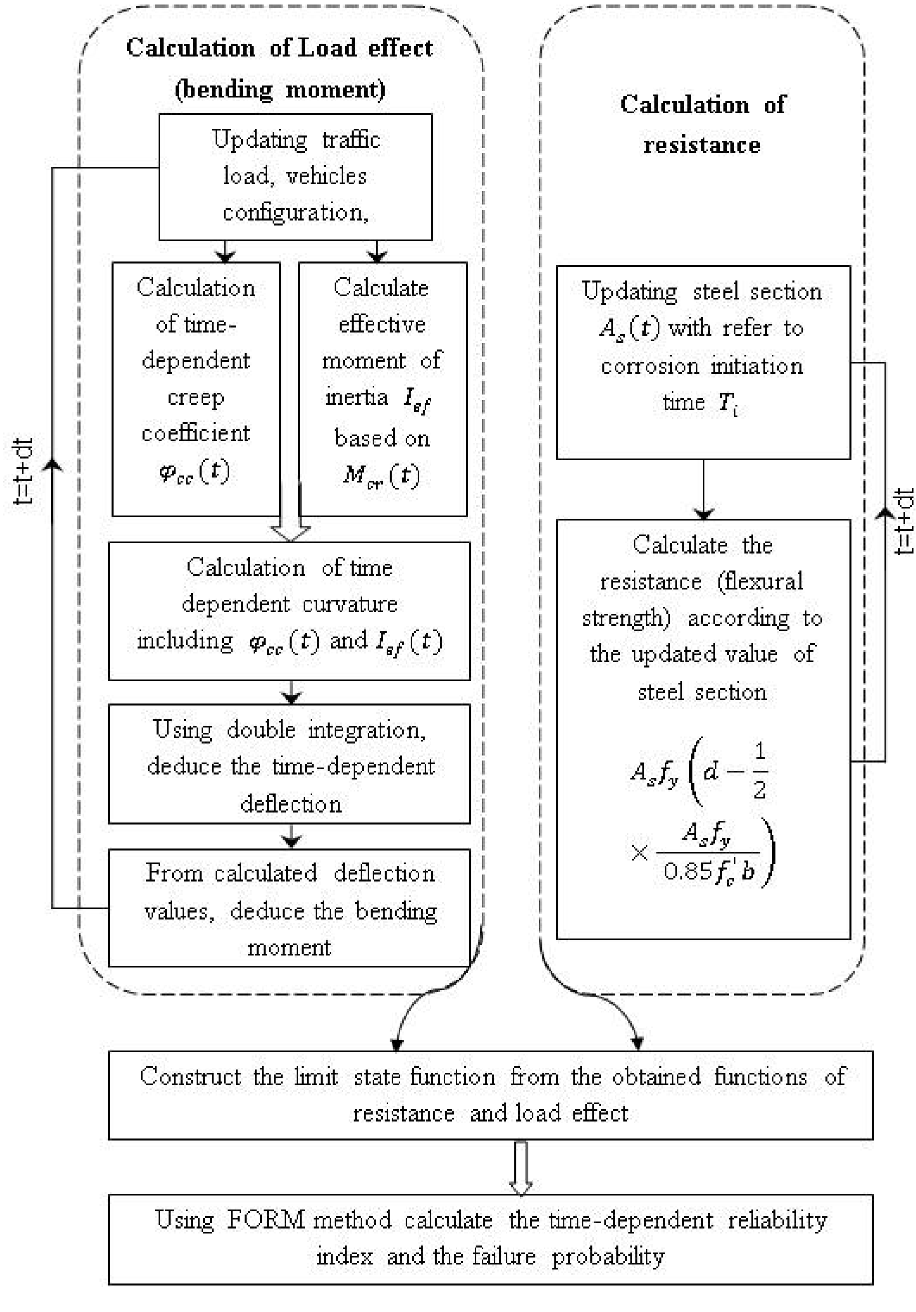

Figure 1: Flowchart of the proposed method.

When no experimental data are available, the corrosion-rate parameter can be calculated by the following eqn [13]:

$$i_{corr}(t) = \left(\frac{32.13 * (1 - w/c)^{-1.64}}{C}\right)(t - Ti)^{-0.29}. \tag{7}$$

In eqn (1), w/c is the concrete water–cement ratio and $C$is the concrete cover thickness.

In case of chloride-induced corrosion, the corrosion initiation time T_i is calculated according to the following eqn [14]:

$$T_i = \frac{C^2}{4D_c}\left[erf^{-1}\left(\frac{C_0 - C_{cr}}{C_0}\right)\right]^{-2}, \tag{8}$$

where $C\ (cm)$ is the concrete cover thickness, $D_c(cm^2/year\ )$ is the chloride diffusion coefficient, $C_0(\%\ weigth\ of\ concrete)$ is the equilibrium chloride concentration at the concrete surface, and $C_{cr}(\%\ weigth\ of\ concrete)$ is the critical chloride concentration.

3.2 Calculation of time-variant load effect

In most of the reliability studies a stationary load process or notional load models are used to simulate traffic loading [9], [15]. In this study, the time-dependent reliability analysis is performed by simulating a realistic traffic loading according to WIM data recorded in some European countries. Reliability analysis under realistic traffic loading has already been done by Hajializadeh et al. [16], but without considering the creep effect under cyclic loading, so in this study the moment applied on the girder cross-section is not directly calculated from the applied traffic load at time t, but it is deduced from the time-dependent deflection using the finite element method; the latter is calculated by considering the effect of tension stiffening, creep and shrinkage on the girder cross-section according to Gilbert [17].

3.2.1 Traffic simulation

For short- to medium-span bridges that have a span length of up to 45 m, free-flowing traffic is simulated, which allows to predict the load configuration at each time step; this model is inspired by WIM data recorded on the Mattstetten motorway in Switzerland in the past 10 years [18]. According to this data, heavy vehicles or trucks (vehicles with a weight of more than 3.5 t) can be grouped into 12 classes with the number of axles ranging from 0 to 6; the distribution of gross vehicle weight for each vehicle class is fitted to bimodal beta distribution (an example is given in Fig. 2). Monte Carlo simulation is then used to generate vehicle queues according to the given distribution for each vehicle class. The distance between vehicles x in the case of free-moving traffic is generated based on traffic volume V (vehicles per hours) using the following density function [19]:

$$f_D(x) = \frac{V}{3600 * 22} exp\left(-\frac{V}{3600 * 22}(x - 5.5)\right). \tag{9}$$

The average hourly flow during the 24 hours of the day is adopted from recorded data in The Netherlands [20].

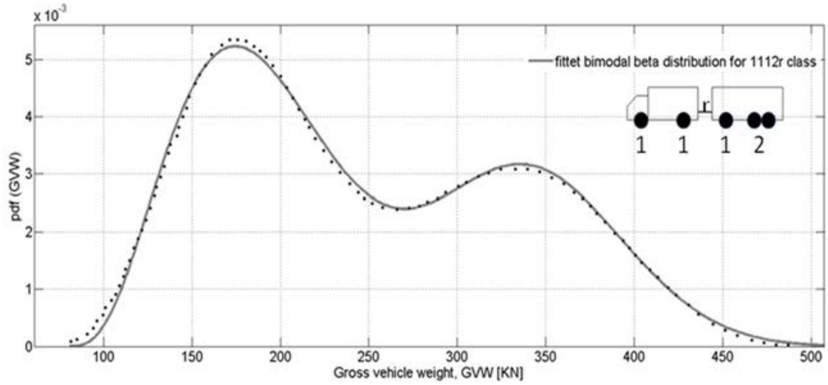

Figure 2: Example of bimodal beta distribution for vehicle class 1112r.

3.2.2 Time-variant applied moment

Time-variant in service moment is deduced from long-term deflection calculation for RC structure according to Gilbert [17]. Pseudo-static analysis is performed assuming sustained in service traffic load at each time step taken as 25 hours in order to cover all possible traffic volumes throughout the day. Using double integration of time-dependent curvature, time-dependent deflection at mid-span can be calculated as follows:

$$\Delta = \frac{l^2}{96}(\kappa_L + 10\kappa_M + \kappa_R), \tag{10}$$

where l is the span length, κ_L, κ_R and κ_M are respectively the curvatures at left support, right support and at the mid-span.

At any cross-section, the total curvature κ is the sum of the load induced $\kappa(t)$ and shrinkage-induced κ_{cs} curvatures, where $\kappa(t) = \kappa_i(1 + \varphi_{cc}/\alpha)$ and $\kappa_{cs} = \kappa_r \varepsilon_{cs}/h$. In $\kappa(t)$, the instantaneous curvature κ_i depends on the maximum applied moment M_s at the beam under sustained in service traffic load at a given time step, so $\kappa_i = M_s/E_c I_{ef}$ where E_c is the concrete young modulus and I_{ef} the effective second moment of inertia which decrease with time by taking into account the gradual reduction of tension stiffening in the calculation of the cracking moment M_{cr}; the creep coefficient at time t, φ_{cc} is calculated according to CEB-FIP Model Code 90 [21] as a function of ambient humidity, composition of the concrete mix and dimensions member, α is related to the effects of cracks. The shrinkage-induced curvature κ_{cs} is a function of the shrinkage strain ε_{cs}, also calculated according to the CEB-FIP Model Code 90 and depends mainly on the concrete strength; h is the overall section depth and κ_r is a term related to the quantity and location of reinforcement.

After calculating the time-dependent deflection which is increasing with time due to shrinkage and creep, the bending moment is then deduced from this increasing deflection using finite element discretization that allows us to calculate the internal forces. This obtained time-dependent moment must, of course, gradually increase with time.

4 CASE STUDY

The proposed methodology is illustrated through an application for a simply supported RC T-beam bridge having a span length of 9.1 m. The nominal values of the cross-section dimensions for a central beam are given in Fig. 3.

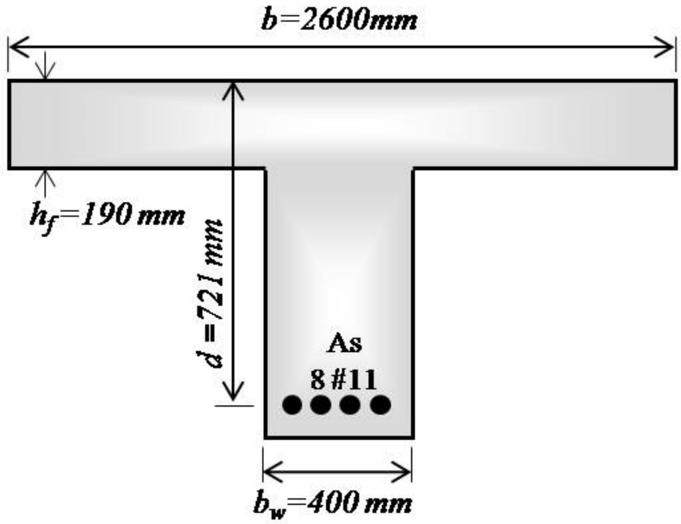

Figure 3: Cross-section of the bridge girder.

Time-dependent reliability analysis is performed for this girder and the bending or flexural limit state is represented by the following limit state function:

$$G(X_i(t)) = M_R(f_c', f_y, d, b, c, \dots) - M_a(M_{ss}, M_{traffic}, \dots) = A_s f_y \left(d - \frac{1}{2} \times \frac{A_s f_y}{0.85 f_c' b} \right) - (M_{DL} + M_{LL})$$

$$= ns * \frac{\pi}{4} \left(D_i - 0.0232 \left(\frac{32.13 * \left(1 - \left(\frac{27}{f_c' + 13.5}\right)\right)^{-1.64}}{C} \right) (t - Ti)^{-0.29} * (t - T_i) \right)^2 - (\alpha * t^2 + \beta * t + \gamma), \quad (11)$$

where ns is the number of reinforcing bars and Ti is the corrosion initiation time given by eqn (8); the distributions of the different variables in the limit state function are shown in Table 1. The calculation of the time-dependent probabilistic distribution of the applied moment is detailed in the following paragraph.

4.1 Time-dependent distribution of applied moment

As mentioned previously in paragraph 3.2.2, the applied moment is deduced from time-dependent deflection using the finite element method; for this purpose, hundreds of simulations for traffic loading are performed, and for each one the moment profile is deduced by fitting the extreme values of the obtained data to a second-order polynomial trend line with the generic equation: $\alpha * t^2 + \beta * t + \gamma$ (see Fig. 4). In this equation, the coefficients α, β and γ seem to be normally distributed (see Fig. 5); their distribution parameters are shown in Table 1. The dead load (self-weight and superstructure) is also included in the moment plot.

For comparison reasons, the moment is also calculated directly based on hundreds of traffic simulations without taking into account the creep effect or the traffic growth; the curve of the stationary moment is shown in Fig. 6. It is also normally distributed with the distribution function shown in the right side of Fig. 6.

Table 1: Distribution of the variables.

Description [source]	Variable	Distribution	Units	Nominal	Bias*	Cov**
Variables related to flexural strength						
Reinforcing steel area [9]	D_i	Lognormal	mm	35.8	1	0.05
Concrete compressive strength [22]	f_c'	Normal	MPa	32	1	0.15
Cover [9]	c	normal	mm	69	1	0.13
Chloride diffusion coefficient [8]	D_c	Lognormal	cm^2/yr	1.29	1	0.1
Surface chloride concentration [8]	C_0	Lognormal	wt%concrete	0.1	1	0.1
Critical chloride concentration [8]	C_{cr}	Lognormal	wt%concrete	0.04	1	0.1
Steel yielding stress [23]	f_y	Normal	MPa	500	1	0.1
Effective height of T beam [10]	d	Normal	mm	721	1.0124	0.0229
Width of the girder flange [9], [10]	b	Deterministic	mm	2600	–	–
Parameter1 related to corrosion rate i_{corr} [9]	A	Deterministic	–	32.13	–	–
Parameter2 related to corrosion rate i_{corr} [9]	B	Deterministic	–	-1.64	–	–
Parameter3 related to corrosion rate i_{corr} [9]	C	Deterministic	–	–0.29	–	–
Variables related to applied moment						
Moment coefficient 1	α	Normal	–	0.042	1	0.24
Moment coefficient 2	β	Normal	–	0.57	1	1.614
Moment coefficient 3	γ	Normal	–	660.206	1	0.0238

*Bias: mean value/nominal value.
**Cov: coefficient of variation.

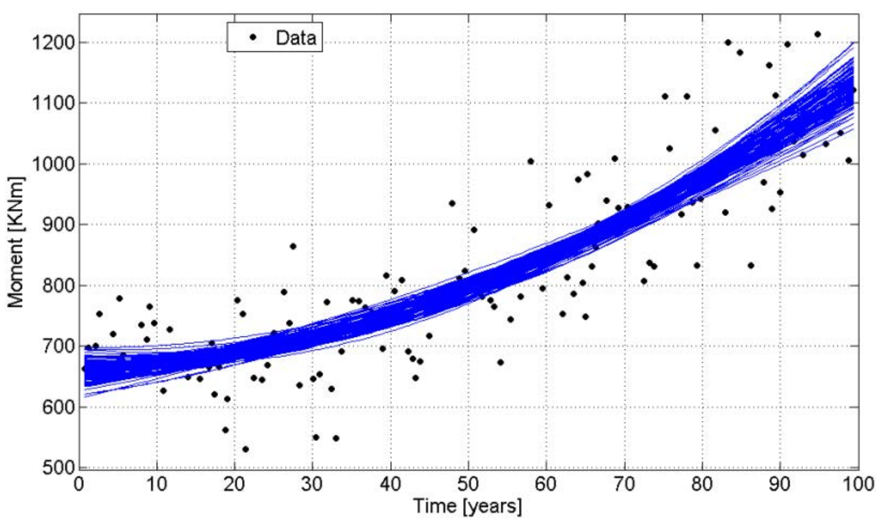

Figure 4: Profiles of the time-dependent applied moment deduced from deflection.

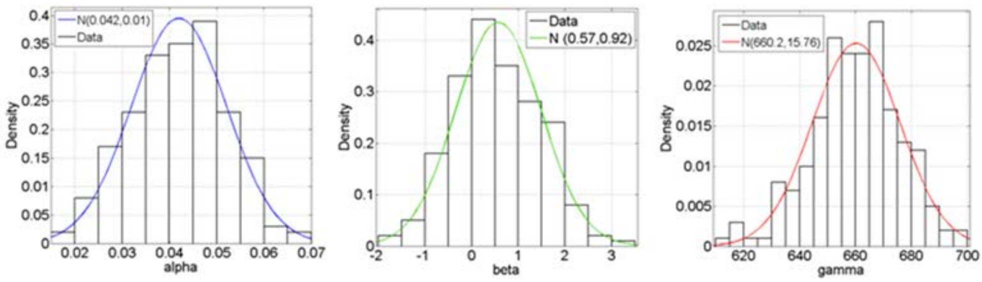

Figure 5: Probability density function for moment coefficients α, β and γ.

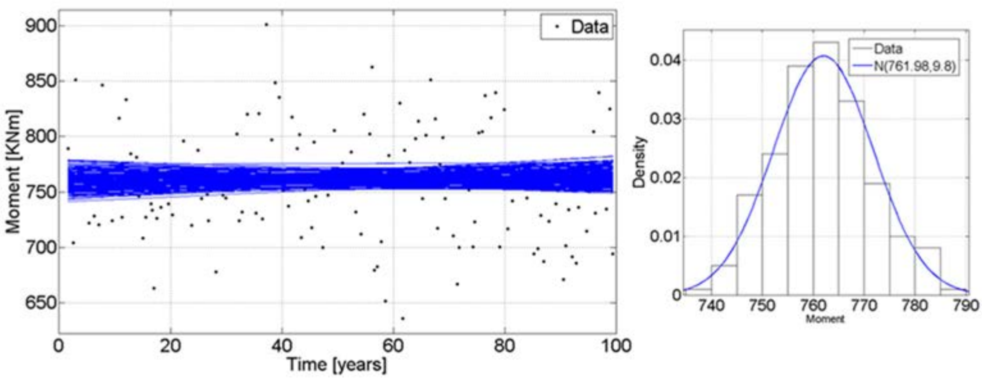

Figure 6: Profiles of the stationary applied moment directly calculated with no degradation.

4.2 Reliability results and discussion

The reliability indices corresponding to the annual probability of failure were calculated for the 100 years of bridge service life in 5-year increments using Matlab with reference to Kostandyan and Sørensen [24]. The evolution of the individual density functions for resistance R and load effect S with time is shown in Fig. 7 for t = 0, 40 and 100 years. It is clear that the beginning the failure zone where R < S was very small, even invisible, but it becomes more visible at 100 years. On the other hand, by considering the reduction of steel section in the resistance function and under realistic traffic loading, including the effect of shrinkage and creep in the load effect function, the reliability index decreases from 7.5 at the beginning to 4.76 after 100 years(see Fig. 8); so, with a rate of 36.5%, which is relatively high compared to the reduction of reliability index calculated under stationary loading by considering only the corrosion effect in the resistance function, the latter decreases from 7.3 to 7 with only 4.1% and the failure probability increases but remains in very small ranks reaching 10^{-12} only (see Fig. 9), which is 100,000 times smaller than the same probability in the first case reaching 8×10^{-7}, as shown in Fig. 8.

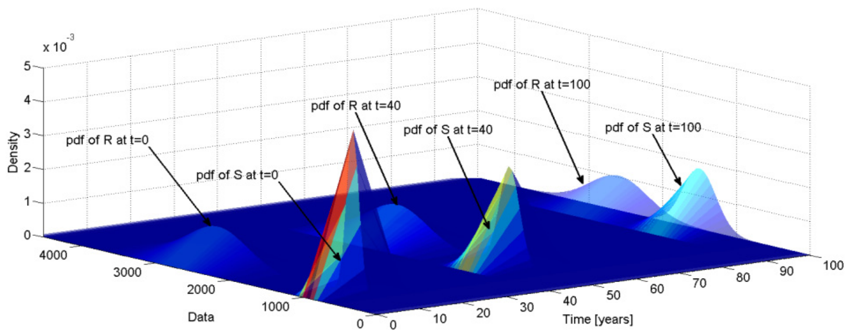

Figure 7: Illustration of the time-varying Pdfs of resistance R(t) and load-effect S(t).

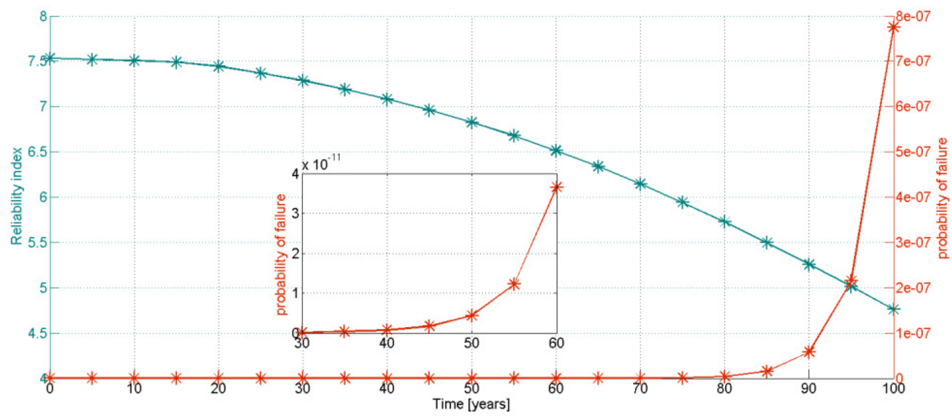

Figure 8: Reliability index and probability of failure including creep and realistic traffic.

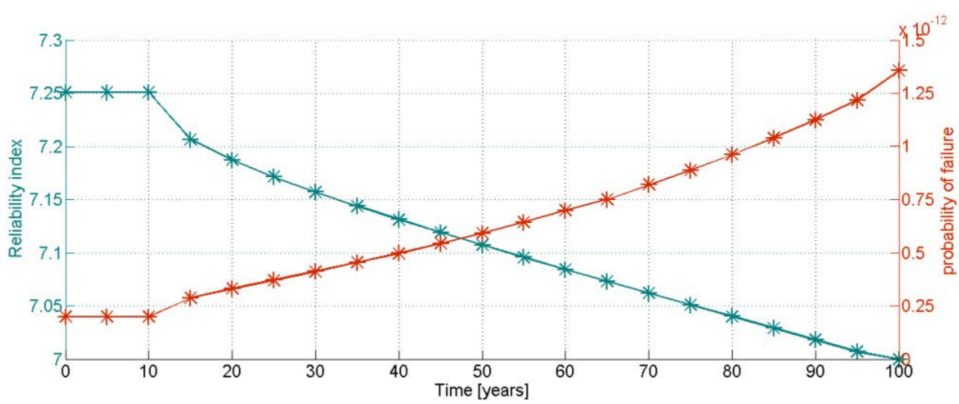

Figure 9: Reliability index and probability of failure under stationary load without creep.

5 CONCLUSIONS

The importance of using a time-dependent applied moment in reliability assessment resulting from realistic traffic simulation and increased by creep and shrinkage effects in addition to the degradation of flexural strength by corrosion is obvious. This will have a considerable influence on the increase of failure probability and the decrease of the reliability index compared to the small evolution of these parameters when considering only the degradation of flexural strength under corrosion with a stationary applied moment.

In both cases the values of the reliability indices, even after 100 years, remain greater than the target reliability index which represents the minimal recommended value for evaluation (3.5 for RC bridges [2]); this depends strongly on the dimensions of the assessed structure because, in fact, reliability analysis is very specific for each structure. In the future, this study will be performed on a group of RC bridges with various characteristics according to their dimensions and materials.

REFERENCES

[1] Bhamare, S.S., Yadav, O.P. & Rathore, A., Evolution of reliability engineering discipline over the last six decades: a comprehensive review. *Int. J. Reliab. Saf.*, **1**(4), p. 377, 2007.

[2] Nowak, A.S. & Szerszen, M.M., Structural reliability as applied to highway bridges. *Prog. Struct. Eng. Mater.*, **2**(2), pp. 218–224, 2000.

[3] O'Brien, E.J., Znidaric, A., Brady, K.C., González, A. & O'Connor, A., Procedures for the assessment of highway structures, 2005.

[4] Stewart, M.G. & Rosowsky, D.V., Time-dependent reliability of deteriorating reinforced concrete bridge decks. *Struct. Saf.*, **20**(1), pp. 91–109, 1998.

[5] Saad, L., Aissani, A., Chateauneuf, A. & Raphael, W., Reliability-based optimization of direct and indirect LCC of RC bridge elements under coupled fatigue-corrosion deterioration processes. *Eng. Fail. Anal.*, **59**, pp. 570–587, Jan. 2016.

[6] Thoft-Christensen, P. & Middleton, C.R., *Reliability assessment of concrete bridges*, Dept. of Building Technology and Structural Engineering, 1997.

[7] Val, D.V., Stewart, M.G. & Melchers, R.E., Effect of reinforcement corrosion on reliability of highway bridges. *Eng. Struct.*, **20**(11), pp. 1010–1019, 1998.

[8] Enright, M.P. & Frangopol, D.M., Probabilistic analysis of resistance degradation of reinforced concrete bridge beams under corrosion. *Eng. Struct.*, **20**(11), pp. 960–971, 1998.

[9] de Cassia Silva, R., *Contribution to the probabilistic analysis of reinforced concrete bridge performance,* Ecole des Ponts Paris Tech, 2004.

[10] Xie, H., Wang, Y. & Zou, R., Reliability analysis of RC T-beam highway bridges in China based on a virtual bridge dataset. *Eng. Struct.*, **104**, pp. 133–140, Dec. 2015.

[11] Thoft-Christensen, P. & Baker, M.J., *Structural Reliability Theory and its Applications*, Springer-Verlag: Berlin, 1982.

[12] American Association of State Highway and Transportation Officials, *AASHTO LRFD Bridge Design Specifications*, 3rd ed., American Association of State Highway and Transportation Officials: Washington, DC, 2005.

[13] Ma, Y., Zhang, J., Wang, L. & Liu, Y., Probabilistic prediction with Bayesian updating for strength degradation of RC bridge beams. *Struct. Saf.*, **44**, pp. 102–109, Sep. 2013.

[14] Thoft-Christensen, P., Jensen, F.M., Middleton, C.R. & Blackmore, A., *Assessment of the reliability of concrete slab bridges,* Dept. of Building Technology and Structural Engineering, 1996.

[15] Mahmoud, H. & Zafar, A., Time-dependent reliability analysis of reinforced-concrete bridges including deterioration effects, pp. 1354–1366, 2016.

[16] Hajializadeh, D., Stewart, M.G., Enright, B. & Obrien, E., Spatial time-dependent reliability analysis of reinforced concrete slab bridges subject to realistic traffic loading. *Struct. Infrastruct. Eng.*, **12**(9), pp. 1137–1152, Sep. 2016.

[17] Gilbert, R.I., Calculation of long-term deflection. Presented at the *CIA seminar*, Brisbane, 2008.

[18] Centre de Recherche, ICOM EPF-Lausanne, Meystre, T., Hirt, M.A. & Office Fédéral des Routes OFROU, *Evaluation de ponts routiers existants avec un modèle de charge de trafic actualisé*, Berne, 2006.

[19] Bailey, S.F., Basic principles and load models for the structural safety evaluation of existing road bridges. *EPFL*, 1996.

[20] Enright, B., *Simulation of traffic loading on highway bridges*, Dublin Institute of Technology, 2010.

[21] Comité Euro-International du Béton, CEB-FIP MODEL CODE 1990, Thomas Telford, 1993.

[22] Bigaud, D. & Ali, O., Time-variant flexural reliability of RC beams with externally bonded CFRP under combined fatigue-corrosion actions. *Reliab. Eng. Syst. Saf.*, **131**, pp. 257–270, Nov. 2014.

[23] Bastidas-Arteaga, E., Bressolette, P., Chateauneuf, A. & Sánchez-Silva, M., Probabilistic lifetime assessment of RC structures under coupled corrosion–fatigue deterioration processes. *Struct. Saf.*, **31**(1), pp. 84–96, Jan. 2009.

[24] Kostandyan, E. & Sørensen, J., Dependent systems reliability estimation by structural reliability approach. *Int. J. Perform. Eng.*, **10**(6), pp. 605–614, 2014.