Optimal and game ship control algorithms for avoiding collisions at sea

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Abstract

This paper introduces the application of optimal and game theory methods in marine navigation. The functional scope of a standard ARPA anti-collision system ends with the simulation of the manoeuvre altering course or speed selected by the navigator. The problem of selecting such a manoeuvre is very difficult as the process of control is very complex and game making in its nature. The most adequate model of the process that has been adopted is a model of a differential game. The control goal is defined firstly, followed by a description of the base model and a presentation of approximated models. For each approximated model, an appropriate method of safe control to support the navigator decision in a collision situation has been assigned. The POSitional TRAJectory (POSTRAJ) and the RISK TRAJectory (RISKTRAJ) control algorithms have been designed. The considerations have been illustrated in examples of computer simulation algorithms to determine the safe ship trajectories in situations when passing many objects.
Keywords: differential games, positional games, matrix games, dual linear programming, safety navigation, transport engineering, safe ship operations.

1 Differential game model of the ship control process

The process of ship control in collision situations, when a great number of objects is encountered, often occurs under the conditions of indefiniteness and conflict, accompanied by an inaccurate co-operation of the objects within the context of the International Regulations for Preventing Collision at Sea (COLREG). The most adequate model of the process that has been adopted is a model of a differential game, in general of *j* tracked ships as control objects [2,8].

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1.1 State equation

The properties of the process are described by the state equation:

$$\dot{x}_i = f_i[(x_0^{\vartheta_0}, x_1^{\vartheta_1}, ..., x_j^{\vartheta_j}, ..., x_m^{\vartheta_m}), (u_0^{\nu_0}, u_1^{\nu_1}, ..., u_j^{\nu_j}, ..., u_m^{\nu_m}), t] \tag{1}$$

$$i = 1, 2, ..., \left(j \cdot \vartheta_j + \vartheta_0\right),\ j = 1,\ 2,\ ...,\ m$$

where

$\vec{x}_0^{\vartheta_0}(t)$ - ϑ_0 dimensional vector of the process state of the own ship determined in time span $t \in [t_0, t_k]$,

$\vec{x}_j^{\vartheta_j}(t)$ - ϑ_j dimensional vector of the process state for the *j-th* object,

$\vec{u}_0^{\nu_0}(t)$ - ν_0 dimensional control vector of the own ship,

$\vec{u}_j^{\nu_j}(t)$ - ν_j dimensional control vector of the *j-th* object, see fig. 1.

Taking into consideration the equations describing the own ship's hydromechanics and equations of the own ship's movement relative to the *j-th* encountered object, the equations of the general state of the process (1) take the following form:

$$\begin{aligned}
\dot{x}_0^1 &= x_0^2 \\
\dot{x}_0^2 &= a_1 x_0^2 x_0^3 + a_2 x_0^3 \left|x_0^3\right| + b_1 x_0^3 \left|x_0^3\right| u_0^1 \\
\dot{x}_0^3 &= a_4 x_0^3 \left|x_0^3\right| \left|x_0^4\right| x_0^4 \left(1 + x_0^4\right) + a_5 x_0^2 x_0^3 x_0^4 \left|x_0^4\right| + a_6 x_0^2 x_0^3 x_0^4 + a_7 x_0^3 \left|x_0^3\right| + \\
&\quad + a_8 x_0^5 \left|x_0^5\right| x_0^6 + b_2 x_0^3 x_0^4 \left|x_0^3 u_0^1\right| \\
\dot{x}_0^4 &= a_3 x_0^3 x_0^4 + a_4 x_0^3 x_0^4 \left|x_0^4\right| + a_5 x_0^2 x_0^4 + a_9 x_0^2 + b_2 x_0^3 u_0^1 \\
\dot{x}_0^5 &= a_{10} x_0^5 + b_3 u_0^2 \\
\dot{x}_0^6 &= a_{11} x_0^6 + b_4 u_0^3 \\
\dot{x}_j^1 &= -x_0^3 + x_j^2 x_0^2 + x_j^3 \cos x_j^3 \\
\dot{x}_j^2 &= -x_0^2 x_j^1 + x_j^3 \sin x_j^3 \\
\dot{x}_j^3 &= -x_0^2 + b_{4+j} x_j^3 u_j^1 \\
\dot{x}_j^4 &= a_{11+j} x_j^4 \left|x_j^4\right| + b_{5+j} u_j^2
\end{aligned} \tag{2}$$

The state variables are represented by the following values:

$x_0^1 = \psi$ - course of the own ship,

$x_0^2 = \dot{\psi}$ - angular turning speed of the own ship,

$x_0^3 = V$ - speed of the own ship,

$x_0^4 = \beta$ - drift angle of the own ship,

$x_0^5 = n$ - rotational speed of the screw propeller of the own ship,

$x_0^6 = H$ - pitch of the adjustable propeller of the own ship,

$x_j^1 = D_j$ - distance to the *j-th* object,

$x_j^2 = N_j$ - bearing of the *j-th* object,

$x_j^3 = \psi_j$ - course of the *j-th* object,

$x_j^4 = V_j$ - speed of the *j-th* object.

where $\vartheta_0 = 6, \vartheta_j = 4$.

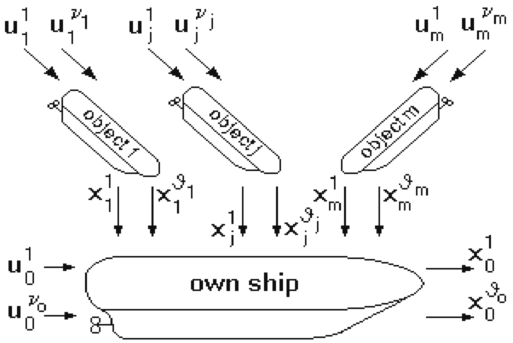

Figure 1: Block diagram of a differential game model including *j* ships.

While the control values are represented by:

$u_0^1 = \alpha_r$ - reference rudder angle of the own ship,

$u_0^2 = n_r$ - reference rotational speed of the own ship's screw propeller,

$u_0^3 = H_r$ - reference pitch of the adjustable propeller of the own ship,

$u_j^1 = \psi_j$ - course of the *j-th* object,

$u_j^2 = V_j$ - speed of the *j-th* object.

where $\nu_0 = 3, \nu_j = 2$.

1.2 Control and state constraints

The constraints of the control and the state of the process are connected with the basic condition for the safe passing of the objects at a safe distance D_s in compliance with COLREG Rules, generally in the following form:

$$g_j(x_j^{\vartheta_j}, u_j^{\nu_j}) \le 0 \tag{3}$$

The constraints referred to as *the ships domains* in marine navigation, may assume a shape of a circle, ellipse, hexagon, or parabola and may be generated – for example – by an artificial neural network, see fig. 2 [1].

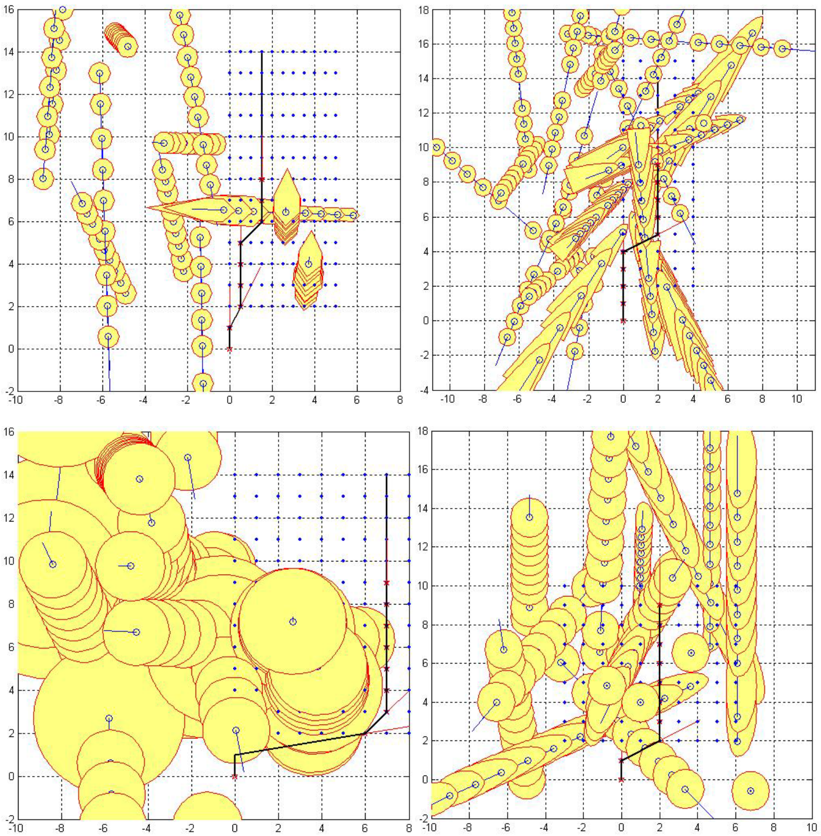

Figure 2: Forms of the neural ships domains.

1.3 Quality index control

The synthesis of the decision making pattern of the object control leads to the determination of the optimal strategies of the players who determine the most favourable, under given conditions, conduct of the process. For the class of non-coalition games, often used in the control techniques, the most beneficial conduct of the own control object as a player with *j-th* object is the minimization of her goal function in the form of the payments – the integral payment and the final one:

$$I_0^j = \int_{t_0}^{t_k} [x_0^{\vartheta_0}(t)]^2 dt + r_j(t_k) + d(t_k) \quad \rightarrow \; min \tag{4}$$

The integral payment represents loss of way by the ship while passing the encountered objects and the final payment determines the final risk of collision $r_j(t_k)$ relative to the *j-th* object and the final deflection of the ship $d(t_k)$ from the reference trajectory [3,5,7].

2 Control algorithms

Each approximated model of the process may be assigned a respective method of safe control of a ship. The multi-stage positional game POSTRAJ and multi-step matrix game RISKTRAJ algorithms of safe ship control will be presented [4,6].

2.1 POSTRAJ algorithm

The safe optimal control of the own ship $u_0^*(t)$, equivalent for the current position $p(t)$ to the optimal positional steering $u_0^*(p)$, is determined by:

- sets of acceptable strategies $U_j^0[p(t_k)]$ for the encountered *j-th* object relative to the own ship,

- a pair of vectors u_j^m and u_0^j,

- the optimal positional strategy for the own ship $u_0^*(p)$ from the condition:

$$I^* = \min_{u_0 \in U_0 = \bigcap_{j=1}^{m} U_0^j} \left\{ \min_{u_j^m \in U_j} \; \min_{u_0^j \in U_0^j(u_j)} S_0\left[x_0(t_k), L_k\right] \right\} = S_0^* \tag{5}$$

where U_0 – sets control of the own ship, U_j – sets control of a met *j-th* object.

Function

$$S_0\left[x_0(t), L_k\right] = \int_{t_0}^{t_{L_k}} u_0(t)\,dt \tag{6}$$

is the own ship's control goal function which characterises the ship's distance at the moment t_0 to the closest point of turn L_k on the assumed voyage route, see fig. 3.

The criterion for the selection of the optimal trajectory of the own ship is achieved by determining the ship's course and speed, which would ensure the smallest loss of way for a safe passing of the encountered objects, at a distance which is not smaller than the assumed value D_s, always with respect to the ship's dynamics in the form of the advance time to the manoeuvre t_m, with element $t_m^{\Delta\psi}$ during course manoeuvre $\Delta\psi$ or element $t_m^{\Delta V}$ during speed manoeuvre ΔV.

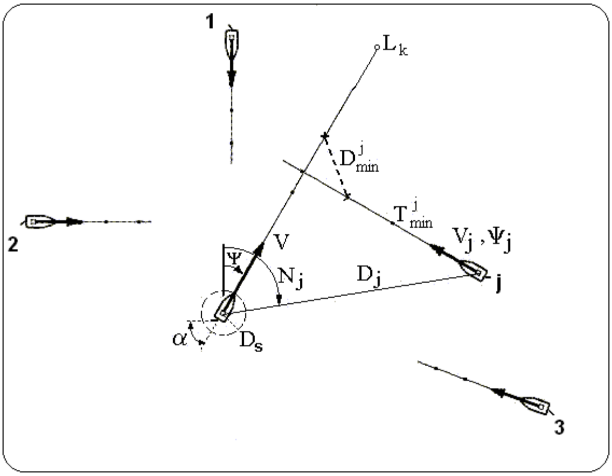

Figure 3: Navigational situation representing the passing of the own ship and *j-th* objects.

2.2 RISKTRAJ algorithm

The safe optimal control of the own ship $u_0^*(t)$ is the solution of a matrix game in which the own ship as player O has a possibility of using s_0 pure various strategies, and a *j-th* met object as player J has s_j various pure strategies. Through the limitation of s_0 and s_j strategies selection the COLREG requirements can be taken into consideration. In a dual problem player O aims to minimize the risk of collision r_j, while player J aims to maximize the risk. The value of the risk of the collision r_j is defined as the reference of the actual approach situation at sea described by parameters D^j_{min} and T^j_{min}, to the assumed assessment of the situation defined as safe determined by safe distance D_s and safe time T_s of the approach:

$$r_j = \frac{1}{\sqrt{w_1\left(\frac{D^j_{min}}{D_s}\right)^2 + w_2\left(\frac{T^j_{min}}{T_s}\right)^2 + w_3\left(\frac{D_j}{D_s}\right)^2}} \tag{7}$$

where the weight coefficients w_1, w_2 and w_3 are dependent on the state of the visibility at sea, dynamic length, the dynamic beam of the ship and the kind of water region.

The components of the mixed strategy express the distribution of the probability of the players using their pure strategies. As a result, for the goal of control quality index in the form:

$$\left(I_0^{(j)}\right)^* = \underset{s_0}{min} \ \underset{s_j}{max} \ r_j \tag{8}$$

probability matrix $\boldsymbol{P}[p_j(s_j, s_0)]$ of applying each one of the particular pure strategies is obtained. Applying the dual linear programming method, the solution of the safe optimal control problem is the strategy representing the highest probability:

$$u_0^*(t) = u_o^{(\nu_0)}\left\{ \left[p_j\left(s_j, s_0\right)\right]_{max} \right\} \tag{9}$$

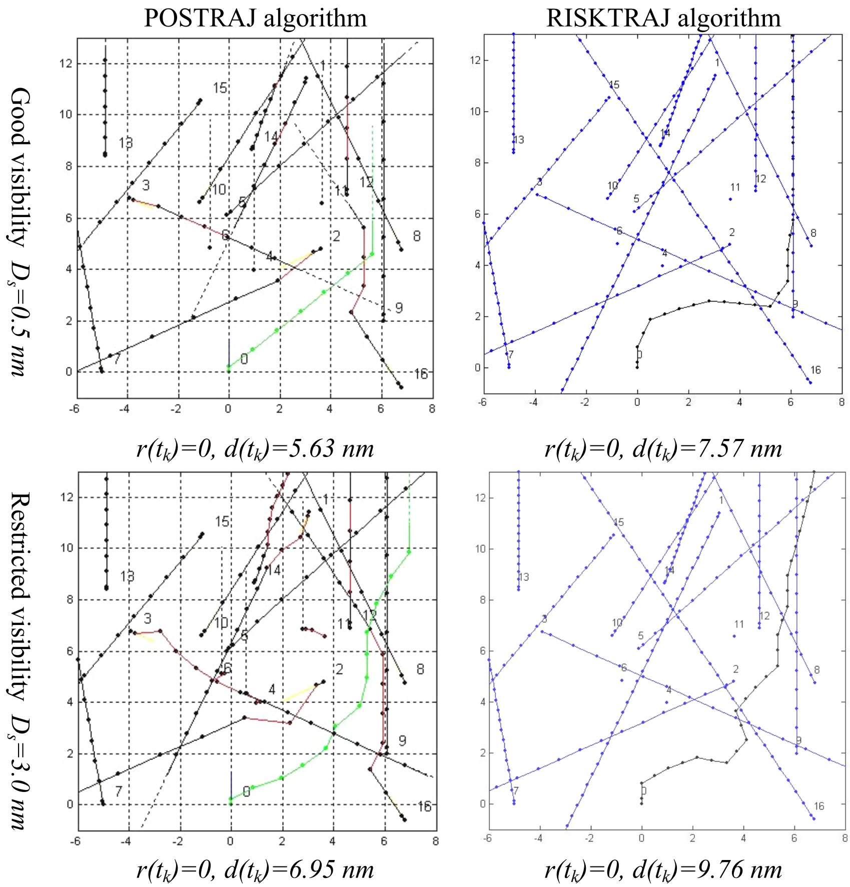

Figure 4: Positional and risk safe optimal game trajectories in situation on the Baltic Sea for *j=16* encountered ships.

3 Computer simulation

Computer simulation of positional trajectory POSTRAJ and risk trajectory RISKTRAJ control algorithms was carried out on in examples of real navigational situations of passing different numbers of encountered objects at sea. See figures 4–6.

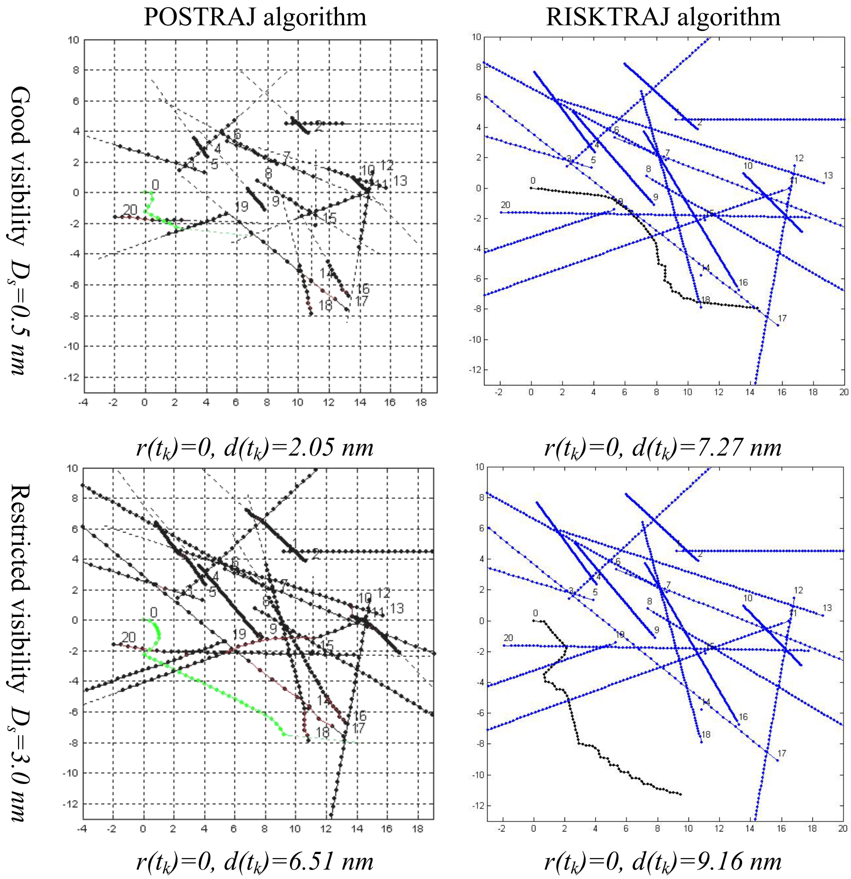

Figure 5: Positional and risk safe optimal game trajectories in situation on the North Sea for $j=20$ encountered ships.

4 Conclusion

POSTRAJ and RISKTRAJ control algorithms represent formal models of mental navigators leading the ship and making manoeuvring decisions. These algorithms can be used for the computer support of navigator safe manoeuvring decisions in collision situations using information from the ARPA anti-collision radar system.

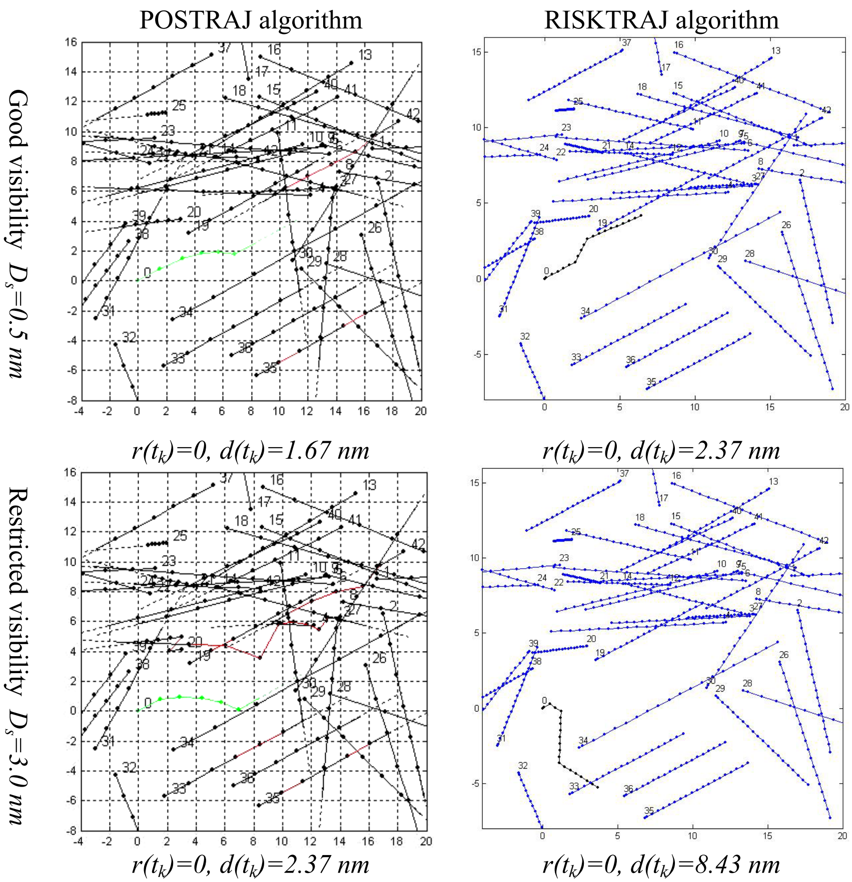

Figure 6: Positional and risk safe optimal game trajectories in situation in the English Channel for *j=42* encountered ships.

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