# AREAL REDUCTION FACTORS FOR DESIGN RAINFALL ESTIMATION IN THE MODDER-RIET RIVER BASIN, SOUTH AFRICA

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ABSTRACT

Design point rainfall estimates assume a uniform distribution of rainfall over a catchment and hence are only representative of a limited area. For larger areas, Areal Reduction Factors (ARFs) are used to convert design point rainfall depths or intensities to an average areal design rainfall depth or intensity for a catchment-specific critical storm duration and catchment area. This paper presents the development of an enhanced methodology to express the spatial and temporal rainfall variability at a quaternary catchment (QC) level by means of geographically-centred and probabilistically correct ARFs. The ARF values presented in this paper are based on observed daily rainfall data as extracted from 223 rainfall stations situated in the Modder-Riet River Basin (MRRB). The methodology adopted is based on a modified version of Bell's geographically-centred approach. Individual sets of ARF values were derived for each of the 23 QCs present in the MRRB by considering various storm durations and corresponding recurrence intervals. The differences in the regional sample ARF values highlight the presence of dominant weather types in each region and also confirm that ARFs are influenced by different rainfall-producing mechanisms, while not being constant for various storm durations and exceedance probabilities or recurrence intervals. It is recommended that the findings from this study and the use of geographically-centred probabilistically correct ARFs be expanded to other regions, both nationally and/or internationally to ultimately facilitate both improved design rainfall and flood estimation.

*Keywords: Areal Reduction Factors, design point rainfall, areal rainfall, geographical-centred.*

## 1 INTRODUCTION

Design point rainfall estimates are only representative of a limited area and for larger areas the areal average design rainfall depth or intensity is likely to be less than the maximum design point rainfall depths or intensities [1]. Areal Reduction Factors (ARFs) are used to describe the relationship between point and areal rainfall by converting design point rainfall depths or intensities to an average areal design rainfall depth or intensity for a catchment-specific critical storm duration and catchment area [2].

Flood-producing rainfall events are mostly characterised by a non-uniform spatial and temporal rainfall distribution. Typically, rainfall storms could have one or more maximum rainfall cores, while displaying for any given period a sensibly smooth non-linear reduction in average areal values with an increasing distance from the maximum rainfall core [2].

## 2 ARF ESTIMATION METHODS

In practice, engineers and/or hydrologists are in most cases concerned with design rainfall, i.e. rainfall information derived from observed rainfall data and which comprises of a depth and duration associated with a given recurrence interval ($T$) or annual exceedance probability (AEP) [3].

ARF estimation methods as applicable to design rainfall, are normally grouped into the following two broad categories.

 
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### 2.1 Analytical methods

Analytical methods are derived mathematical algorithms used to characterise the spatial and temporal rainfall variability by incorporating simplified assumptions that are not entirely true descriptions of the actual rainfall process [1], [4]. The fact that the actual rainfall processes are partially ignored is a cause for concern and this is further compounded by the often limited amount of actual rainfall data used during the verification of these methods. In response to these inherent shortcomings, several new analytical methods to estimate ARFs have been proposed during the last three decades such as storm movement [5], crossing properties [6], spatial correlation structure [7] and scaling relationships [8].

### 2.2 Empirical methods

Empirical methods can either be based on a geographically-centred or storm-centred approach. The geographically-centred approach describes the relationship between areal average design rainfall over a geographically fixed area (e.g. catchment area) and a corresponding design point rainfall value representative of the area under consideration. In other words, ARFs are used for percentage reduction, which relates to the statistics of point and areal design rainfall and considers the uniform temporal and spatial distribution of rainfall over a catchment area.

In the storm-centred approach, the estimation of areal design rainfall is not limited to a fixed geographical area, but rather associated with the extent of individual storm rainfall events and the way in which the rainfall intensity decreases with distance from the central maximum rainfall core [2], [4].

The empirical and/or analytical estimation of ARFs on a large scale is basically limited to the United Kingdom [9], United States of America [10], [11] and Australia [1]. Omolayo [12] also highlighted that, apart from these studies, not much research has been conducted in other parts of the world and ascribed this to insufficient rainfall-monitoring networks and a lack of short duration (sub-daily) rainfall data. Furthermore, the inconsistency present in the ARF results obtained from using the above data-intensive empirical and/or analytical methods internationally, is also a major concern and could be ascribed to the variation in predominant weather types, storm durations, seasonal factors and recurrence interval [13]–[15].

According to Svensson and Jones [4], the level of agreement between the empirical and analytical methods currently in use is limited to a specific scaling regime, i.e. short storm durations and small catchment areas. Thus, these methods are inappropriate for use with a comprehensive set of temporal and spatial scales such as at a quaternary catchment (QC) level. On the other hand, a number of these empirical (storm-centred) and analytical (correlation-based and annual maxima-centred) methods do not provide probabilistically correct areal design rainfall estimates since it is assumed that the AEP of both the point and areal design rainfall is similar [4].

In South Africa, the estimation of ARFs is limited to the storm-centred approaches of Van Wyk [16] and Wiederhold [17], while Alexander [18] also developed a geographically-centred approach based on the UK FSR methodology and observed daily rainfall data limited to the 1980s [9]. There has also been a concern in some sections of the hydrological community in South Africa that the UK FSR results may not be appropriate for South African conditions [19]. Moreover, some studies (e.g. [1], [12]) have conclusively shown that ARFs are dependent on the average AEP of rainfall.

Based on the shortcomings highlighted above, it is clearly evident that the development of ARFs appropriate to South Africa is a high-priority research area in design flood estimation. Hence, the primary objective of this study is to derive geographically-centred and probabilistically correct ARFs representative of the different rainfall-producing mechanisms at a QC level in the Modder-Riet River Basin (MRRB), South Africa as pilot study area. The focus is on the development of probabilistically correct ARFs, in other words, to establish the relationships between $T$-year areal design rainfall estimates and weighted average $T$-year design point rainfall estimates.

## 3 STUDY AREA

South Africa is divided into 22 primary drainage regions, which are further delineated into 148 secondary drainage regions [20]. The MRRB, as shown in Fig. 1, is situated in the C5 secondary drainage region within the primary drainage Region C and covers 34 795 km$^2$ [21]. The MRRB consists of two tertiary drainage regions, the Riet River (C51) and Modder River (C52) catchments, which are further sub-divided into 23 QCs.

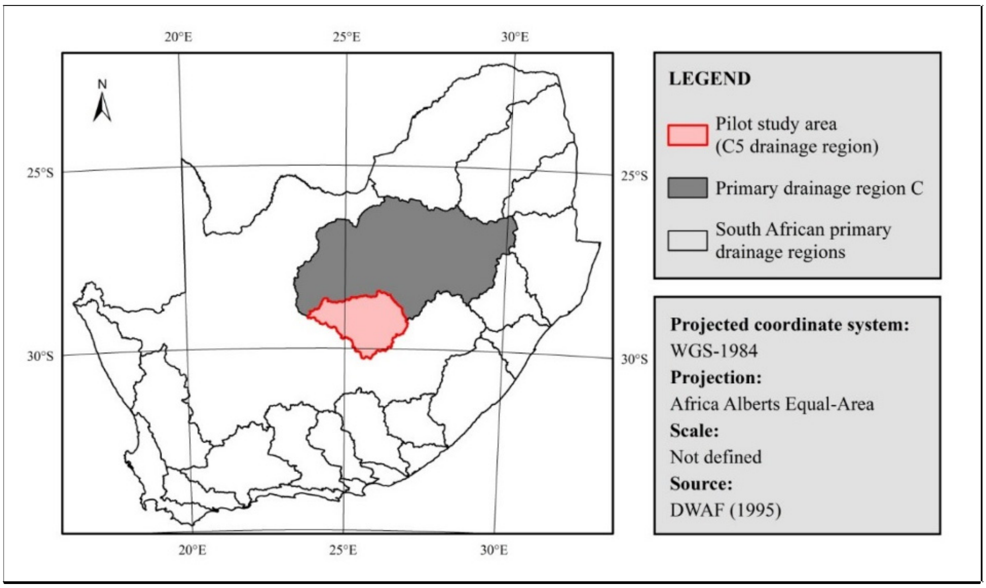


Figure 1: Location of the pilot study area [22].

The MRRB is predominantly characterised by convective rainfall associated with high rainfall intensities and thunder activity during the summer months. The Mean Annual Precipitation (MAP) is 424 mm, ranging from 275 mm in the west to 685 mm in the east [23]. The 185 South African Weather Services (SAWS) daily rainfall stations located within the boundaries of the MRRB are shown in Fig. 2. The rainfall monitoring network is in general denser in the mid-eastern parts of the study area as opposed to the north-western parts.

The overall distribution and location of the individual rainfall stations are regarded as sufficient for the purpose of this study. However, when point rainfall depths are converted to rainfall depths over an area using averaging techniques, e.g. Thiessen polygon method [24], denser rainfall monitoring networks are preferred. Consequently, the 38 neighbouring rainfall stations of the MRRB were therefore also considered in this study.

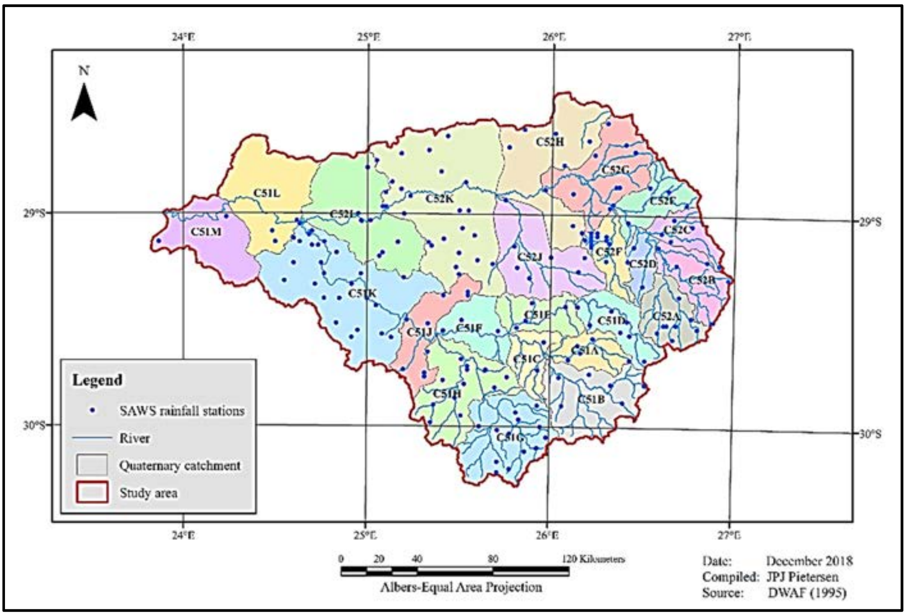


Figure 2: Location of the daily SAWS rainfall stations in the pilot study area.

## 4 DERIVATION OF GEOGRAPHICAL-CENTRED ARFs

This section presents the methodology adopted and estimated geographically-centred ARF results representative of the different rainfall-producing mechanisms at a QC level in the MRRB, South Africa.

### 4.1 Analysis of rainfall data

A daily rainfall database was established by evaluating, preparing and extracting daily rainfall data from the SAWS rainfall stations present in the MRRB as well the data from neighbouring rainfall stations. The Daily Rainfall Extraction Utility (DREU) [23] was used for the extraction and infilling of all the daily rainfall data series. Each rainfall station identified with the DREU was evaluated in terms of record length (≥ 30 years), data quality and geographical location in relation to the specific QC under consideration. The daily point rainfall annual maximum series (AMS) and areal AMS were established and extracted from the observed rainfall data for the purpose of probabilistic analyses. Infilling of daily rainfall values were necessary in some cases to obtain a minimum record length of 30 years. The observed rainfall data represents 15,791 years in total, as opposed to the 6,053 infilled years; hence, 72.3% of the total record lengths used are based on observed data.

### 4.2 Conversion and scaling factors

The application of conversion and/or scaling factors were considered to convert/scale the fixed 1-day rainfall to a continuous 24-hour rainfall series. Each daily observed point rainfall data series recorded at a fixed 1-day interval was converted/scaled to a continuous 24-hour rainfall series by making use of the Adamson [25] conversion and Smithers and Schulze [26] scaling factors, respectively.

The conversion factors applied to the point and areal AMS remained constant for the various durations (*D*) under consideration, e.g. 0.6 (*D* = 1-hour), 0.90 (*D* = 8-hour), 0.96 (*D* = 16-hour), 1.11 (*D* = 24-hour), 1.05 (*D* = 72-hour) and 1.02 (*D* = 168-hour). The estimated average scaling factors for each duration typically ranged from 0.56 (*D* = 1-hour), 0.92 (*D* = 8-hour), 1.05 (*D* = 16-hour), 1.19 (*D* = 24-hour), 1.38 (*D* = 72-hour) to 1.71 (*D* = 168-hour). A comparative example between the conversion and scaling factors as applicable to QC C51M is shown in Fig. 3.

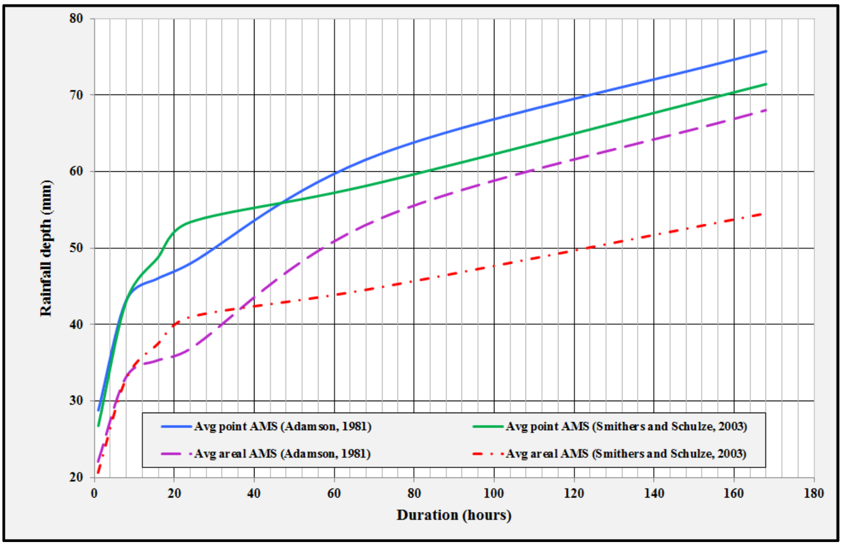


Figure 3: Comparative example of conversion and scaling factors in QC C51M.

The data presented in Fig. 3 are based on a mutual data period of 70 years applicable to seven rainfall stations located in QC C51M. From Fig. 3, it is evident that the scaling factors [27] tend to increase at a constant rate for durations > 24-hour.

### 4.3 Averaging of observed rainfall

Various methods proposed for the averaging of point rainfall depths over an area were considered in this study. The results obtained by [3] confirm the even spatial distribution of the rainfall stations and the relatively flat topography of the MRRB. Based on these findings, the large amount of data and computations necessary, and the preferential use of the Thiessen polygon method in various international ARF studies, e.g. Bell [28], Stewart [29] and Siriwardena and Weinmann [1], the Thiessen polygon method was selected as the most suitable method to use. The generated Thiessen polygons in the MRRB are shown in Fig. 4.

### 4.4 Probabilistic Analyses of Weighted AMS

The quintile estimates of point and areal rainfall are more reliable when a longer period of record is used. The probabilistic analyses of point and areal rainfall AMS were conducted separately to result in separate point and areal design rainfall frequency curves. Probabilistic analyses of the point rainfall AMS were conducted at a QC level to result in one

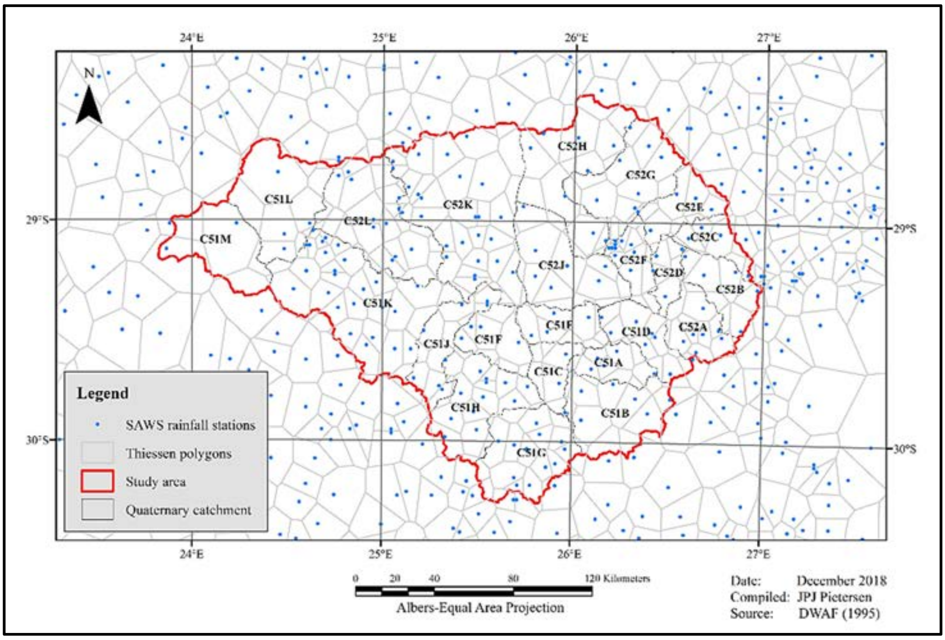


Figure 4: Layout of the Thiessen polygons in the MRRB.

representative frequency curve condensing information from all the point rainfall data series within a particular QC. Similarly, probabilistic analyses of the areal rainfall AMS (extracted and weighted at a daily time interval within a particular QC), were also conducted at a QC level to result in one representative areal frequency curve which condenses information from all the areal design rainfall data series within a particular QC.

The selection of the most suitable theoretical probability distribution was based on the statistical properties (mean, standard deviation, skewness and coefficient of variation) of each point and/or areal rainfall AMS. Typically, the Log-Normal (LN), Log-Pearson Type 3 (LP3), General Extreme Value (GEV) and Generalised Logistic (GLO) probability distributions were considered for the frequency analyses and probabilistic curve fitting. However, Smithers and Schulze [26] highlighted that the GEV probability distribution is regarded as the most suitable distribution to estimate 1-day design rainfall values in South Africa.

A typical example of the probabilistic plot results based on the ranked point AMS values in QC C51M for various durations is illustrated in Fig. 5.

### 4.5 Estimation of ARFs

The estimation of ARFs is based on a modified version of Bell’s method [28], since the AMS of point and areal rainfall were used as opposed to the partial duration series (PDS). One set of sample ARFs was estimated for each QC with durations of 1, 8, 16, 24, 72 and 168 hours with corresponding *T*-intervals of 2, 5, 10, 20, 50, 100 and 200 years. Fig. 6 is illustrative of the typical ratio between point and areal design rainfall estimates used when individual sample ARFs need to be estimated. The variation of sample ARFs with the corresponding *T*-intervals is also shown.

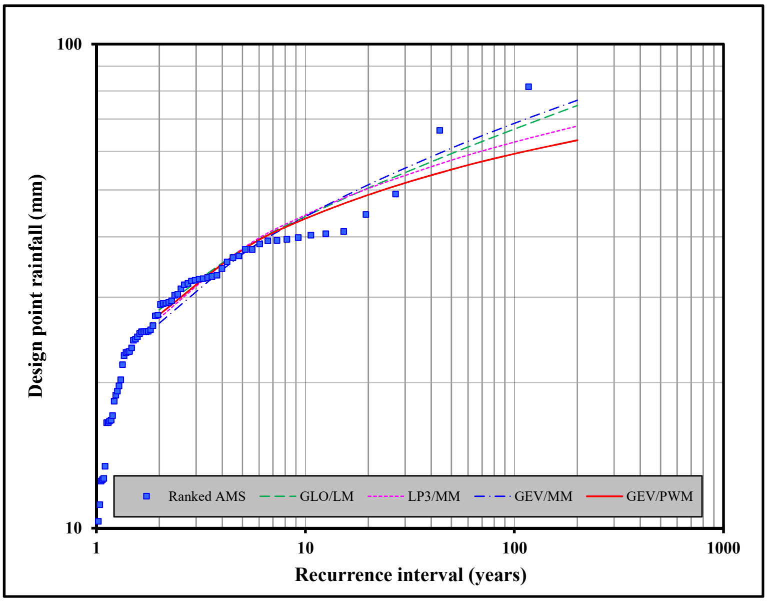


Figure 5: Example of probabilistic curve fitting to estimate design point rainfall in QC C51M.

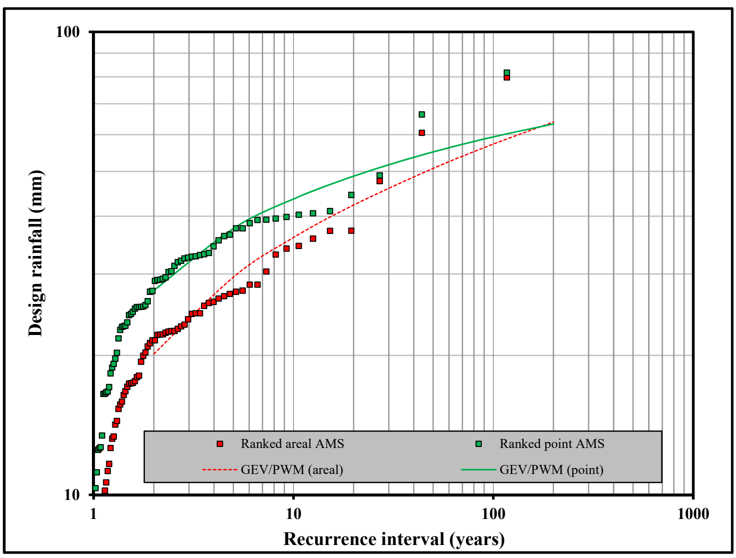


Figure 6: Example of 1-hour point and areal design rainfall in QC C51M.

It is evident from Fig. 6 that individual sample values of the geographically-centred ARFs can be expressed as the ratio between the areal catchment design rainfall and single station design point rainfall estimates for corresponding *T*-intervals using eqn (1)

$$ARF_{sample} = \frac{Areal\ design\ rainfall}{Design\ point\ rainfall}. \quad (1)$$

The ARFs estimated using eqn (1) in each QC in the MRRB were pooled together to estimate average sample ARFs for a combination of different storm durations and *T*-interval values. The latter results are listed in Table 1.

Table 1: Geographically-centred sample ARF value ranges in the MRRB.

| | ***T* (years)** | **Storm duration (hours) < 24** | **72** | **168** |
|---|---|---|---|---|
| **Riet River Catchment (C51)** | **2** | 0.61–0.73 | 0.79–0.85 | 0.83–0.89 |
| | **5** | 0.67–0.79 | 0.84–0.90 | 0.87–0.92 |
| | **10** | 0.68–0.83 | 0.85–0.93 | 0.89–0.94 |
| | **20** | 0.68–0.87 | 0.86–0.95 | 0.91–0.97 |
| | **50** | 0.68–0.93 | 0.86–0.97 | 0.92–0.99 |
| | **100** | 0.68–1.00 | 0.86–1.00 | 0.93–1.00 |
| | **200** | 0.68–1.00 | 0.86–1.00 | 0.93–1.00 |
| **Modder River Catchment (C52)** | **2** | 0.58–0.79 | 0.78–0.89 | 0.84–0.93 |
| | **5** | 0.62–0.80 | 0.83–0.92 | 0.88–0.95 |
| | **10** | 0.65–0.82 | 0.86–0.94 | 0.91–0.97 |
| | **20** | 0.68–0.84 | 0.87–0.97 | 0.93–0.98 |
| | **50** | 0.69–0.89 | 0.88–0.98 | 0.94–0.99 |
| | **100** | 0.69–0.93 | 0.89–1.0 | 0.95–1.00 |
| | **200** | 0.69–0.97 | 0.89–1.0 | 0.96–1.00 |

In terms of recurrence interval, it was evident that the sample ARF values as listed in Table 1, are not constant and tend to increase with an increase in recurrence interval. The average sample ARF values ranged from 0.58 ($T$ = 2-year; $D \leq$ 24-hour) to 1.0 ($T \geq$ 100-year; applicable to most of the durations). In some cases, the average sample ARF values deviated from the expected norm, i.e. an increase in catchment area with decreasing ARF values. Typically, larger ARF values are evident in some of the larger QCs as opposed to some of the smaller QCs.

In most of the QCs under consideration, the ratio between point and areal design rainfall for various durations, e.g. the sample ARFs, equalled unity (≈ 1) for $T \geq$ 100-year. Apart from the possible presence of uniform rainfall events for the larger *T*-intervals, the areal average design rainfall depth or intensity is likely to be less than the maximum design point rainfall depths or intensities. This is confirmed by the results and also highlighted that design point rainfall estimates are only representative for a limited area.

## 5 CONCLUSIONS AND RECOMMENDATIONS

In this study, an enhanced methodology to express the spatial and temporal rainfall variability at a QC level by means of probabilistically correct ARFs was developed. The geographically-centred ARFs are representative of the different rainfall-producing mechanisms at a QC level in the MRRB. In addition to this, the ARF values are regarded as being probabilistically correct seeing that the relationships between *T*-year areal rainfall estimates and weighted average *T*-year point rainfall estimates for various catchment areas, storm durations, recurrence intervals and MAP values at a QC level were established.

The major findings of the study are as follows:

a) Design point rainfall estimates are only representative of a limited area and for larger areas the areal average design rainfall depth or intensity is likely to be less than the maximum design point rainfall depths or intensities.
b) ARFs vary according to predominant weather types, storm durations, climatological factors and recurrence intervals.
c) The use of a geographically-centred approach based on a modified version of Bell's method has proved to be appropriate for the study undertaken bounded within a "fixed" catchment area, namely, at a QC level.

In view of the results obtained from this study, the methodology should be expanded to other catchments in South Africa and/or internationally by incorporating the: (i) derivation of empirical ARF equations using multiple regression analysis, (ii) regionalisation of ARF equations, (iii) estimation of ARF index values to enable the transfer of hydrological information from gauged to ungauged sites, and (iv) development of a software interface to enable practitioners to apply and use the regionalised ARF equations.

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