Bank erosion and planimetric evolution of alluvial meandering streams

A. M. Ferreira da Silva & M. Ebrahimi
Department of Civil Engineering, Queen's University, Canada

Abstract

In this paper, an attempt is made to explain why, in the absence of geological constraints, some rivers remain regular, in which case they tend to closely follow sine-generated curves, while others become irregular. The considerations rest on the present understanding of physical mechanisms determining both meandering bed deformation and bank erosion, as well as laboratory experiments carried out at Queen's University. The experimental observations suggest that the degree of erodibility of the bank plays an essential role in determining the "planimetric fate" of the stream.
Keywords: meandering, bed deformation, bank erosion, plan shape, planimetric evolution.

1 Introduction

Alluvial meandering streams are very dynamic systems, which strive to achieve an equilibrium, or stable, state. When the flow and the stream are, either naturally or as a result of human intervention, in a nonequilibrium state, morphological changes (adjustments of equilibrium) occur. These include bed morphological changes as well as plan shape changes, the bed and bank adjustments progressing with the passage of time as the stream aims to reach the equilibrium (or developed) state. The plan shape changes determining the stream planimetric evolution are known to involve both downstream migration and expansion of meander loops (Friedkin [1], Yalin and da Silva [2]).

One of the most intriguing questions related to the planimetric evolution of meandering rivers is the following: why do some rivers remain regular in plan shape as their planimetric evolution takes place, in which case their plan shape tends to closely follow the sine-generated curve proposed by Leopold and

WIT Transactions on Ecology and The Environment, Vol 172,
www.witpress.com, ISSN 1743-3541 (on-line)
doi:10.2495/RBM130231

Langbein [3], while others acquire irregular plan shapes? Here we are only interested in the case where the irregularity is not imposed by geological formations in the terrain, but rather the case where the irregularity occurs as an intrinsic characteristic of the meandering phenomenon. A well-known form acquired by irregular meandering rivers is the Kinoshita curve (Kinoshita [4], Abad and Garcia [5]). The sine-generated curve is given by $\theta = \theta_0 \cos(2\pi l_c / L)$ (see the definition sketch in fig. 1, as well as the List of Symbols). The significance of this question is highlighted in the recent works by Seminara [6], Frascati and Lanzoni [7] and Pittaluga and Seminara [8], dealing with aspects of the planimetric evolution of meandering streams.

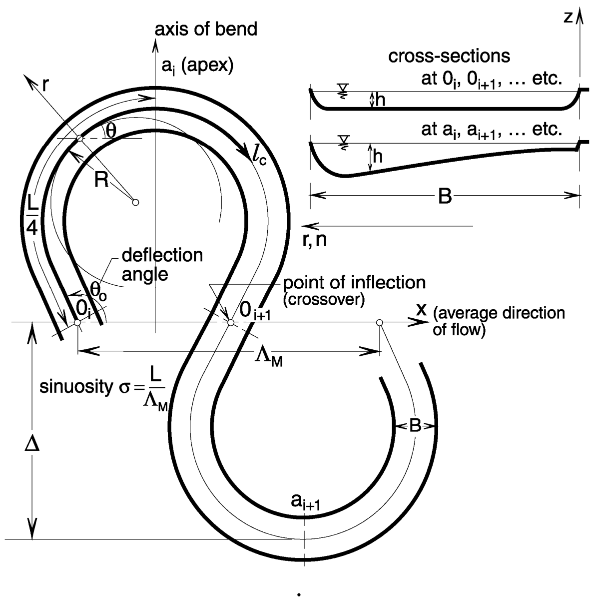

Figure 1: Definition sketch.

In this paper, this question is considered in view of the present understanding of the physical mechanisms of bed and bank deformation, as well as recent laboratory experiments carried out at Queen's University. The findings in this paper suggest that the degree of erodibility of the banks plays a central role in determining the planimetric evolution of a river and, ultimately, whether a river remains regular or becomes irregular.

In accordance with the majority of current publications on the topic, the present study rests on the following assumptions: 1. The stream is imbedded in a cohesionless alluvium which is homogeneous and isotropic; and 2. The width-to depth-ratio of the stream, B/h, is "large" (larger than ≈ 10, say, as in most natural streams).

An important implication of assumption 2 is that the role of cross-circulation (Γ) becomes negligible in determining both bed and bank deformation (see the relation (5.34) in Yalin and da Silva [2], and related discussion). The irrelevance of Γ with regard to the formation of wide natural streams has been independently pointed out in the past by many eminent field-research engineers (such as Leliavsky [9], Matthes [10, 11], Kondratiev *et al.* [12], Velikanov [13], Makaveyev [14]). (More on the topic in Chapter 5 of Yalin and da Silva [2]).

2 Convective flow, sediment transport, and bed development

2.1 Theoretical considerations

i) Consider the flow in a wide meandering channel at the beginning of experiment (at the time $t = 0$): the channel bed is *flat* (it is the graded surface of a mobile bed); the stream is assumed to follow a sine-generated curve. The vertically-averaged streamlines s of this (initial) flow are not parallel: in some parts of the flow-plan they converge, in some others they diverge from each other (fig. 2). Let s_* be the streamline separating two (inner and outer) zones, each conveying half the flow rate ($Q/2$). It can be demonstrated, on the basis of longitudinal periodicity of meandering flow (see Yalin and da Silva [2]), that s_* is, at the same time, the line which separates the laterally adjacent zones of converging and diverging flows. For example, in fig. 2, s_* separates the converging (convectively accelerating) flow-zone on the side of bank 2, from the diverging (convectively decelerating) flow-zone on the side of bank 1.

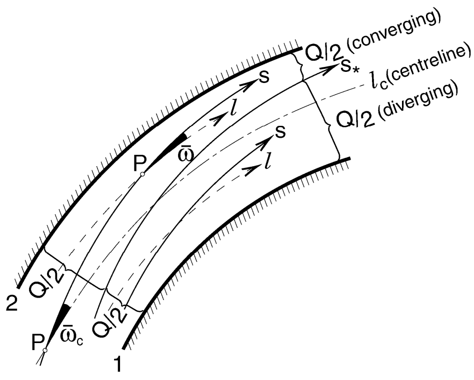

Figure 2: Convergence-divergence zones of meandering flows.

From earlier measurements and computation of flows in sine-generated streams (e.g. Whiting and Dietrich [15], da Silva [16], Termini [17], Shimizu [18], Smith and McLean [19]), it is known that the pattern formed by the streamlines in plan view varies periodically along l_c: the flow characteristics in the cross-sections separated by the meander length L (see fig. 1) are identical, those in the cross-sections separated by $L/2$, are antisymmetrically identical.

ii) Since the local sediment transport rate q_s is an increasing function of the (local) vertically-averaged flow velocity $\bar{U}$, the convective variation of $\bar{U}$ in a flow zone must inevitably cause the corresponding convective variation of q_s in that zone; i.e. it must cause the scalar $\nabla\vec{q}_s$ to acquire a non-zero value. But $\nabla\vec{q}_s \neq 0$ must, in turn, induce the displacement of the bed surface in vertical direction (z_b), as required by the sediment transport continuity equation

$$W = (1-p)\frac{\partial z_b}{\partial t} = -\nabla\vec{q}_s \tag{1}$$

where W is the vertical displacement velocity of the bed surface, p is bed porosity and z_b is bed surface elevation measured with regard to an arbitrary datum. This equation indicates that if the flow is accelerating, then $\nabla\vec{q}_s > 0$, and $W < 0$ (i.e. erosion occurs at the bed); and that if the flow is decelerating, then $\nabla\vec{q}_s < 0$, and $W > 0$ (i.e. deposition occurs at the bed). Clearly, the elevation z_b of the bed surface can remain unchanged ($W = 0$) only in the locations where the flow is parallel, and thus $\nabla\vec{q}_s = 0$.

It can also be shown, as done by Yalin and da Silva [2], that the maximum displacement $(z_b)_T$ of the bed surface at a location of flow plan is an increasing function of the intensity of convective behaviour of flow at that location. Here $(z_b)_T$ denotes bed surface elevation at time $t = T$, where T is bed development time.

iii) The deformed bed of a meandering stream consists of a longitudinal sequence of laterally adjacent "deeps" and "hills" (fig. 3). Each (deep) + (hill) complex can be viewed as one erosion-deposition zone (in short [ED]). From the preceding point it should be clear that each [ED] is brought into being by a (corresponding) convergence-divergence zone (in short, by [CD]) of the initial flow. Hence the length of each [ED] must be the same as that of each [CD], viz $L/2$. The channel cross-sections separating the successive [CD]'s (and [ED]'s) are identifiable by the fact that the streamlines s at those sections must be parallel to each other, i.e. we must have at those sections, $\bar{\varpi} = 0$, where $\bar{\varpi}$ is vertically-averaged streamline deviation angle (see fig. 2). Moreover, the deeps and hills forming an [ED] of infinitesimal amplitude, just after $t = 0$, must be expected to be separated from each other by the same streamline s_* which separates the convergence-divergence zones of the initial flow (fig. 3a). The meandering streams under study are periodic along the streamwise direction x, and therefore the consecutive [ED]s are antisymmetrically identical.

The process of bed deformation is solely due to the transfer of bed material from the area of deeps to the area of hills (as determined by the convective flows over these areas). These locally transferred transport rates, which occur only during $0 < t < T$, are superimposed on the (always present) "transit" transport rate. In wide natural meandering streams, the material eroded from a deep (and/or banks) is deposited predominantly on the *next* hill (bank) on the same side of the flow: "only a small portion of eroded material crosses the channel" (Leliavsky [9], Matthes [10, 11], Kondratiev *et al.* [12], Velikanov [13], Makaveyev [14]).

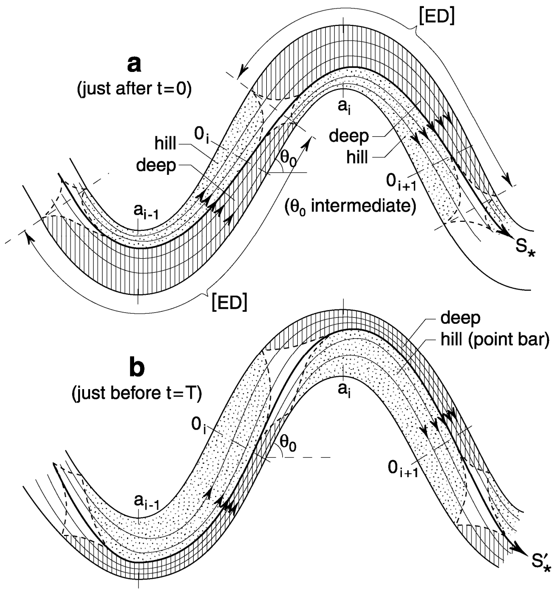

Figure 3: Schematic representation of the streamline s_*, [CD]'s and [ED]'s in a channel having intermediate θ_0: a) just after the beginning of experiment; b) near equilibrium.

It should be noted that, as shown by da Silva *et al.* [20], for the same remaining conditions, the plan location of the converging-diverging flow zones, and consequently, that of the erosion-deposition zones varies depending on the sinuosity of the meandering stream, i.e. on the value of the deflection angle θ_0. It is, however, beyond the scope of this paper to enter this topic. For the sake of simplicity, all considerations that follow are illustrated by resorting to streams having an "intermediate" value of sinuosity.

2.2 Experimental observations

The processes discussed above were confirmed through a series of bed deformation experiments carried out in a 70° sine-generated channel (Binns and da Silva [21]). A schematic of the channel and its hydraulic circuit is shown in fig. 4. Fig. 5 shows the result of one of these experiments. In this test, the bed was flat at the beginning of the experiment. The hydraulic conditions were as follows: flow rate Q = 12.2 l/s, flow depth h = 4.34 cm and bed slope S = 1/150. The bed was allowed to naturally deform until the equilibrium state was reached at about $t \approx 75$ min. The bed deformation was monitored throughout the experiment. Figs. 5a),b) show the deformed bed at t = 10 min and t = 75 min, respectively. Complete details of these experiments are given by Binns and da Silva [21, 22]. Note the similarities between the schematical representation of the bed morphology in fig. 3 and observations in fig. 5.

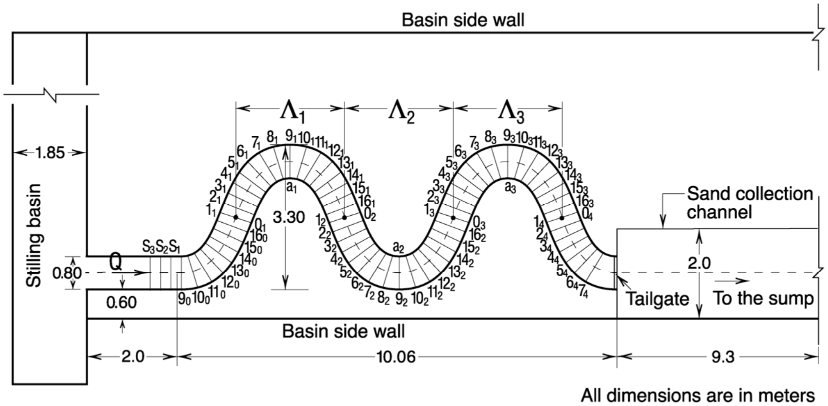

Figure 4: Schematic of experimental facility.

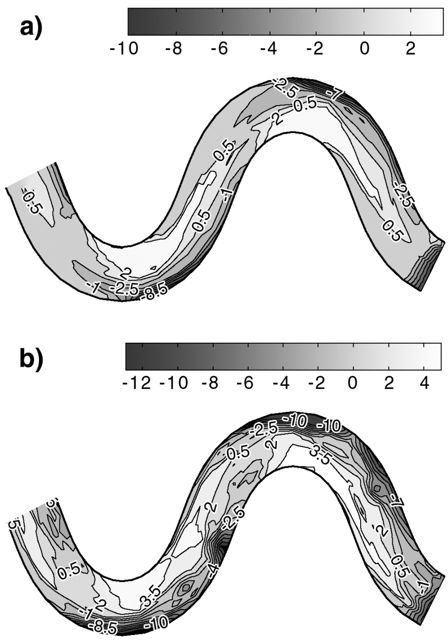

Figure 5: Bed-contours plots from a bed deformation test in a 70° sine-generated channel (Q = 12.2 l/s, h = 4.34 cm and S = 1/150). Contours based on Δz_b values; units are in cm; flow from left to right: a) at t = 10 min; b) at t = 75 min.

3 Action of flow on the banks

3.1 Theoretical considerations

Because of the bank slope, the material eroded at any given point on the bank tends to slide down the bank, thus being removed from it. The phenomenon is no longer governed by the sediment transport continuity equation (unless the bank slope is very small). Instead, erosion is directly related to the flow velocity (or bank shear stress), with maximum erosion tending to occur toward the location of maximum velocity (or shear stress). This concept is behind the well-known equations of bank erosion by, e.g., Arulanandan *et al.* [23], Ikeda *et al.* [24], and Parker [25]. Mosselman [26] modified these equations by adding a term that would take into account the bank slope, yielding a model that combines the effect of both, velocity and flow acceleration, on the rate of erosion at any given location. The equation by Mosselman [26] implies that for small bank slopes, bank erosion may occur somewhere in between the location of maximum erosion at the bed and maximum velocity in the proximity of the bank.

The difference in plan location (plan shift) of the "points" of maximum erosion at the bed and at the bank are illustrated in the next section with the aid of a special experiment carried out in the same meandering channel as the experiment reported in Section 2.2. In Section 3.3, it is discussed how this plan shift may lead to deviations from regular meander plan shapes.

3.2 Experimental observations

The present experiment was carried out in the 70° sine-generated channel previously mentioned. A sand channel (D_{50} = 0.65 mm) with cross-section as shown in fig. 6 was installed inside the channel. The hydraulic conditions were as follows: flow rate Q = 7.44 l/s, flow depth h = 4.50 cm, bed slope S = 1/500. The experiment was conducted in two stages. In the first stage, the movable boundary was fixed (with the cement-based method introduced by Ebrahimi and da Silva [27]), to enable detailed measurements of the initial flow field. In the second stage, only the bed was fixed, and the temporal bank deformation was monitored.

The initial flow field is shown in fig. 7. Here Φ_u is flow velocity, normalized by the channel averaged velocity. Fig. 8 shows the gradient field of dimensionless velocities. This provides a measure of local flow acceleration/

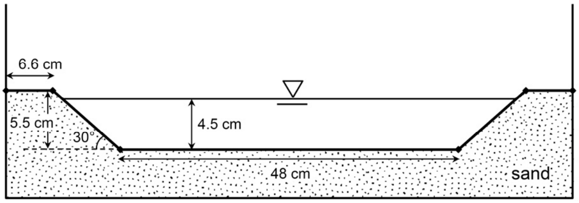

Figure 6: Schematic of cross section inside the meandering channel.

deceleration. The temporal bank deformation can be inferred from fig. 9, showing the measured temporal variations of the bank crest width.

Observe from fig. 8 that the maximum acceleration and deceleration of flow occurs immediately downstream of the apexes. This is in agreement with the bed pattern in fig. 5, where maximum erosion and deposition occur downstream of the apexes. However, in the experiment with mobile banks, the maximum bank erosion occurred mainly at the crossovers, i.e. much closer to the location where maximum flow velocity occurs (see fig. 7).

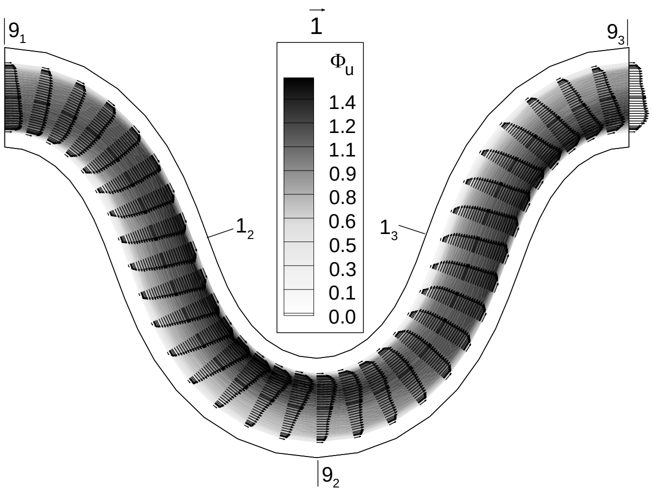

Figure 7: Dimensionless measured velocity field.

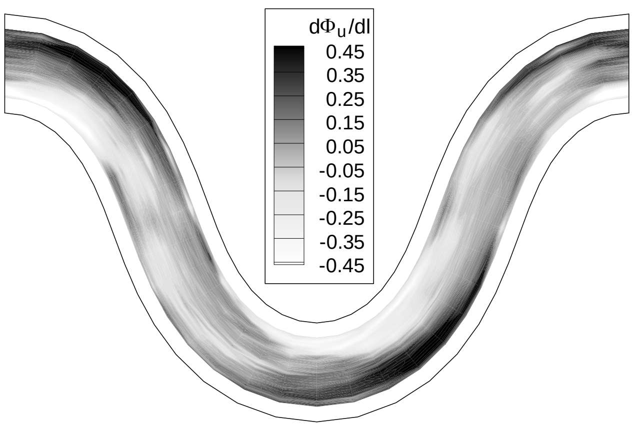

Figure 8: Calculated gradient field (m^{-1}) of dimensionless velocities.

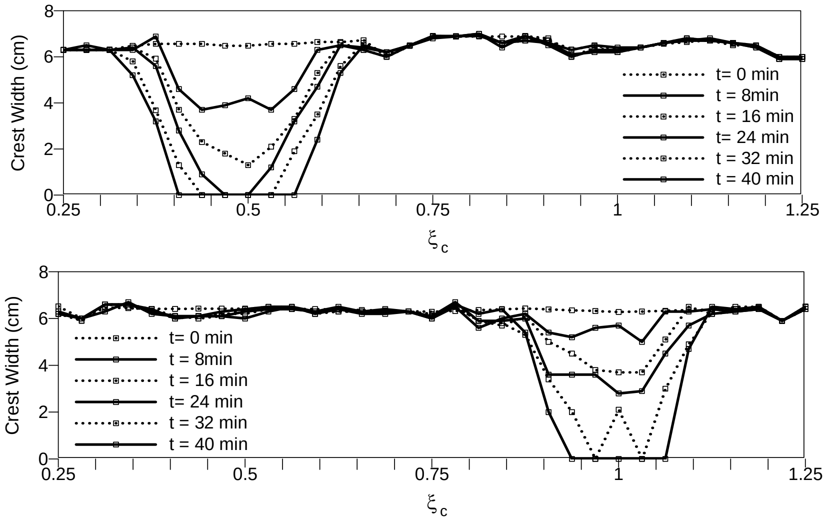

Figure 9: Bank crest width variations of left (top) and right (bottom) banks (looking downstream) with the passage of time.

3.3 Meander planimetric adjustments

The observations above suggest the following. If the banks are rather unerodible, then only bed erosion occurs. This will lead to bank to scour at the same location where bed erosion occurs, which eventually may lead to bank failure at the same location.

However, if the banks are quite erodible, then an episodic event of sudden bank failure (at the location of maximum bed scour) will temporarily eliminate the scour at the bed. This will change the flow pattern, which will then become more consistent with the flow over a flat initial bed, and thus able to attack the banks substantially further downstream. This will lead to deviations from a regular plan shape, and likely the more so the more erodible will the bank be. The present experiments suggest that such process can indeed determine the planimetric fate of the stream.

4 Conclusions

Bed deformation is associated with the convective acceleration/deceleration of flow (in wide streams), while bank erosion is primarily related to flow velocity. Because of this, for non-deformed or only weakly deformed beds, there is a shift between the plan location of maximum bed deformation, and maximum bank deformation (bank erosion). This shift appears as an efficient mechanism leading to deviations from regular plan shapes. The present analysis suggests that the degree to which such deviations will happen depends on the degree of erodibility of the banks.

Acknowledgements

This research was supported by funds from the Natural Sciences and Engineering Research Council of Canada through a Discovery Grant to the first author. The authors thank Ms Regina Huber, MSc student at the Technical University of Munich, for the help she provided to the execution of the tests mentioned in Section 3.2, during her 2012 stay at Queen's University as a Visiting Research Student.

List of symbols

a_i	apex section;
B	flow width;
D_{50}	representative grain size;
h	flow depth;
L	meander length (measured along l_c);
l	longitudinal coordinate; $l = 0$ at crossover O_i;
l_c	longitudinal coordinate along the centerline of a meandering flow ($l_c = 0$ at O_i);
n	radial coordinate of meandering flow; $n = 0$ at flow centerline;
O_i	crossover section;
p	porosity of granular material;
Q	flow rate;
$\vec{q}_s$	specific volumetric bed-load rate vector; $\nabla\vec{q}_s$ = divergence of $\vec{q}_s$;
R	local radius of curvature;
r	radial coordinate of meandering flow; $r = 0$ at center of channel curvature;
S	bed slope;
s	streamline;
s_*	streamline that divides flow rate Q in two equal (left and right) parts;
T	duration of bed development of deformed bed of meandering stream;
t	time;
$\overline{U}$	magnitude of local vertically averaged velocity vector;
W	displacement of the bed surface in vertical direction;
x	streamwise direction;
z_b	bed elevation measured with regard to an arbitrary reference datum;
Δz_b	difference between z_b at any time and the flat bed initial elevation;
$(z_b)_T$	bed elevation at time $t = T$;
Γ	cross-circulation;
θ, θ_0	deflection angle for meandering flow at any l_c and at $l_c = 0$, respectively;
Λ_M	meander wavelength;
ξ_c	dimensionless counterpart of l_c ($\xi_c = l_c/L$);
σ	sinuosity of a meander flow ($\sigma = L/\Lambda_M$);
Φ_u	dimensionless counterpart of flow velocity;
$\overline{\omega}$	local depth-averaged deviation angle.

References

[1] Friedkin, J.F., A Laboratory Study of the Meandering of Alluvial Rivers, Report, US Waterways Experiment Station, Vicksburg, 1945.
[2] Yalin, M.S. and da Silva, A.M.F., *Fluvial Processes*, IAHR Monograph: IAHR, Delft, 2001.
[3] Leopold, L.B. and Langbein, W.B., River meanders, *Scientific American*, **214(6)**, pp. 60–70, 1966.
[4] Kinoshita, R. Investigation of Channel Deformation in Ishikari River, Report of the Bureau of Resources, Tokyo, 1961.
[5] Abad, J.D. and Garcia, M.H., Experiments in a high-amplitude Kinoshita meandering channel: 1. Implications of bend orientation on mean and turbulent flow structure, *Water Resources Research*, **45(2)**, 2009.
[6] Seminara, G., Meanders, *Journal of Fluid Mechanics*, **554**, pp. 271–297, 2006.
[7] Frascati, A. and Lanzoni, S., Morphodynamic regime and long–term evolution of meandering rivers, *Journal of Geophysical Research*, **114**, 2006.
[8] Pittaluga, M.B. and Seminara, G., Nonlinearity and unsteadiness in river meandering: a review of progress in theory and modelling, *Earth Surface Processes and Landforms*, **36(1)**, 20–38, 2011.
[9] Leliavsky, S., *An introduction to fluvial hydraulics*, Constable and Company, London, 1959.
[10] Matthes, G.H. Basic aspects of stream meanders, *Transactions of American Geophysical Union*, **22(3)**, 632–636, 1941.
[11] Matthes, G.H, Mississippi River cutoffs, *Transactions of the American Society of Civil* Engineers, **113(1)**, 1–15, 1948.
[12] Kondratiev, N., Popov, I. and Snishchenko, B, Foundations of hydromorphological theory of fluvial processes (in Russian), *Gidrometeoizdat*, Leningrad, 1982.
[13] Velikanov, M.A., Dynamics of Alluvial Streams. Vol. II. Sediment and Bed Flow, State Publishing House for Theoretical and Technical Literature, Moscow. 1955.
[14] Makaveyvev, N.I., River Bed and Erosion in Its Basin, Press of the Academy of Sciences of the USSR, Moscow, 1975.
[15] Whiting, P.J. and W.E. Dietrich, Experimental studies of bed topography and flow patterns in large–amplitude meanders: 2. Mechanisms, *Water Resources Research*, **29(11)**, 3615–3622, 1993.
[16] da Silva, A.M.F. Turbulent flow in sine-generated meandering channels, *Ph.D. thesis*, Queen's University, Kingston, 1995.
[17] Termini, D., Evolution of a meandering channel with an initial flat bed: Theoretical and experimental study of the channel bed and the initial kinematic characteristics of flow (in Italian), *Ph.D. thesis*, University of Palermo, Palermo, 1996.

[18] Shimizu, Y., A study on prediction of flows and bed deformation in alluvial streams (in Japanese), Civil Engineering Research Institute, Hokkaido Development Bureau, Sapporo, 1991.
[19] Smith, J.D and McLean, S.R., A model for flow in meandering streams, *Water Resources Research*, **20(9)**, pp. 1301–1315, 1984.
[20] da Silva, A.M.F., El–Tahawy, T. and Tape, W.D., Variation of flow pattern with sinuosity in sine–generated meandering streams, *Journal of Hydraulic Engineering*, **132(10)**, pp. 1003–1014, 2006.
[21] Binns, A. and da Silva, A.M.F., On the quantification of the bed development time of alluvial meandering streams, *Journal of Hydraulic Engineering*, **135(5)**, pp. 350–360, 2009.
[22] Binns, A. and da Silva, A.M.F., Rate of growth and other features of the temporal development of pool–bar complexes in meandering streams, *Journal of Hydraulic Engineering*, **137(12)**, pp. 1565–1575, 2011.
[23] Arulanandan, K., Gillogley, E. and Tully, R., Development of a quantitative method to predict critical shear stress and rate of erosion of natural undisturbed cohesive soils, Report GL–80–5, US Army Corps of Engineers, Waterways Experiment Station, Vicksburg, 1980.
[24] Ikeda, S., Parker, G. and Sawai, K., Bend theory of river meanders. Part 1. Linear development, *Journal of Fluid Mechanics*, **112**, pp. 363–377, 1981.
[25] Parker, G., Theory of meandering bend deformation, River Meandering, Proceedings of the Conference River '83, pp. 722–732, ASCE, Reston, 1983.
[26] Mosselman, E., A review of mathematical models of river planform changes, *Earth Surface Processes and Landforms*, **20**, pp. 661–670, 1995.
[27] Ebrahimi M. and da Silva, A.M.F., A cement–based method to fix sand in laboratory channels, *Journal of Hydraulic Research* (in press), 2013.

www.witpress.com, ISSN 1743-3541 (on-line)