Energy and momentum coefficients for wide compound channels

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Abstract

Experiments were conducted in a straight smooth compound trapezoidal main channel flanked by two symmetrical floodplains having width ratio value ≈12. The point velocities were measured throughout the compound cross section and isovel patterns were analyzed to determine the values of kinetic energy coefficient (α) and momentum coefficient (β) under varying flow conditions of Froude no. between 0.277 to 0.444 and relative depth of between 0.11 and 0.43. The values obtained for α and β are 2.09 and 1.39 respectively, which are significantly higher than their respective values reported by previous researchers for compound cross sections of different size and shape. New relations are suggested to determine approximate values of α and β with R2 values more than 0.97. It is hoped that the present findings will provide new guidelines for design and research in trapezoidal compound channels
Keywords: compound channel, velocity distribution. kinetic energy coefficient, momentum coefficient.

1 Introduction

Accurate knowledge of velocity distribution in channels is very important for flood studies, afflux studies for bridge and culvert design [1] and estimation of stage discharge curve in natural channels. Often for simplicity in river engineering practice the velocity is considered uniform and analysis is carried out considering energy or momentum approach. But any deviation from such theoretical uniformity are usually accounted for by the introduction of two coefficients, namely kinetic energy correction coefficient α (also termed Coriolis coefficient) [2] and momentum coefficient (β) (also termed Boussinesq coefficient) [3]. The former is applied when the energy principle is adopted for

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computation while the latter is introduced in case of momentum approach for computation. Also all the 1-D river engineering packages such as HEC-RAS etc use the values of α and β for dealing with 3-D influences and channel shape for computation of water surface profile and other design and estimation purpose. When the velocity distribution in a channel section in lateral and vertical direction is considered uniform then both α and β are taken as unity. However, in view of wide variations in velocity distributions reported throughout literature which in turn has been known to be influenced by a host of factors e.g. cross sectional shape and complexity, alignment, depth of flow, channel slope and roughness, etc. [4, 5], it is really important to determine the numerical values of both coefficients with sufficient accuracy. Ignoring the effect of such variations often leads to considerable error in predicting flow behavior, stage discharge prediction, afflux studies and other related analysis in the case of both natural and artificial channels. Also, there is a tendency to include some predefined values as reported in literature for kinetic energy correction coefficient (α) and momentum correction coefficient (β) by field engineers. Although in channels of simple geometry such an approach works well but in channels of compound cross sections where the main channel carrying deeper and faster flow is flanked by one or two shallow berms or flood plains carrying shallow and slower flow, detailed analysis to determine exact numerical values is warranted to minimize design errors.

2 Theory and background

Due to scarce data reported in literature about the velocity distributions in compound channels it becomes imperative that more research be undertaken in this area to augment the existing scant database in this field. Generally summing up the kinetic energy flux passing through the elementary sections in a channel section and equating it to the energy flux passing through the entire section, α is computed as

$$\alpha = \frac{\int v^3 \Delta A}{V^3 A} = \frac{\sum v^3 \Delta A}{V^3 A} \tag{1}$$

And similarly considering momentum flux for passing fluid in the channel section β can be computed as

$$\beta = \frac{\int v^2 \Delta A}{V^2 A} = \frac{\sum v^2 \Delta A}{V^2 A} \tag{2}$$

where v is the point velocity measured in an elemental area ΔA of the whole cross sectional area A through which flow takes place and V is the cross sectional mean velocity found by dividing $\sum v\,\Delta A$ with A.

In practice, different methods are suggested for the computation of α and β viz. approximate method [6]; graphical method [7]; theoretical method [8]. For details the interested reader is referred to [4] and [8].

Previously, many studies were undertaken to determine both kinetic energy correction coefficient and momentum energy correction coefficient for channels of simple geometrical shapes and of single section. Useful references are available in [4, 7–9] etc. However, studies relating to channels of compound cross sections are relatively fewer in comparison to simple channels. Significant research contributions are those due to [10–14]. Kolupaila [10], based on limited research at that point of time, recommended average value of α as 1.75 and that of β as 1.25 for over flooded river valleys or channels flanked by floodplains. Due to the possibility of variation in the main channel and floodplain combination regarding their shape, position, symmetry and other factors like roughness, etc., the variations in values of α and β are also reported. Seckin *et al.* [14], based on their experimental results for a symmetrical rectangular compound channel of width ratio {($\boldsymbol{\alpha}$) =3.046} and results of experiments conducted by [11, 12] for an asymmetrical channel of width ratio {($\boldsymbol{\alpha}$) =3.60} reported average values of α and β as 1.156 and 1.056 respectively. In both cases, the main channels were of rectangular section. Similarly, Al-Khatib and Gogus [13] conducted a series of experiments on a rectangular compound channel with width ratio 3.35 and reported values of α in the range of 1.023–1.063 and β in the range of 1.005–1.034 under various flow and geometrical conditions.

Due to the presence of floodplains, excessive momentum transfer usually takes place between the deep main channel and shallow floodplains. This has been well reported in literature [15–19] etc. On account of this momentum transfer, the lateral distribution of velocity becomes non uniform in addition to natural non uniform velocity distribution of flow between the channel bed and free surface in a vertical direction. This leads to different kinetic and momentum correction coefficients for compound channels as compared to single channels. Although FCF channels experiments were conducted for a width ratio up to {($\boldsymbol{\alpha}$) =6.67} but no systematic estimation of α and β are reported in literature to the best knowledge of the authors. Taking into context the relevance of these aforementioned coefficients (α and β) in river engineering and to fill existing gaps in our knowledge base in this emerging field of compound channels, we report here the outcome of experimentations conducted in the Fluid Mechanics and Hydraulics Laboratory of NIT, Rourkela, India the details of which are enumerated in the next section.

3 Experimental details and methods

For carrying out research in compound channels, an experimental setup was built in the Fluid Mechanics and Hydraulics Laboratory of NIT, Rourkela. A compound channel having trapezoidal main channel (bottom width 0.33m, depth 0.065m and side slope 1:1) and wide symmetrical rectangular floodplains having a total width of 3.95m (width ratio, $\boldsymbol{\alpha} \approx 12$, Pl (see Fig. 1)) was built inside a steel tilting flume of around 15m in length. The bed and wall of the channel was made with Perspex sheet (6 to 10 mm thick and having Manning's n value=0.01). Water was supplied to the flume from an underground sump via an overhead tank by a centrifugal pump (15 hp) and recirculated to the sump after flowing through the compound channel and a downstream volumetric tank fitted with closure valves for calibration purposes. Water entered the channel bell mouth section via an upstream rectangular notch. An adjustable vertical gate along with flow straighteners were provided in the upstream section sufficiently ahead of the rectangular notch to reduce turbulence and velocity of approach in the flow near the notch. At the downstream end another adjustable tail gate is provided to control the flow depth and maintain a quasi uniform flow in the channel. A movable bridge was provided across the flume for both spanwise and streamwise movements. Fig. 2(a)–(d) shows photos of some distinct components of the experimental set up.

The dimensions of the compound channel were adopted keeping in view the larger research goals of velocity and boundary shear distribution in wide compound channels (width ratio **α** ≈12 and aspect ratio =5 as seen in natural channels, where the width ratio is the ratio of total width of channel to width of main channel and the aspect ratio is the ratio of width of main channel to depth of main channel). Point velocities were measured along verticals spread across the main channel and flood plain so as to cover the width of the entire half cross section. Also, point velocities were taken at a number of horizontal layers in each vertical. Due to symmetry, measurements were only made on one half of the span from the centre of the main channel to the edge of the floodplain. The lateral spacing of grid points over which measurements were taken was kept 4cm inside the main channel and 8cm on the flood plain. Velocity measurements were taken by pitot static tube (outside diameter 4.77mm) and two piezometers fitted inside a transparent fiber block fixed to a wooden board and hung vertically at the edge of flume the ends of which were open to atmosphere at one end and connected to a total pressure hole and static hole of pitot tube by long transparent PVC tubes at other ends. Steady uniform discharge was maintained in each run of the experiment and altogether six runs were conducted for overbank flow with relative depth **β** varying between 0.109 and 0.434, where the relative depth is the ratio of depth of flow over floodplain to the total flow depth. The discharge varied between 13.543lit/s and 106.181lit/s and Froude's no. was kept between 0.277 and 0.444, i.e., a subcritical flow condition all throughout the test runs.

Figure 1: Definition sketch of compound cross section for overbank flow. (All dimensions are in cm.)

Details of experimental runs are given in Table 1. After obtaining the point velocities at various grid points representing the whole compound flow cross section velocity contours were drawn for all six flow cases (a sample contour diagram for overbank depth 7.5cm is given in Fig. 3). Using a suitable graphic package, the areas between successive isovels were planimetered digitally and summed up to find the total area of the flow cross section. Any error from the true geometric area was distributed among all slices in a weighted average method. Then computations were performed to obtain values of α and β for individual runs.

Table 1: Experimental details of compound channel.

Sl. No.	Q in lit/sec	Overall depth (H) in cm	Relative depth(β)	α	β
1	13.543	7.3	0.109589041	2.092723961	1.385219155
2	17.482	7.5	0.133333333	1.965256023	1.32089785
3	36.396	8.8	0.261363636	1.261415481	1.082895399
4	53.546	10.1	0.356435644	1.045017162	1.015842773
5	60.282	10.5	0.380952381	1.038031224	1.014550085
6	106.181	11.5	0.434782609	1.036523387	1.012695217

(a) (b) (c) (d)

Figure 2: (a)–(d), clockwise starting top left: (a) U/S rectangular notch entrance bell mouth section; (b) Series of pitot static tube fitted to stand with transparent pipes and point gauge; (c) Set of Piezometers fitted to wooden board to record pressure with spirit level, digital clock etc.; (d) D/S tail gate and volumetric tank.

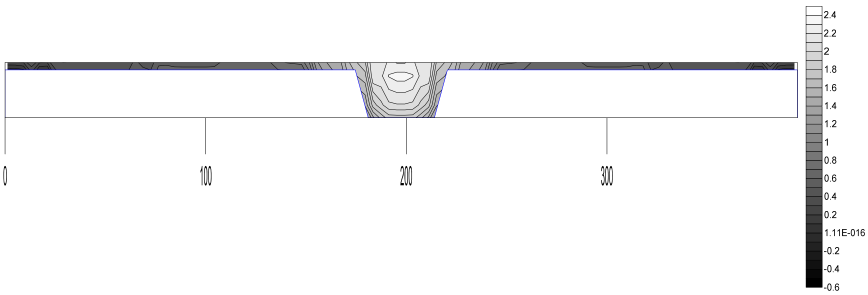

Figure 3: Velocity contour (isovel) diagram for overbank depth 7.5cm drawn with point velocities 'v' normalized by sectional mean velocity V.

4 Results and discussion

The values of α and β tabulated in Table 1 reveal some interesting features only characteristic of a compound channel. It can be clearly seen that both the kinetic energy and momentum coefficients reach high values with low overbank flow or at a small relative depth (**β**). At low overbank depths, due to large widths of flood plain attached to a comparably small trapezoidal main channel, the slow moving flow in floodplains interact with the fast moving main channel flow intensely and considerable momentum exchange takes place giving rise to large non uniformity in lateral velocity distribution. Such intense exchange of momentum between faster and deeper main channel flow and slower and shallower floodplain flow at low relative depth in the case of a compound channel, has been well researched and documented in literature in the past [20–23]. As the discharge increased with a corresponding rise in relative depth value the intensity of the interaction diminishes considerably.

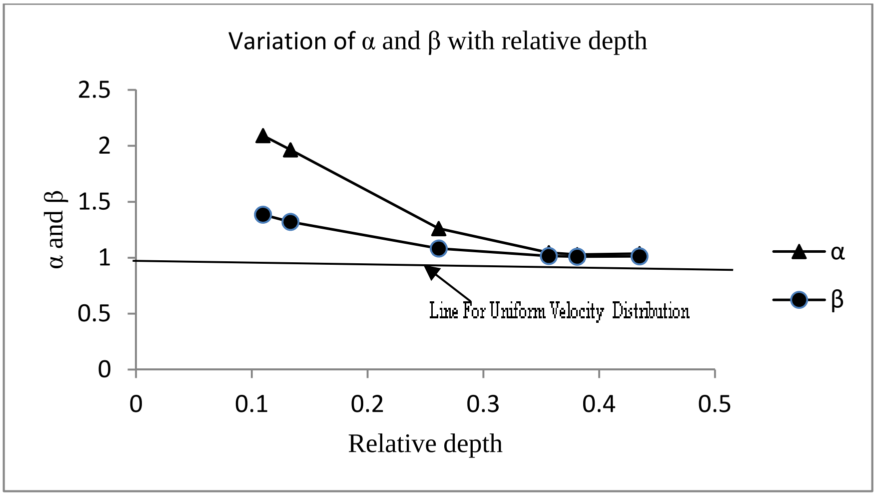

Figure 4: Variation of α and β with relative depth.

However, due to the high width ratio (**α**) value of nearly 12 in our experimental runs even at a relative depth value of 0.434, the flood plain flow interacts with the main channel flow to a far greater extent than in the case of simple channels resulting in a comparably higher value for both α and β.

For a comparison purpose when α and β were computed as explained earlier but without the contributions from floodplains for planimetering and summing the intercepted areas between the successive isovels, their values in each case decreased substantially from their earlier or originally computed values (illustrated in Table 2). In Table 2, the modified α and β are shown with a prime. This adequately corroborates the fact that due to the flood plains the main channel flow is always affected and the velocity distribution in the channel usually becomes distorted resulting in more non uniformity than in the case of simple cross sections.

Another important feature which also emerged from these experimentations is that in comparison to experimental results obtained by other researchers, such as like [11, 13, 15], α and β values in the present case are much higher than the respective values reported by them for rectangular compound sections having smaller width ratio **α** (in the range of 3–4). It can be due to a much larger area of floodplains (**α** ≈12) having a severe interaction on the main channel in the present case. Since, in an ideal case, both α and β are considered unity, so their deviation from the ideal value in the present case was examined (Pl see Fig. 4) and on regression analysis it is found that

$$\alpha - 1 = 2.898(\beta - 1) \tag{3}$$

It is also revealed from Fig. 4 that α is more sensitive than β, to change in velocity values which in turn varies with relative depth (**β**), also evident from eqns. (1) and (2).

Table 2: Details of kinetic energy and momentum coefficients of compound channel by different approaches

Sl. No.	Relative depth (**β**)	α	β	ε	α′	β′	ε′
1	0.109589041	2.09272396	1.38521915	0.83182817	1.05121	1.017278	0.194345
2	0.133333333	1.96525602	1.32089785	0.79294188	1.03356	1.011412	0.149741
3	0.261363636	1.26141548	1.08289539	0.52590871	1.02783	1.009539	0.109917
4	0.356435644	1.04501716	1.01584277	0.17355006	1.02467	1.008512	0.118024
5	0.380952381	1.03803122	1.01455008	0.15659564	1.02223	1.007684	0.069691
6	0.434782609	1.03652338	1.01269521	0.13800543	1.00929	1.003168	0.080903

In addition to this α is related to β by the equation following a straight line relation as given below.

$$\alpha = 2.898\beta - 1.894 \tag{4}$$

In both the above regression analyses coefficient of determination R^2 value was 0.998.

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α and β are also known to be related to a different parameter ε [6] where, $\varepsilon = \frac{v_m}{V} - 1$ where v_m is maximum velocity in the flow cross section and V is cross sectional mean velocity. For an approximate value both α and β are usually found by the following relations [6].

$$\alpha = 1 + 3\varepsilon^2 - 2\varepsilon^3 \tag{5}$$

$$\beta = 1 + \varepsilon^2 \tag{6}$$

These expressions are usually found to apply to cases of wide rough channels of simple sections assuming logarithmic distribution of velocity in the channel. Since, in the present case, only a smooth channel compound section having relatively very large floodplains was used, new expressions are seemed to apply on the basis of regression analysis as given below.

$$\alpha = 1 - 0.4\varepsilon^2 + 2.36\varepsilon^3 \tag{7}$$

$$\beta = 1 + 0.54\varepsilon^2 \tag{8}$$

The R^2 values in the case of eqns. (7) and (8) are found to be 0.99 and 0.97 respectively. However, if the revised value of ε is taken as ε' for the compound section without flood plains and a relationship was derived to calculate the approximate value of α' and β' then the following relations eqns. (9) and (10) in similar form as eqns. (5) and (6) respectively are found.

$$\alpha' = 1 + 2.14\varepsilon'^2 - 5.27\varepsilon'^2 \tag{9}$$

$$\beta' = 1 + 0.33\varepsilon'^2 \tag{10}$$

5 Conclusions

The present study aims at supplementing the scarce data base for design values of kinetic energy coefficient (α) and momentum coefficient (β) that are applicable for a smooth straight trapezoidal main channel flanked by two smooth wide symmetric floodplains on the basis of experiments conducted in the Hydraulics and Fluid Mechanics Laboratory of the National Institute of Technology, Rourkela. Several pertinent points of specific interest to researchers and designers in the field of river hydraulics emerged, as enumerated below.

(a) The velocity distribution is excessively non uniform in a lateral or span wise direction in the case of a compound channel in comparison with regular single channel sections as evident from high values for both the kinetic energy coefficient and momentum coefficient found in the present case (Table 1).

(b) In the case of low overbank depth or small relative depth values the slow moving liquid in wide floodplain area (width ratio≈12) has a much greater impact on faster moving liquid in the main channel, thereby causing the velocity distribution in a span wise direction to be intensely nonuniform. The α and β values are much higher than what has been reported for compound channels of smaller width ratio earlier [11–14].

(c) As the relative depth goes on increasing, the values of α and β gradually reduce (Fig. 3) but still are higher than those reported earlier for lesser wide compound channels at nearly the same relative depth.

(d) The formulas to find approximate values for α and β suggested previously [4] seem to fail in the present case. In their place, new relations (eqns. (7) and (8)) are given with coefficient of determination R^2 value of 0.99 and 0.97 respectively.

In view of the above findings it is suggested to design engineers in the field of river hydraulics to incorporate appropriate values for α and β while analyzing the flow phenomena in energy or momentum approach respectively. Further similar study must be carried out by researchers for rough compound sections with floodplains of various widths to investigate the influence of roughness in determination of both coefficients.

Notations used

The following symbols are used in this paper.

α = Kinetic energy coefficient
β = Momentum coefficient
α’= Modified kinetic energy coefficient
β’= Modified momentum coefficient
α = Width ratio
β = Relative depth
ε = Non-dimensional parameter
ε’= Revised non-dimensional parameter
Q = Discharge
v_m= Maximum cross-sectional velocity
V = Cross-sectional mean velocity.

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