Flow modelling of dual permeability systems: the case of the Vigolana Massif (Trento, Northern Italy)

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Abstract

The paper deals with the water circulation in rock masses, with particular reference to those karstic/fractured systems in which the porous matrix contribution to flow cannot be neglected. The state of the art proposes different numerical solutions to study dual permeability systems, while analytical solutions are pointed out only for dual porosity model. In this study, three different models were developed to study typical dual permeability systems: a numerical code written in Matlab for solving 1-D partial differential equations of flow in dual permeability system, a Single Continuum Model implemented in MODFLOW-2005, a combined model developed in MODFLOW-2005 using the Conduit Flow Process Package. First, the sensitivity of the model responses to different input parameters was evaluated. Then, the different approaches were applied to the case of Vigolana Massif (Trento, Northern Italy), evaluating their ability to reproduce the response of this typical dual permeability system. Modeling results showed that the Single Continuum Model gives a quite good response with a relatively low cost (in terms of required data and computational time), even if it is not properly representative of the examined phenomenon, because it is not able to return and shape the water transfer between matrix and conduit. On the contrary, the Combined Model, solving two flow equations, is perfectly able to reproduce the response of a dual permeability system, but it obviously requires more detailed parameters, often not easy to define. Finally Matlab 1-D code does not reproduce correctly the delay of the matrix response, due to the mono-dimensionality of flow.
Keywords: dual permeability, karstic system, Italy, numerical models.

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1 Introduction

In rock masses, intact rock is considered as a continuum medium with almost no permeability and low porosity, whereas fracture and karst network is a non-continuum medium where discontinuities determine the hydraulic behavior of the whole. Often portions of intact rock, separated by discontinuities, may present a quite high porosity (storage coefficient) with very low hydraulic conductivity (lack of flow); while fractures have high hydraulic conductivity and small storage volumes. Dual porosity models take into account, at the same time, the water storage within intact rock (primary porosity), the water flow along the discontinuity network (secondary porosity) and the flow exchanged between the two media [1]. The delay in the hydraulic response of rock masses is well described by the dual porosity models as a consequence of the high storage coefficient of the rocky matrix. Yet, this phenomenon often depends also on the presence of small but very frequent fractures in the rock matrix, especially nearby master joints. Therefore, rock matrix achieves a not negligible permeability with the possibility of local flow and water exchanges with the master joints. In this case “dual permeability models” can better describe the phenomenon (Fig. 1).

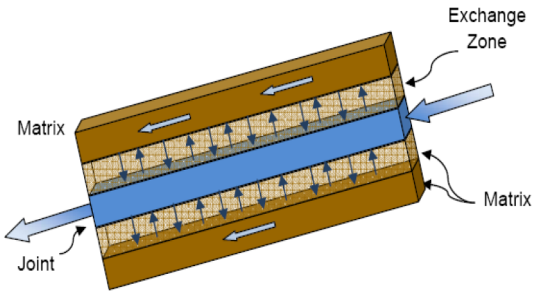

Figure 1: Flow scheme in a dual permeability medium.

The purpose of this note is to study and to reproduce the behavior of rock masses in presence of a permeable matrix, by using dual permeability models. Based on the state of the art, it has been possible to define the analytical system describing a dual permeability rock mass, and therefore to implement a specific code for 1-D numerical resolution of the system. Afterwards, the capability of this tool to reproduce the physical phenomenon was verified by carrying out a sensitivity analysis in very simple case studies. Results were compared with the ones of the numerical commercial code MODFLOW-2005 [2]. This latter was implemented with a recently developed package, the Conduit Flow Process (CFP), which allows the overlapping of two distinct media, one for the matrix and one for the karst conduit network. Finally, a real case study was analyzed: the Vigolana Massif (Trentino Alto Adige, Northen Italy), where a spring is supplied by a typical dual permeability system. Once the conceptual model of the system was defined, the simulation was carried out by using different approaches:

1) a single continuum approach, by using standard MODFLOW-2005, with the definition of different permeability zones in the same continuum;
2) a combined approach, by using the Conduit Flow Process (CFP) package in MODFLOW-2005, which can simulate both laminar and turbulent flow, as well as fluid exchange between the matrix and conduit networks in a dual conductivity model.

Results obtained were compared to identify the capability of the different approaches to reproduce the observed discharges curve.

2 State of the art

The dual permeability medium scheme provides the contemporary presence of two media throughout water flows: fractures and rock matrix interested by a micro-fractured network. Considering the rock matrix hydraulic conductivity negligible respect to the hydraulic conductivity of fractures, the system can be considered a dual porosity medium. The first dual porosity approach applied to a fractured aquifer was introduced into petroleum reservoir engineering [3]. Afterwards, it was developed by other authors [4], which combined the simplification of Darcy's models with the complexity of the discrete models, distinguishing the water flow within intact rock mass (primary porosity) and the water flow in fractured network (secondary porosity). Dual porosity model may be obtained using volume averaging techniques [5–7] or with the homogenization method [8–9]. A great number of models using the dual porosity was proposed to forecast flow and solute transport in a fractured reservoir [10–12], in unsaturated fractured rocks [13, 14], and fissured groundwater systems [3, 15, 11]. As far as the equation system solution is concerned, some authors [16] neglected the conductivity of the matrix, and considered the storage of the matrix and the quasi-steady water transfer between fractures and pores blocks. Then, an analytical solution for 1-D flow, was proposed [17], with porosity weighted continua and assuming a first-order exchange kinetics between two porous media ruled by the exchange term. For dual permeability systems numerical solutions are prevalent. In particular, a numerical 1-D solution for flow and transport model was proposed for simulating preferential movement of water and solutes in structured porous media [18]. It assumed that Richards' equation governs the flow in both matrix and macro-pore and that water transfer between two pore systems depends on the hydraulic conductivity and on the pressure gradient at the interface of the two regions. Some authors tried to apply theories for dual porosity media to dual permeability developing 2-D approach [19, 20], that allowed to consider the spatial variability of the hydraulic properties and improved the simulation of water and solute movement in naturally heterogeneous field soils. Finally, numerical 3-D model has recently been developed in MODFLOW-2005 [2], through the definition of a new package, the Conduit Flow Process, that incorporates the multi-porosity and the multi-permeability components of groundwater flow [21–23].

3 Numerical modelling of dual permeability systems

3.1 1-D modelling with Matlab

It is possible to provide a 1-D numerical solution of the dual permeability model, solving the Gerke and van Genuchten system [18] of two partial differential equations that describes groundwater flow in a dual permeability medium, by introducing some simplifying assumptions [17]. The starting equations system is:

$$\begin{cases} S_m \dfrac{\partial h_m}{\partial t} = \dfrac{\partial}{\partial z}\left(K_m \dfrac{\partial h_m}{\partial z} - K_m \right) + \dfrac{\Gamma_w}{1-w_f} - P_m. \\ S_f \dfrac{\partial h_f}{\partial t} = \dfrac{\partial}{\partial z}\left(K_f \dfrac{\partial h_f}{\partial z} - K_f \right) + \dfrac{\Gamma_w}{w_f} - P_f. \end{cases} \tag{1}$$

where h_m and h_f *(L)* are the hydraulic heads for matrix and fracture, Γ_w (T^{-1}) is the space and time dependent exchange term, P_m and P_f (T^{-1}) are sink terms to account for root water extraction for matrix and fracture, S_m and S_f (L^{-1}) are the specific storage coefficients for matrix and fracture, and w_f is a volumetric weighting factor given by V_f / V_t (where V_f and V_t are the volumes of respectively the fractured pore system and the whole system). In this study matrix and fracture hydraulic conductivities (respectively, K_m and K_f *(L/T)*) are constant. The hydraulic heads depend on the spatial coordinate x and the temporal variable t, $h_m=h_m(x,t)$ and $h_f=h_f(x,t)$, α *(-)* is lumped parameter, evaluated as

$$\alpha = \alpha^* \cdot K_a = \frac{\beta}{a^2} \cdot K_a \cdot \gamma_w. \tag{2}$$

where β *(-)* depends on the geometrical properties of system, and *a (L)* is the distance from the center of a fictitious matrix block to the fracture boundary, γ_w *(-)* is an empirical coefficient equal to 0.01, K_a *(L/T)* is the apparent hydraulic conductivity of transfer term, evaluated as

$$K_a = 0.5 \cdot \lfloor K_a(h_f) + K_a(h_m) \rfloor \tag{3}$$

and $\alpha = (h_f - h_m)$. is the exchange term between matrix and fracture.

From hydraulic head, the exchange flow q_{ex} (L^3/T) between matrix and fracture can be expressed as

$$q_{ex} = -\alpha \left[h_m(t) - h_f(t) \right]. \tag{4}$$

At t = 0, initial head condition is the same for fracture and matrix, whereas boundary condition in x = 0 is an head time dependent function applied to the fracture. A sensitivity analysis on the different parameters was then carried out (Table 1 a, Fig. 2 a, b). For low values of α, matrix head isn't influenced by the hydraulic head in the fracture, and if α increases, hydraulic head in the matrix increases too. The peak is characterized by a time delay, typical of dual permeability systems. If matrix size increases, α tends to infinity ($\beta \rightarrow 0$), hydraulic head of matrix tends to flatten and water transfer decreases because of attenuation of the interaction between two porous media. The α parameter is also strictly dependent on interface hydraulic conductivity K_a. For high values of K_a, that corresponds to high fracture conductivity, model flow rate and exchange flow increase, while reducing K_a value ($K_f \rightarrow K_m$), flow rate decreases and system loses its dual permeability feature.

Table 1: Parameters for: (a) dual permeability Matlab code; (b) scm.

(a)			(b)	
	Matrix	Conduit	K_m (m/s)	1E-5-1E-3
S_s (1/m)	1E-04-1E-2	1E-2-1E-1	K_f (m/s)	1E-2-10
K (m/s)	1E-5-1E-3	5E-1-5	S_m (1/m)	1E-4-5E-2
α (m/ms)	8E-5-3.2E-3	8E-5-3.2E-3	S_f (1/m)	1E-1-2E-1

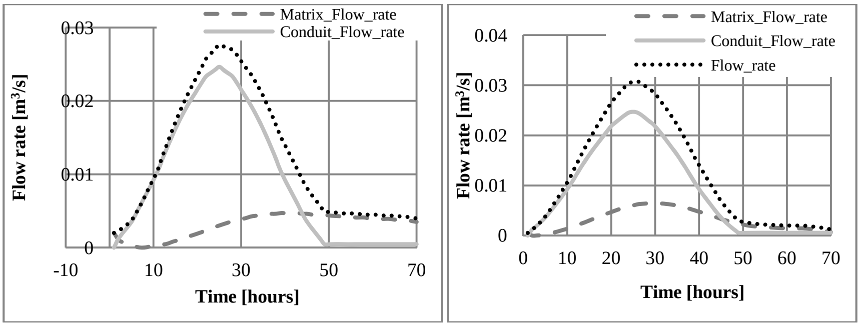

Figure 2: Flow rates with (a) 1-D, α=0.0002 h^{-1}; (b) 1-D, α=0.001h^{-1}.

3.2 1-D modelling with single continuum model (scm)

Single continuum groundwater flow models are often used to simulate groundwater flow in karst aquifers, even if they don't account for turbulent flow, or fluid exchange between the matrix and the conduit networks. Therefore, a simple one dimensional code was reproduced in MODFLOW-2005, which can be compared with the numerical one dimensional model previously described. In the single continuum model, fracture and matrix were reproduced with different hydraulic properties (Table 1 b). Initial and boundary conditions were considered the same of the previously described model. Sensitivity analysis showed that single continuum model is very sensitive to matrix specific storage. For high values of S_m, matrix flow rate is reduced and more delayed, whereas for low values of S_m, matrix flow rate constitutes a more important contribute to total flow rate and matrix response time is more similar to fracture/conduit response time (Fig. 3 a, b).

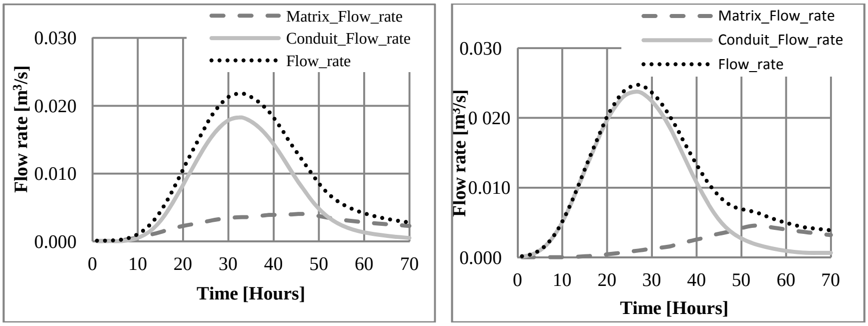

Figure 3: Flow rates with (a) scm, S_m=0.0005 m^{-1}; (b) scm, S_m=0.001 m^{-1}.

3.3 1-D modelling with combined model (cm)

For considering also a transfer function, ruling the exchange flow between matrix and fracture, a dual conductivity model was implemented using MODFLOW-2005 using the Conduit Flow Process Mode 1. The latter requires the definition of more detailed hydraulic characteristics and geometric parameters (Table 2). Fluid exchange is expressed by a linear exchange term and its direction is typically head-dependent. If there are fractures or conduits into the system, in combined models finite element grids are used, which are more robust at handling irregularly. The CFP considers the non Darcian components of flow, coupling permeable matrix (laminar flux) to conduit network (turbulent flux), defining the following new parameters (Table 2):

- conduit geometrical properties (diameter, roughness, tortuosity);
- conduit wall permeability;
- critical Reynolds Number, that defines the transition from laminar to turbulent flow.

Table 2: Conduit properties in the combined model.

	Value		Value
Diameter (m)	2.0-4.0	Bottom elevation (m)	75
Wall permeability (m/s)	5E-2-1E-1	Lower Reynolds No.(-)	20
Tortuosity (-)	1	Upper Reynolds No.	1000
Roughness (m)	1E-1-2E-2	Percent Recharge (%)	100

Boundary and initial conditions are the same previously described for the other approaches. The sensitivity analysis has interested in particular the conduit wall permeability (Fig. 4 (a), (b)). For increasing values of conduit wall permeability, the exchange flow between matrix and conduit becomes higher causing an increase of matrix flow rate.

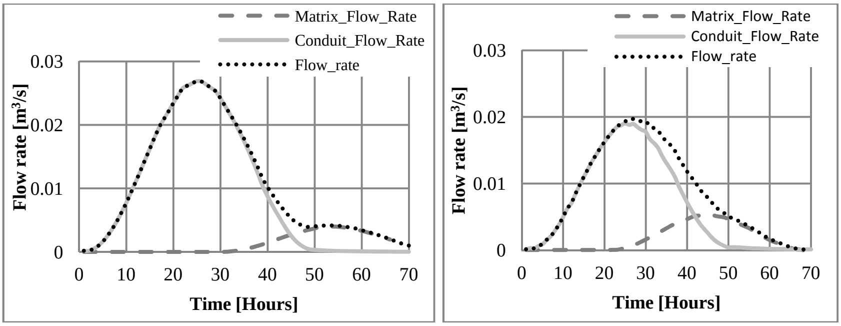

Figure 4: Flow rates with cm: conduit wall (a) K=0.01 m/s; (b) K=0.5 m/s.

3.4 Comparison between different approaches

The comparison between the results from the previously described modeling approaches, shows that the Matlab code is characterized by a slight delay and overestimates flow rates (Fig. 5(a),(b)). The contribute of matrix flow rate is more evident in the combined Modflow model, while it results flattened for single continuum model. In particular combined model implemented in MODFLOW-2005 CFP Mode 1, is able to reproduce an evident second peak that is not distinguishable in the other ones. The discrepancies in the responses may be attributed to the different basic approach of these three models:

- single continuum model considers only one continuum in which different permeability zones are defined, and applies only one flow equation;
- dual permeability model in Matlab solves two distinct flow equation considering two overlapped continua;
- finally, only combined model solves two distinct flow equations: one for a continuum medium, matrix, and the other for a discrete/dis-continuum medium, conduit.

Unfortunately, Matlab code can reproduce only one dimensional flow, which is not properly representative of hypothetical real phenomena.

4 The real case application of Vigolana Massif

The different approaches to dual permeability modeling previously described were finally applied to a real case study, located in the Vigolana Massif. This latter consists of a mountain area, placed near Trento (Northern Italy, Fig. 6(a)).
Vigolana Massif is characterized by the presence of Dolomia Principale (Upper Triassic) and lithologies belonging to the group of Calcari Grigi (Lower Jurassic) [24] (Fig. 6(a)).

Water circulation in the Vigolana Massif is due to a fracturing permeability. So water can flow in the rock masses following preferential paths represented by structural discontinuities (fractures and stratification): permeability value is related to the fracturing degree and to the composition of the rock mass. More in detail, the concerned hydrogeological system is characterized by a sub-vertical fracturing zone. The site is characterized by the presence of several karst springs. One of these inflows is inside an exploratory horizontal tunnel. The in continuum monitoring of its flow rate allowed to reconstruct the flow curve (Fig. 7(a), the blue line), characterized by a main peak in spring, resulting from snowmelt, followed by secondary peaks not directly correlate to precipitation.

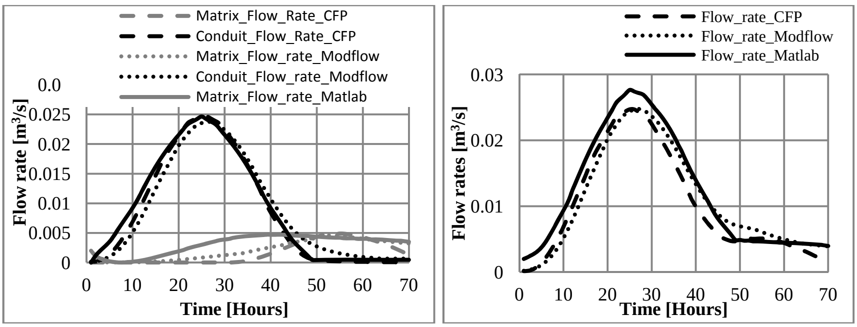

Figure 5: Comparison among the results of the different numerical models (a) conduit and matrix flow rates; (b) total flow rates.

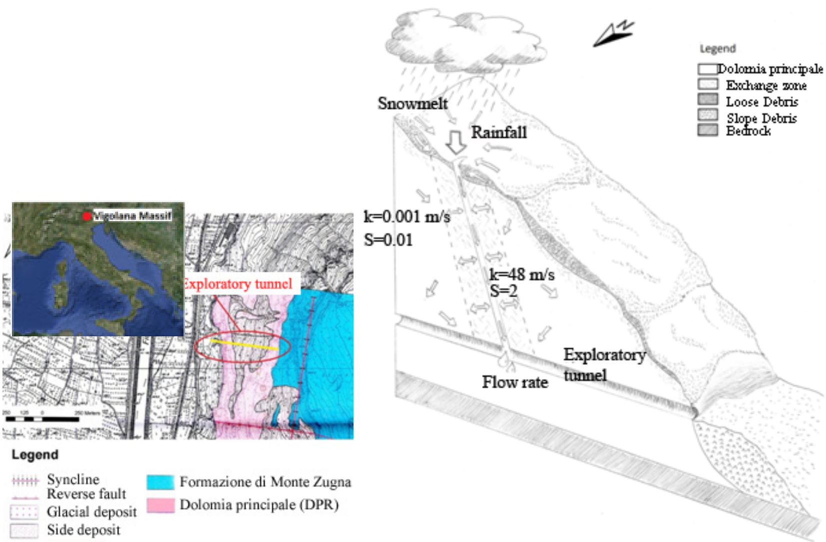

Figure 6: (a) Geological map and (b) hydrogeological conceptual scheme of the Vigolana Massif.

4.1 Hydrogeological conceptual model

A conceptual scheme of the system was reconstructed to point out the main properties and features which have to be considered for a dual permeability system (Fig. 6(b)). The domain includes a single sub-vertical conduit, which is connected at the bottom with an exploratory tunnel, surrounded by a fractured rock matrix (Dolomia Principale), interested by the exchange flow with the fracture. The system inflows are

- a recharge, in terms of rainfall;
- a seasonal flow rate due to snow-melting.

The flow rate in situ measurements was carried out where fracture intersects the exploratory tunnel. The hydraulic conductivities for matrix and conduit was evaluated on the basis of in situ measurements along the exploratory tunnel, whereas matrix porosity values were defined from laboratory tests on rock samples (Table 3(a)).

4.2 Numerical modelling in MODFLOW-2005

In MODFLOW-2005, a 3-D model with three layers was implemented according to the conceptual model previously described (Fig. 6(b)). The middle layer represents the mainly fractured zone, while the other layers composed the matrix (Table 3(a)). The grid has dimension 400x200 m^2 with cells having dimensions ranging from 1 m to 5 m. Simulations were carried out in transient state for one year, using 365 time steps and introducing the daily rainfall. Modeling results showed that with a single continuum model the second peak is no longer distinguishable, losing the feature of dual permeability. The system actually stores a large part of water, from precipitations and snow-melting but it cannot drain all stored water during the following period.

Table 3: Parameters in (a) scm; (b) cm.

(a)			(b)	
	Matrix	Fracture		Value
K_s (m/s)	1E-3	48	Diameter (m)	4.0
K_v (m/s)	1E-3	48	Wall permeability (m/s)	5E-1
K_z (m/s)	1E-3	48	Tortuosity (-)	1
S_{sx}	1.36E-05	2E-2	Roughness (m)	1E-2
S_{sv}	1.36E-05	2E-2	Lower Reynolds No.(-)	20
Porosity	1.36E-05	2E-2	Upper Reynolds No.	1000
			Percent Recharge (%)	100

The use of the CFP (Table 3(b)) allowed to better reproduce the flow rate, as it can compute the exchange flow, not only along conduit in each node, but also for every stress period. Overall, the two models return results characterized by relatively low errors, ensuring a certain degree of reliability of the models (Fig. 7). As a verification of the fact that the studied case is a dual permeability system, a dual porosity model was implemented, considering a negligible value of matrix hydraulic conductivity, about 5E-07 m/s, without changing the other parameters (Fig. 7). As expected, the effect is to flatten the secondary peaks, in particular the first minor peak, that is more probably due to the dual permeability feature of the system. The flow rate presents a slight delay compared with the observed data and simulated flow rate with the dual permeability approach. Overall, a dual porosity model cannot reproduce observed flow rate, specifically the secondary peaks. This confirms that the measured discharges are the response of a dual permeability system.

4.3 Results discussion

The single continuum model is not a satisfactory representation of structured rock system interested by a significant karstic network. The Modflow combined model includes a permeable continuum for the matrix and a discrete network for karstic conduits, and it considers the water exchange between matrix and conduits. These analyses confirm that the conduit wall permeability, or linear exchange term, between the matrix and conduit network is an important component of numerical simulations for dual permeability systems. It is one of the most sensitive model parameter in combined model. The comparison with an hypothetical dual porosity model demonstrated that it is not able to reproduce the responses of the case study. This example may be considered as a typical dual permeability system. Laminar single continuum model simulates the observed discharges with an average error equal to 4%, whereas the combined model reaches an average error equal to 1%. Specifically during drought period, the error of the dual conductivity model decreases until 0.02%, whereas the single continuum model increase its error to 5%.

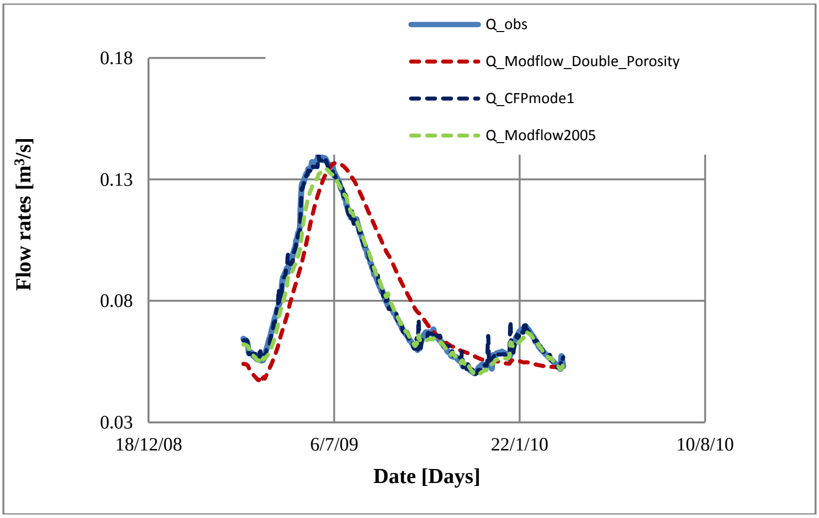

Figure 7: Dual porosity model response compared to dual permeability models responses.

5 Conclusions

Starting from Gerke and Van Genuchten Eqns (1), a numerical solution for solving 1-D dual conductivity systems was derived. This study was useful for a better comprehension of physical mechanism of hydraulic heads and groundwater flow in dual permeability system, showing advantages and disadvantages of each approach. Although the Matlab code is simple to be applied, it presents the following limitations: it reproduces a mono-dimensional flow and the initial input must be assigned only as continuous functions of hydraulic head. On the other hand, even if single continuum model is relatively low-cost, it cannot simulate the exchange term between matrix and fractures. The combined model requires more detailed data about the conduit network characteristics, but distinguishing the two continua, matrix and fractures, it can simulate the exchange term and reproduce the turbulent/non Darcian components of flow. Overall, despite the combined model provides the best results, it may not be convenient to choose a combined approach as all models offer a good degree of precision and then the choice of one rather than the other may depend on the time and costs foreseen for modeling as well as the availability data.

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