Synthesis of trusses using the MINLP optimization approach

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Abstract

This paper presents the synthesis of trusses using Mixed-Integer Non-Linear Programming (MINLP). The discrete/continuous non-convex and non-linear optimization problems are solved by the Modified OA/ER algorithm. The Hiearchical superelement approach (HSA) is introduced for the discrete topology optimization of trusses in order to reduce the number of topology alternatives. For the solution of truss synthesis problems, which include a high number of discrete/binary variables, we applied multilevel MINLP strategies. The Sequential Two-Phase (STP) strategy was developed for topology, shape and standard dimension optimization of trusses. A numerical example at the end of the paper shows the suitability of the proposed approach.

1 Introduction

The paper presents the Mixed-Integer Non-Linear Programming (MINLP) optimization approach to the synthesis of trusses. The solution of discrete/continuous and non-linear optimization problems is discussed with respect to the simultaneous topology, shape and standard dimension optimization of trusses.

In the context of truss synthesis, the discrete optimization problems can be divided into two main spheres of activity. The first one is topology optimization with the object of obtaining the optimal number and configuration of structural elements (bars), while the second one is a discrete dimension optimization problem, where cross-sections of elements are forced to have discrete, in many cases standard dimensions. A wide range of different optimization techniques has been employed in order to solve the topology optimization problem of trusses, e.g. Genetic Algorithm (GA) [1], [2], the GA linked with Non-Linear

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Programming (NLP) [3], the Tabu Search [4], Shape Annealing [5], the Random Cost Method [6], etc. Many attempts have also be made to solve the discrete dimension truss optimization problems, using GA [7], the Random Search Method [8], Sequential Linear Discrete Programming [9], etc.

In our approach, the simultaneous topology, shape and standard dimension optimization is performed by the MINLP approach, which deals with continuous and discrete variables simultaneously. While continuous variables are defined for the continuous optimization of parameters (continuous dimensions, stresses, strains…), discrete binary 0-1 variables are used to express discrete decisions, i.e. existence/non-existence of alternative structural elements, as well as for the selection of discrete/standard dimensions. Since the discrete and continuous optimizations are carried out simultaneously, the MINLP approach also finds optimal continuous parameters (shape) as well as discrete components (topology, standard dimensions) simultaneously.

The mass of the truss structure is minimized under stress, local buckling and displacement constraints. Finite element equations are used for the calculation of internal forces and displacements. The design conditions are defined in accordance with Eurocode 3 [10]. Logical relations are defined to provide the kinematical stability of the structure, as well as to prevent overlapping of elements and appearance of needless elements.

The MINLP synthesis of trusses is performed through three steps [11]: the process is started with the *generation of truss superstructure*, followed by the *development of a special MINLP model formulation* and the final step is the *solution of the defined MINLP problem*. All three steps are briefly described in the following sections.

The discrete/continuous, non-convex and non-linear problems are solved using the Modified Outer-Approximation/Equality Relaxation (OA/ER) algorithm [12]. Topology optimization of trusses is in general a high combinatorial/expansive problem and needs a high number of alternative arrangements of elements to be defined in the truss superstructure. The Hierarchical Superelement Approach (HSA) was thus developed with the object of reducing the combinatorial expanse of the problem.

Beside many non-convexities and non-linearities, the main problem of truss synthesis is that a very high number of discrete/binary variables, particularly those defining discrete/standard dimensions can be included in the optimization. In order to be able to solve such comprehensive optimization problems, we apply multilevel strategies. The so-called Sequential Two-Phase (STP) strategy was developed specially for truss synthesis problems and is introduced in the paper.

A numerical example is presented at the end of the paper in order to demonstrate the efficiency and the suitability of the proposed approach.

2 Generation of MINLP truss superstructure

The first step of the truss synthesis is the generation of a MINLP truss superstructure, which includes all possible topology/structure alternatives to compete for a feasible and optimal solution.

The truss superstructure consists of the provided set of nodes and their interconnections, which represent the elements of a truss, i.e. bars. The notation *i-j* defines an element connecting nodes *i* and *j*. Special logical relations between the elements have been defined in order to provide the kinematical stability of the structure and prevent the undesirable overlapping of elements involved in different topology/structure alternatives. In general, all nodes and elements are alternative (optional), i.e. they can be selected or excluded from the superstructure. Only nodes, which represent the supports and where nodal loads are applied, are always included in the superstructure, i.e. the later compose all the defined alternative structural designs. The fixed nodes together with some fixed elements reduce the combinatorial expanse of the topology optimization problem.

2.1 The Hierarchical Superelement Approach (HSA)

The discrete topology optimization problems of large-scale trusses generally comprise a high number of alternative structural elements and their configurations. A number of them will never be selected to compose the optimal topology/structure. In order to reduce the combinatorics of the discrete optimization, it is preferred for such elements to be removed from the set of the defined alternatives. For this reason, we propose a special topological formation, the so-called Hierarchical Superelement Approach (HSA), by which a truss superstructure is in a hierarchical manner composed from a number of different superelements, see Fig. 1.

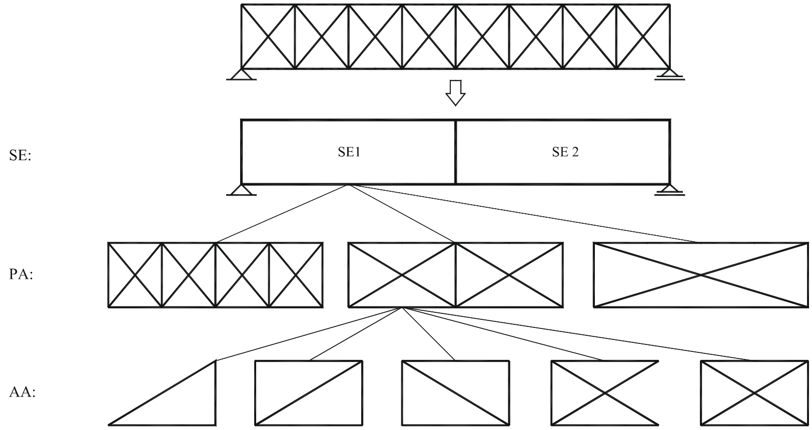

Figure 1: The Hierarchical Superelement Approach (HSA).

The highest level of superelements is segments (SE), which are then further partitioned into one of the pre-defined alternative numbers of internal partitions - panels (PA). The panels represent the medium level of superelements. Each alternative panel can additionally be formed from any possible alternative arrangement (AA) of bracing members which assure the kinematical stability of the structure. Alternative arrangements comprise the lower level of superelements. While the upper levels, i.e. the segments and panels, represent the quantity of the alternative superelements incorporated in the superstructure, the lower level determines their quality. The number of segments is fixed and defined before the optimization. Though different topology alternatives (active/non-active nodes and elements) are independently determined and optimized inside each SE, the entire superstructure is optimized simultaneously, all the defined SEs included.

3 The MINLP model formulation for truss superstructure

The general non-linear and non-convex discrete/continuous optimization problem can be formulated as an MINLP problem in the form:

$$
\begin{aligned}
&\min \quad z = \boldsymbol{c}^{\mathrm{T}} \boldsymbol{y} + f(\boldsymbol{x}) \\
&s.t. \ \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0} \\
&\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{0} \qquad\qquad \textbf{(MINLP)} \\
&\boldsymbol{B}\,\boldsymbol{y} + \boldsymbol{C}\,\boldsymbol{x} \leq \boldsymbol{b} \\
&\boldsymbol{x} \in X = \{ \boldsymbol{x} \in R^n : \boldsymbol{x}^{\mathrm{LO}} \leq \boldsymbol{x} \leq \boldsymbol{x}^{\mathrm{UP}} \} \\
&\boldsymbol{y} \in Y = \{ 0,1 \}^m
\end{aligned}
$$

where $\boldsymbol{x}$ is a vector of continuous variables specified in the compact set X and $\boldsymbol{y}$ is a vector of discrete, mostly binary 0-1 variables. Functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ are non-linear functions involved in the objective function z, equality and inequality constraints, respectively. Finally, $\boldsymbol{B}\,\boldsymbol{y} + \boldsymbol{C}\,\boldsymbol{x} \leq \boldsymbol{b}$ represents a subset of mixed linear equality/inequality constraints. The set of continuous variables $\boldsymbol{x}$ represents the continuous parameters (stresses, deflections, nodal coordinates etc.), while the set of discrete variables $\boldsymbol{y}$ contains parameters for discrete decisions (standard dimensions, elements).

The MINLP model formulation for truss superstructure was developed on the basis of the continuous NLP truss optimization model, which was previously successfully used for sizing and shape optimization of steel trusses [13], composite trusses [14] and timber trusses by considering joint flexibility [15]. In the proposed truss synthesis model formulation, the objective function defines the mass of the truss. The main components of the nonlinear equality constraints are the finite element equations for the calculation of nodal displacements, reactions and member forces. The nonlinear inequality constraints include the tensional and compressive/buckling resistance conditions of elements, as well as the displacement conditions. All the design conditions are defined in accordance with Eurocode 3.

Independent continuous variables include nodal coordinates and sizing variables. The cross sections of bars are considered to be steel circular hollow sections. The cross section of each element i-j is thus defined by the variables $d_{i,j}$

and $t_{i,j}$, which represent the diameter and the wall thickness of the tube, respectively. The binary 0-1 variables are divided into topological binary variables and binary variables defining standard dimensions. The topological binary variables subjected to the currently active truss elements take the value 1, while the rest of them are given the value zero. The binary variables subjected to standard dimensions are defined for each standard value that can be valued to any cross-section dimension of each element *i-j*.

4 Solution of the MINLP truss synthesis problem

4.1 Multilevel MINLP strategies

For the solution of non-linear and non-convex problems we used the Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm. Beside many non-convexities and non-linearities, truss synthesis problems involve a high number of discrete/binary variables. Such problems are thus hard to solve in a single MINLP phase, where all the included binary variables are initialized in a single full set. For this reason we applied multilevel strategies [16], which are performed through a hierarchic decomposition of binary variables into more subsets. From a number of previously developed multilevel strategies, two were applied to truss synthesis: the Two-Phase (TP) MINLP approach [17] and the Linked Two-Phase (LTP) strategy [18].

The TP approach performs topology, shape and standard dimension optimization separately in two phases. In the first phase, simultaneous topology, shape and sizing optimization is performed with dimensions of cross-sections being temporarily relaxed into continuous parameters. When the optimal topology is obtained, the discrete dimensions are re-established and the process continues with the second phase, where the shape and standard dimension optimization is performed until the optimal solution is obtained. In the second phase, the optimization is carried out at the fixed optimal topology, obtained in the first phase. The main disadvantage of the TP strategy lies in he fact that the final solution is not necessarily the optimal one since topology and standard dimensions are optimized sequentially. A simultaneous consideration of standard dimensions may cause a change in the optimal topology. In order to avoid the disadvantages of the TP approach, the LTP strategy was developed. While the first phase of the LTP strategy is identical to the first phase of the TP approach, the second phase differs in the fact that by using the LTP strategy, the topology is not fixed. The second phase of the LTP strategy thus performs simultaneous topology, shape and standard dimension optimization.

Both described multilevel MINLP strategies were applied to truss synthesis problems. The TP approach yielded good solutions in few MINLP iterations. By the use of the LTP strategy, however, no feasible solutions were obtained in reasonable CPU times. In order to allow the changing of the topology caused by the influence of discrete dimensions and to obtain a feasible optimal solution in a reasonable CPU time, we proposed a Sequential two-phase (STP) strategy, discussed in the following section.

4.2 The Sequential Two-Phase (STP) strategy

The Sequential two-phase MINLP strategy involves the running of a sequence of two-phase algorithms, see Fig. 2. After the initialization with the standard dimensions currently relaxed into continuous parameters, the process starts with the first TP procedure with the goal to obtain the first continuous and the first discrete solution. At this point, the continuous solution OBJ_C corresponds to the currently best continuous solution OBJ_{CB}. Later, the standard dimensions are relaxed into continuous parameters again and the process proceeds with the second TP in the search of a new continuous solution OBJ_C. On such basis, the second TP proceeds to give the next discrete solution given that the convergence condition is satisfied. This condition is satisfied, when the value of the current continuous solution OBJ_C is better/lower then the current best solution OBJ_{CB}. However, the results have shown, that a better continuous solution does not necessarily assure a better discrete solution. And vice-versa, a less favourable continuous solution may lead to a better discrete result. For this reason the convergence condition to continue the optimization of standard dimensions was modified. The condition is fulfilled also if the OBJ_C is a bit worse/higher than the OBJ_{CB}, but the difference would have to be smaller than a reasonably defined value ε. In general, a few successive MINLP iterations (n_{succ}) are needed in order to obtain the following continuous solution, which would satisfy the defined convergence condition. In order to limit the number of these iterations, a reasonable terminal number nfin has to be defined. The optimization using STP strategy is completed when the number of not satisfied successive MINLP iterations n_{fin} is attained.

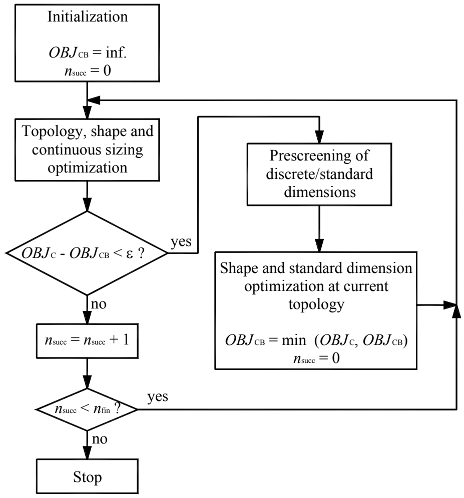

Figure 2: The Sequential Two-Phase (STP) strategy.

As mentioned above, the majority of discrete binary variables are associated with standard dimensions. After the completion of each continuous phase, a prescreening procedure is applied in order to reduce the number of active binary variables. This way only those binary variables are included in the discrete phase, which define the standard dimensions close to the optimal continuous dimensions, obtained in the previous phase. The set of active binary variables becomes essentially smaller than the full initial set, which leads to shorter and solvable optimization. A shorter CPU time and fewer MINLP iterations are needed for reaching the final solution.

The optimization model was developed using General Algebraic Modelling System (GAMS) [19]. The optimization was performed by the MINLP computer package MIPSYN, the extension of PROSYN [12]. GAMS/CONOPT [20] has been used to solve the NLP subproblems and GAMS/CPLEX [21] to solve the MILP master problems.

5 Numerical example

As a numerical example the synthesis of a simply supported truss girder over the span of 20 m is presented. The simultaneous topology, shape and sizing optimization was performed by the use of the proposed Hierarchical superelement approach. The defined truss superstructure consists of two equal segments, each of them may be further partitioned into 1, 2, 3 or 4 panels. The proposed superelement is shown in Fig. 3. The superstructure includes 50 nodes, 46 of which are alternative (optional), and 93 alternative elements. The truss is subjected to a uniformly distributed load of 15 kN/m, acting on the top chord (see Fig. 4). The applied load is considered as design load with a partial safety factor already included in the defined value. The structure is designed in accordance with Eurocode 3. The buckling lengths of the truss elements are considered as being equal to the system lengths of the elements for both in-plane and out-of-plane buckling. The bars are designed from circular hollow sections made of S235 steel. The vectors of discrete/standard alternative values for the diameter d and wall thickness t of cross-sections are given as follows: $\mathbf{d}$ = {42.4, 48.3, 60.3, 76.1, 88.9, 108.0, 114.3, 133.0, 139.7, 159.0, 168.3, 219.1, 273.0, 323.9, 355.6, 406.4, 457.0, 508.0, 558.8, 609.6} [mm] and $\mathbf{t}$ = {2.0, 2.9, 3.2, 4.0, 5.0, 6.3, 7.1, 8.0, 10.0, 12.5, 14.2, 16.0} [mm]. With respect to the available standard cross sections, the lower/upper bounds of the wall thickness (t^{LO}/t^{UP}) for each individual value of diameter d of cross section are given in Table 1. Considering these bounds, 103 alternative standard cross-sections are defined for each element of the truss. The cross sections of the chords are forced into being constant through the entire span. There are thus 57 different cross-sections for 55 alternative bracing members plus 2 chords defined by 114 independent sizing variables. The vertical coordinates of top chord joints were defined as geometric variables with their lower and upper bounds of 200 and 700 cm. Two examples of the simply supported truss synthesis were performed:

- example ***SST20a***: the height of the truss is constant through span (all top chord joints have equal vertical coordinates)
- example ***SST20b***: the height of the truss varies through span (each top chord joint can take a different value of its vertical coordinate)

Since the loading of the defined truss is symmetric, symmetry of topology with respect to the vertical axis through the midspan of the structure is requested. The topology is thus optimized within the segment SE1, while SE2 represents its mirror image. It should be noted, that the defined superstructure includes 425 alternative topologies. All alternative elements were included in the initial topology.

Table 1: Standard cross sections – bounds on wall thickness.

d [mm]	t^{LO} [mm]	t^{UP} [mm]	d [mm]	t^{LO} [mm]	t^{UP} [mm]	d [mm]	t^{LO} [mm]	t^{UP} [mm]
42.4	2.0	4.0	133.0	2.9	5.0	355.6	5.0	16.0
48.3	2.0	4.0	139.7	3.2	5.0	406.4	5.0	16.0
60.3	2.0	4.0	159.0	4.0	5.0	457.0	6.3	16.0
76.1	2.9	5.0	168.3	4.0	8.0	508.0	6.3	16.0
88.9	3.2	5.0	219.1	4.0	10.0	558.8	6.3	16.0
108.0	2.9	5.0	273.0	4.0	12.5	609.6	7.1	16.0
114.3	2.9	5.0	323.9	5.0	14.2			

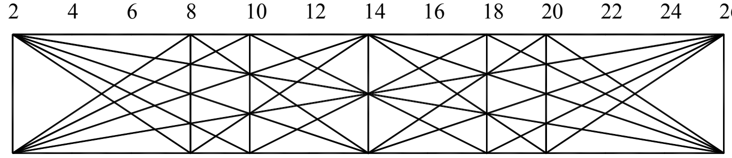

Figure 3: The proposed superelement for 20 m simply supported truss.

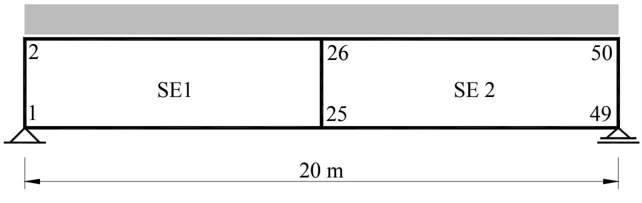

Figure 4: The superstructure for 20 m simply supported truss.

Regarding the 46 optional nodes, the 93 alternative elements as well as the 21 standard alternatives of section diameters $d_{i,j}$ and the 12 alternatives of wall thicknesses $t_{i,j}$ (both dimension alternatives are attributed to each of the 57 independent bar groups), the problem involves 2020 binary variables altogether. The convergence condition for the STP strategy is satisfied when the difference between the current (OBJ_C) and the currently best (OBJ_{CB}) continuous solution is less than $\varepsilon = 0.01 \cdot OBJ_{CB}$ and the allowed number of not satisfied successive MINLP iterations is $n_{fin} = 5$.

The progress of the STP strategy applied to examples ***SST20a*** and ***SST20b*** is presented in Tables 2 and 3, respectively. Two upper neighbouring standard alternatives regarding the previously obtained continuous tube diameter $d_{i,j}$ and one upper and one lower standard alternative regarding the obtained continuous wall thickness $t_{i,j}$, i.e. only four binary variables for each active element were used for standard dimension optimization. The optimal and some intermediate solutions are graphically presented in Figs. 5-9. The obtained optimal standard cross sections of both examples are listed in Tables 4 and 5.

Table 2: Progress of STP strategy, example SST20a.

TP cycle	Sizing variables	Value of obj. fun. [kg]	Num. of active dicr. var.
	Initialization	1446.117	2020
1	Continuous	747.117	139
	Discrete	816.256	16
2	Continuous	583.824	139
	Discrete	**572.625**	36
3	Continuous	580.193	139
	Discrete	616.227	52

Table 3: Progress of STP strategy, example SST20a.

TP cycle	Sizing variables	Value of obj. fun. [kg]	Num. of active dicr. var.
	Initialization	1446.117	2020
1	Continuous	747.117	139
	Discrete	816.256	16
2	Continuous	509.639	139
	Discrete	**502.801**	52
3	Continuous	507.103	139
	Discrete	506.493	68

In both discussed examples the optimal solution was obtained in the 2^{nd} TP cycle. Analysing the progress of the truss synthesis (Tables 2 and 3) it can be observed that in both cases the topology of the 3^{rd} TP cycle yielded the best solution with continuous cross-section dimensions. However, the optimal solutions with standard cross-sections were obtained at topologies, which yielded a somewhat worse continuous solution. Therefore the presented numerical example has proven, that inclusion of discrete/standard cross-section dimensions can have an important influence on the final/optimal structural topology.

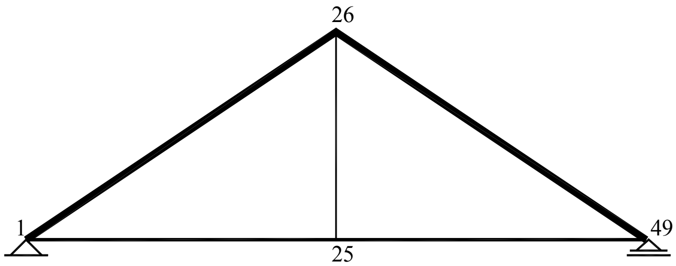

Figure 5: Examples ***SST20a*** and ***SST20b***, 1st TP cycle (m = 816.256 kg, h = 669.51 cm).

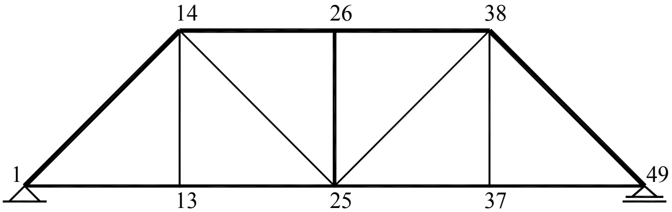

Figure 6: Example ***SST20a***, 2nd TP cycle (optimal solution) (m = 572.625 kg, h = 502.13 cm).

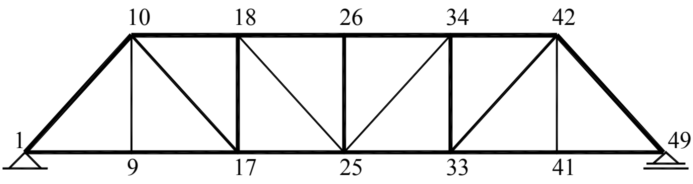

Figure 7: Example ***SST20a***, 3rd TP cycle (m = 616.227 kg, h = 364.37 cm).

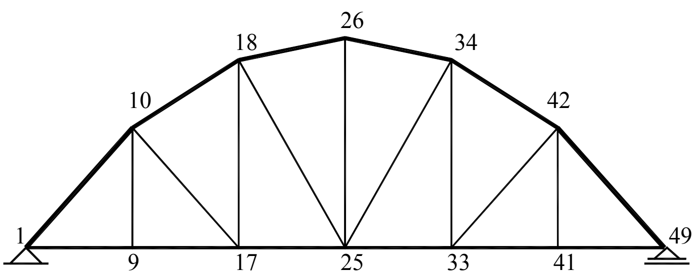

Figure 8: Example ***SST20b***, 2nd TP cycle (optimal solution) (m = 502.801 kg, h_{max} = 656.46 cm).

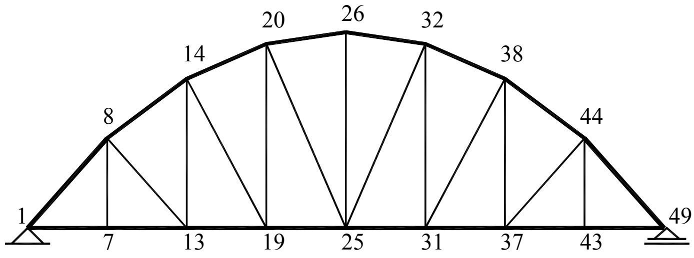

Figure 9: Example ***SST20a*** 3rd TP cycle (m = 507.013 kg, h_{max} = 621.53 cm).

Table 4: Optimal standard cross-sections, example SST20a (2nd TP cycle).

Element	Diameter d [mm]	Wall thickness t [mm]
1-13, 13-25, 25-37, 37-49	88.9	4.0
14-26, 26-38	133.0	3.2
1-14, 38-49	159.0	4.0
13-14,14-25, 25-38, 37-38	42.4	2.0
25-26	108.0	2.9

Table 5: Optimal standard cross-sections, example ***SST20b*** (2nd TP cycle).

Element	Diameter d [mm]	Wall thickness t [mm]
1-9, 9-17, 17-25, 25-33, 33-41, 41-49	88.9	4.0
10-18, 18-26, 26-34, 34-42	114.3	3.2
1-10, 42-49	139.7	3.2
9-10, 10-17, 17-18, 18-25, 25-26, 25-34, 33-34, 33-42, 41-42	42.4	2.0

6 Conclusions

The synthesis of trusses using Mixed-Integer Non-Linear Programming (MINLP) approach was presented. The non-convex and non-linear problems have been solved by the Modified OA/ER algorithm. The Hierarchical superelement approach (HSA) was introduced for the discrete topology optimization of trusses in order to reduce the number of topology alternatives.

Alongside several involved non-linearities and non- convexities, the main difficulty of truss synthesis is that a very high number of discrete variables, particularly those associated with standard dimensions, may be included in the optimization. The Sequential Two-Phase (STP) strategy was developed specially for solving topology, shape and standard dimension optimization of trusses. The additional reduction in the number of currently active binary variables for standard dimensions is attained by the use of a special prescreening procedure. The suitability and efficiency of the proposed method was shown through a numerical example. The influence of discrete/standard cross-section dimensions on the final/optimal structural topology was also proved.

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