MARINE SHIPS' CONTROL FAULT DETECTION BASED ON DISCRETE H_2-OPTIMIZATION

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ABSTRACT

This paper is devoted to the problem of the H_2-optimal discrete observer-filter, meant for detection of slowly varying additive faults in marine ship control processes. Sensitivity of the observer to the external disturbance is to be minimized by the choice of its parameters. Fault detection is a hot area of research, but most of the published papers do not concern a case with initially given spectral features of external disturbances. External disturbances, which are considered in this research, can be presented as a sum of harmonic oscillations with given central frequency (sea wave disturbance) and a constant signal (ocean currents and wind). The novel discrete H_2-optimization algorithm is proposed to solve the problem of polyharmonical disturbance suppression. This approach guaranties nonuniqueness of the optimal solution that often makes possible to provide additional useful properties, such as insensitivity of the residual signal to the constant external disturbance, i.e. its integral action. Let us note that it is possible (not always) only if at least two sensors are used. It is notable, that the observer-filter can be designed in a multipurpose structure consisting of basic and auxiliary observers and a dynamical corrector that makes possible the retuning of the basic item in real time on-board. The novel discrete algorithm of adaptive fault detection observer analytical design in a multipurpose structure is proposed, and its effectiveness is demonstrated by the numerical example of the fault detection process with implementation of the MATLAB package.

Keywords: linear-quadratic functional, H2-optimazation, discrete, fault detection, optimal control, spectral approach, stability, integral action.

1 INTRODUCTION

Most of controlled plants can be subject of various malfunctions which may cause loss of the control effectiveness and, at worst, instability of the closed-loop system. One can see that it significantly increases possibility of the shipwreck, especially in case of underwater vehicles. As a result, improvement of the systems reliability is paid serious attention nowadays. The first step in fault diagnosis is fault detection, i.e. binary decision process, determining whether a fault has occurred or not. There are two main groups of approaches to fault detection problem: model-free (or data-based) methods (e.g. PCA [1]), and model-based ones, using equations, describing dynamics of the plant, and including asymptotic observers.

Model-based fault detection has been paid serious attention for the past decades [2], but, in our attention, there are aspects, requiring further research. For example, most of published papers and monographs do not use information about initially known features of the external disturbance, but there are some situations, where disturbances can be considered as output of a shaping filter or sum of harmonic oscillations. The most obvious example of such dynamic systems are various marine vehicle, affected by sea waves, wind, sea currents, etc. One can see that filtering properties of the fault detection observers can significantly increase their effectiveness. Suppression of the harmonic disturbance is considered as H_2 optimization problem, which is solved with application of the discrete version of the spectral approach in frequency domain, presented in Aliev et al. [3] and successfully applied in Veremey and Knyazkin and works by Veremey [4]–[7]. This approach does not include such complicated procedures, as linear matrix inequalities solving, and guaranties nonuniqueness of the solution that enables to provide such useful features of the observer, as integral action

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relatively to wind and sea currents. The second main feature of this research is the multipurpose structure of the fault detector, including a dynamic corrector, which should provide the desirable filtering properties.

The paper consists of six sections. The next one introduces the problem and its formal statement. Suppression of polyharmonic oscillations effect, based on discrete spectral H_2 optimization, is described in Section 3. In Section 4, multipurpose structure of the observer is presented, and choice of the dynamical corrector, which represents the optimal solution is demonstrated. In Section 5, the proposed approach is illustrated by the practical example of the fault detection observer synthesis for a transport ship with the displacement of about 4 000 t. Finally, Section 6 concludes this paper by discussing the overall results of the investigation.

2 PROBLEM STATEMENT

Let us consider a discrete linear time invariant system

$$\begin{aligned} &\mathbf{x}[k+1] = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\delta[k] + \mathbf{E}\mathbf{f}[k] + \mathbf{H}d[k] \\ &\delta[k+1] = \delta[k] + T_S\mathbf{u}[k], \mathbf{y}[k] = \mathbf{C}\mathbf{x}[k] \end{aligned} \tag{1}$$

where $\mathbf{x} \in E^n$ is the state space vector, $\delta \in E^{n_r}$ is the rudder angle, $\mathbf{u} \in E^{n_r}$ is the control signal, $d(t)$ is the scalar external disturbance, $\mathbf{f} \in E^{n_f}$ is the slowly varying fault, i.e. $\dot{\mathbf{f}} \approx 0$, and $\mathbf{y} \in R^m$ is output measured signal. All components of the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{E}, \mathbf{H}$ are known constant values, the pairs $\{\mathbf{A}, \mathbf{B}\}$ and $\{\mathbf{A}, \mathbf{C}\}$ are controllable and observable respectively. The system in eqn (1) has the sample time T_s.

External disturbance d consist of sea waves one, ocean currents and wind effect. It can be presented as sum of a few harmonics and a step function:

$$d(k) = \sum_{i=1}^{N_d} A_{di} \sin(\sigma_i T_S k + \phi_i) + 1(t)d_0, \tag{2}$$

where N_d is the number of harmonics and A_{di}, σ_i, φ_i are their amplitudes, frequencies and phases respectively, d_0 is a constant value and $1(t)$ is Heaviside step function. The central frequency of the sea wave disturbance is $\omega = \omega_0$.

Adaptive fault detection observer has the structure

$$\begin{aligned} &\hat{\mathbf{x}}[k+1] = \mathbf{A}\hat{\mathbf{x}}[k] + \mathbf{B}\delta[k] + \mathbf{v}[k] \\ &\mathbf{v}(z) = \mathbf{L}(z)(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}) \\ &r = \tilde{\mathbf{C}}_r(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}) = \mathbf{C}_r(\mathbf{x} - \hat{\mathbf{x}}), \end{aligned} \tag{3}$$

where $\mathbf{v} \in R^n$ is the corrective signal, $\mathbf{L}(z)$ is the transfer matrix, and scalar r is the residual signal. If r is not close to zero, then the plant operation is affected a malfunction. Denote new variables characterizing deviation from the reference behaviour

$$\mathbf{e}_x = \mathbf{x} - \hat{\mathbf{x}}, \ \mathbf{e}_y = \mathbf{C}\mathbf{e}_x = \mathbf{y} - \mathbf{C}\hat{\mathbf{x}}, \tag{4}$$

then the error dynamics is given by

$$\begin{aligned} &\mathbf{e_x}[k+1] = \mathbf{A}\mathbf{e_x}[k] + \mathbf{E}\mathbf{f}[k] + \mathbf{H}d[k] - \mathbf{v}[k] \\ &\mathbf{e_y} = \mathbf{C}\mathbf{e_x}, \mathbf{v}(z) = \mathbf{L}(z)\mathbf{e_y} \\ &r = \tilde{\mathbf{C}}_r\mathbf{e_y} = \mathbf{C}_r\mathbf{e_x}. \end{aligned} \tag{5}$$

Effectiveness of sea wave filtration and sensitivity to fault are expressed by the following values

$$J = J_1 / J_2, \; J_1 = \min_{\omega\in\Omega_{\mathbf{f}}} \left\| \mathbf{F}_{r\mathbf{f}}(e^{j\omega T_s}) \right\|^2, \; J_2 = \max_{\omega\in\Omega_d} \left| F_{rd}(e^{j\omega T_s}) \right|^2, \tag{6}$$

where $F_{rd}(z)$, $\mathbf{F}_{r\mathbf{f}}(z)$ are the transfer functions from the external disturbance d and fault $\mathbf{f}$ to the residual signal r:

$$\begin{aligned} F_{rd}(z) &= \mathbf{C}_r \left(z\mathbf{I} - \mathbf{A} + \mathbf{L}(z)\mathbf{C} \right)^{-1} \mathbf{H} \\ \mathbf{F}_{r\mathbf{f}}(z) &= \mathbf{C}_r \left(z\mathbf{I} - \mathbf{A} + \mathbf{L}(z)\mathbf{C} \right)^{-1} \mathbf{E}. \end{aligned} \tag{7}$$

Ω_d, $\Omega_{\mathbf{f}}$ are areas of ω_0 and 0 respectively. One can see that J is a functional of the transfer matrix $\mathbf{L}(z)$ and choice of its parameters should maximize the value J:

$$J(\mathbf{L}) \to \max_{\mathbf{L}\in\Omega_{\mathbf{L}}}, \; J_1(\mathbf{L}) \to \max_{\mathbf{L}\in\Omega_{\mathbf{L}}}, \; J_2(\mathbf{L}) \to \min_{\mathbf{L}\in\Omega_{\mathbf{L}}}, \tag{8}$$

where $\Omega_{\mathbf{L}}$ is a set of $\mathbf{L}(z)$, guarantying stability of the designed closed-loop system eqn (7). Similarly to all control problems, concerned with marine vehicles, one of the main requirements is integral action relatively to the external disturbance, i.e.

$$\mathbf{F}_{\mathrm{rd}}(1) = 0. \tag{9}$$

Firstly, it is necessary to minimize the value J_2. Consider the transfer function $F_{rd}^T(z)$ and the auxiliary DLTI plant

$$\mathbf{x}_1[k+1] = \mathbf{A}_1\mathbf{x}_1[k] + \mathbf{B}_1\mathbf{u}_1[k] + \mathbf{H}_1 d_1[k], \tag{10}$$

where $\mathbf{x}_1 \in R^n$, $\mathbf{u}_1 \in R^m$, $\mathbf{A}_1 = \mathbf{A}^T$, $\mathbf{B}_1 = -\mathbf{C}^T$, $\mathbf{H}_1 = \mathbf{C}_r^T$ and $d_1 \in R^1$ presents a wave with the central frequency ω_0. The control $\mathbf{u}_1$ is to be designed in the following tf-form:

$$\mathbf{u}_1 = \mathbf{L}^T(z)\mathbf{x}_1 = \mathbf{W}(z)\mathbf{x}_1 = \mathbf{W}(z)_2^{-1}\mathbf{W}_1(z)\mathbf{x}_1, \tag{11}$$

where $\mathbf{W}_1(z)$, $\mathbf{W}_2(z)$ are the ($m \times n$) and ($m \times m$) polynomial matrix functions. Let us introduce the mean-square functional

$$\tilde{J}_2(\mathbf{W}) = \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N} (\mathbf{x}_1^T\mathbf{R}\mathbf{x}_1 + c^2\mathbf{u}_1^T\mathbf{Q}\mathbf{u}_1) = \lim_{N\to\infty} \frac{1}{N} \int_0^N (e^T e + c^2\mathbf{u}_1^T\mathbf{Q}\mathbf{u}_1)dt, \tag{12}$$

where $\mathbf{R} = \mathbf{H}\mathbf{H}^T$, $e = \mathbf{H}^T\mathbf{x}_1$, $\mathbf{Q}$ is symmetrical positive definite matrix, and c is small positive value. It is evident that $\mathbf{F}_{ed_1}(z) = \mathbf{F}_{rd}^T(z)$, and problem of the polyharmonical disturbance suppression can be considered as minimization of the functional eqn (12):

$$\tilde{J}_2(\mathbf{W}) \to \min_{\mathbf{W}\in\Omega_\mathbf{W}}, \tag{13}$$

where $\Omega_\mathbf{W}$ is the set of the stabilizing controllers, such that that all the roots of the characteristic polynomial

$$\Delta(z) = (A_s(z))^{1-m}\det(\mathbf{W}_1\mathbf{B}_{1z} - \mathbf{W}_2(s)A_z(z)),$$

where

$$\mathbf{B}_{1z}(z) \equiv A_z(z)(z\mathbf{I} - \mathbf{A}_1)^{-1}\mathbf{B}_1,\ A_z = \det(\mathbf{A}_1), \tag{14}$$

are located in the open unit disk on complex plane.

3 SPECTRAL DISCRETE H$_2$ OPTIMIZATION

The mean-square functional eqn (12) can be minimized with implementation of discrete version of spectral approach, presented in Veremey and Knyazkin [4]. Firstly, let us present $\tilde{J}_2$ in frequency domain

$$\tilde{J}_2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathrm{trace}\left[\mathbf{F}^*_{\mathbf{x}_1}(e^{i\omega T_s})\mathbf{R}\mathbf{F}_{\mathbf{x}_1}(e^{i\omega T_s}) + k^2\mathbf{F}^*_{\mathbf{u}_1}(e^{i\omega T_s})\mathbf{Q}\mathbf{F}_{\mathbf{u}_1}(e^{i\omega T_s})\right]S_{d_1}(\omega)d\omega, \tag{15}$$

where $\mathbf{F}^*(z) = \mathbf{F}^T(z^{-1})$, $S_{d_1}(\omega) = \mathbf{I}\cdot\delta(\omega-\omega_0)$, and $\mathbf{F}_{\mathbf{x}_1}(z)$, $\mathbf{F}_{\mathbf{u}_1}(z)$ are transfer matrices from d_1 to $\mathbf{x}_1$ and $\mathbf{u}_1$ correspondently,

$$\begin{aligned}\mathbf{F}_{\mathbf{x}_1}(z) &= (z\mathbf{I} - \mathbf{A}_1 - \mathbf{B}_1\mathbf{W})^{-1}\mathbf{H}_1\\ \mathbf{F}_{\mathbf{u}_1}(z) &= \mathbf{W}(z\mathbf{I} - \mathbf{A}_1 - \mathbf{B}_1\mathbf{W})^{-1}\mathbf{H}_1,\end{aligned} \tag{16}$$

We can overcome difficulties concerned with nonlinear dependency of $\tilde{J}_2$ from the $\mathbf{W}(z)$ by parameterization technique, proposed in Aliev et al. [3] for the first time. There is a dependency between transfer matrices $\mathbf{F}_{\mathbf{x}_1}(z), \mathbf{F}_{\mathbf{u}_1}(z)$, and it can be expressed by the parameter

$$\Phi(z) = \alpha(z)\mathbf{F}_{\mathbf{x}_1}(z) + \beta(z)\mathbf{F}_{\mathbf{u}_1}(z), \tag{17}$$

where $\alpha(z)$, $\beta(z)$ are $(m\times n)$ and $(m\times m)$ polynomial matrices, which can be calculated by the formulae proposed in Aliev et al. [3]. Firstly, let us solve the discrete Riccati equation:

$$\mathbf{A}_1^T\mathbf{S}\mathbf{A}_1 - \mathbf{S} - \mathbf{A}_1^T\mathbf{S}\mathbf{B}_1(\mathbf{B}_1^T\mathbf{S}\mathbf{B}_1 + k^2\mathbf{Q})^{-1}\mathbf{B}_1^T\mathbf{S}\mathbf{A}_1 + \mathbf{R} = 0, \tag{18}$$

and calculate $\alpha(z)$ and $\beta(z)$

$$\alpha(s) = \alpha_0 = (c^2\mathbf{Q} + \mathbf{B}_1^T\mathbf{S}\mathbf{B}_1)^{-1}\mathbf{B}_1^T\mathbf{S}\mathbf{A}_1,\ \beta(s) = \beta_0 = \mathbf{I}. \tag{19}$$

Then we introduce the notations

$$\Theta(z) = A_s(z)\beta_0 + \alpha_0 \mathbf{B}_{1z}(z),$$
$$\mathbf{H}_{1z}(z) \equiv A_z(z)(z\mathbf{I} - \mathbf{A}_1)^{-1}\mathbf{H}_1,\ \mathbf{P}(z) \equiv (z\mathbf{I} - \mathbf{A}_1), \tag{20}$$

and consider presentation of the plant eqn (10) in the form

$$(z\mathbf{I} - \mathbf{A}_1)\mathbf{F}_{\mathbf{x}_1} - \mathbf{B}_1\mathbf{F}_{\mathbf{u}_1} = \mathbf{H}_1,$$

that results in the following equations:

$$\begin{pmatrix} \mathbf{F}_{\mathbf{x}_1} \\ \mathbf{F}_{\mathbf{u}_1} \end{pmatrix} = \mathbf{M}_\Phi^{-1} \begin{pmatrix} \mathbf{H}_1 \\ \Phi \end{pmatrix},\ \mathbf{M}_\Phi = \begin{pmatrix} z\mathbf{I} - \mathbf{A}_1 & -\mathbf{B}_1 \\ \alpha_0 & \beta_0 \end{pmatrix}. \tag{21}$$

Finally, let us express the transfer matrices $\mathbf{F}_{\mathbf{x}_1}$, $\mathbf{F}_{\mathbf{u}_1}$ as functions of $\Phi(z)$:

$$\mathbf{F}_{\mathbf{x}_1} = \mathbf{F}_{\mathbf{x}_1}(\Phi) = \mathbf{H}_{1z}(z)/\mathrm{A}_s(z) + \mathbf{B}_{1z}(z)\Theta^{-1}(z)\tilde{\Phi}(z). \tag{22}$$

$$\mathbf{F}_{\mathbf{u}_1} = \mathrm{F}_{\mathbf{u}_1}(\Phi) = \mathrm{A}_z(z)\Theta^{-1}(z)\tilde{\Phi}(z), \tag{23}$$

where

$$\Theta(z) = \mathrm{A}_z(z)\beta_0 + \alpha_0\mathbf{B}_1(z),\ \tilde{\Phi} = \left(\Phi - \alpha_0\mathbf{P}^{-1}\mathbf{H}_1\right).$$

Then we denote

$$\mathbf{B}_{1\delta} = D_{1z}(z\mathbf{I} - \mathbf{A}_1 + \mathbf{B}_1\alpha_0)^{-1}\mathbf{B}_1,\ D_{1z} = \det(s\mathbf{I} - \mathbf{A}_1 + \mathbf{B}_1\alpha_0), \tag{24}$$

and present $\Theta^{-1}(z)$ in more convenient form for calculations:

$$\Theta^{-1}(z) = \left(\alpha_0\mathbf{B}_{1z} + \mathbf{I}A_{1z}\right)^{-1} = A_{1z}^{-1}\mathbf{I} - A_{1z}^{-1}\alpha_0(\mathbf{B}_{1z}\alpha_0 + \mathbf{I}A_{1z})^{-1}\mathbf{B}_{1z} =$$
$$= A_{1z}^{-1}\mathbf{I} - A_{1z}^{-1}\alpha_0\left(\mathbf{I} + \mathbf{P}^{-1}\mathbf{B}_1\alpha_0\right)^{-1}\mathbf{P}^{-1}\mathbf{B}_1 = \frac{1}{A_{1z}D_{1z}}(D_{1z}\mathbf{I} - \alpha_0\mathbf{B}_{1\delta}).$$

Integrand in eqn (14), having a form

$$\mathbf{F}_0^*\mathbf{F}_0 = \mathbf{F}_{\mathbf{x}_1}^*\mathbf{R}\mathbf{F}_{\mathbf{x}_1} + \mathrm{c}^2\mathbf{F}_{\mathbf{u}_1}^*\mathbf{Q}\mathbf{F}_{\mathbf{u}_1},$$

can be presented as linear-quadratic function of the parameter $\tilde{\Phi}(z)$.

$$\mathbf{F}_0^*\mathbf{F}_0 \equiv (\mathbf{T}_1^* + \tilde{\Phi}^*\mathbf{T}_2^*)(\mathbf{T}_1 + \mathbf{T}_2\tilde{\Phi}) + \mathbf{T}_3,$$

where

$$\mathbf{T}_1(\mathrm{s}) = (\sqrt{\mathrm{c}^2\mathbf{Q} + \mathbf{B}_1^{\mathrm{T}}\mathbf{S}\mathbf{B}_1})^{-1}\Theta_*^{-1}\mathbf{B}_{1z}^*\mathbf{R}\mathbf{H}_{1z}/\mathrm{A}_z(z)z^n, \mathbf{T}_2(\mathrm{s}) = \sqrt{\mathrm{c}^2\mathbf{Q} + \mathbf{B}_1^{\mathrm{T}}\mathbf{S}\mathbf{B}_1},$$

$$\mathbf{T}_3(\mathrm{s}) = (\mathbf{H}_{1z}^*\mathbf{R}\mathbf{H}_{1z})/\mathrm{A}_z(z)\mathrm{A}_z(z^{-1}) - \mathbf{T}_1^*(z)\mathbf{T}_1(z). \tag{25}$$

$$\mathbf{F}_0^*\mathbf{F}_0 \equiv (\mathbf{T}_1^* + \widetilde{\Phi}^*\mathbf{T}_2^*)(\mathbf{T}_1 + \mathbf{T}_2\widetilde{\Phi}) + \mathbf{T}_3 \text{ , where}$$
$$\mathbf{T}_1(s) = (\sqrt{c^2\mathbf{Q} + \mathbf{B}_1^T\mathbf{S}\mathbf{B}_1})^{-1}\Theta_*^{-1}\mathbf{B}_{1z}^*\mathbf{R}\mathbf{H}_{1z} / A_z(z)z^n, \mathbf{T}_2(s) = \sqrt{c^2\mathbf{Q} + \mathbf{B}_1^T\mathbf{S}\mathbf{B}_1}, \quad (25)$$
$$\mathbf{T}_3(s) = (\mathbf{H}_{1z}^*\mathbf{R}\mathbf{H}_{1z}) / A_z(z)A_z(z^{-1}) - \mathbf{T}_1^*(z)\mathbf{T}_1(z).$$

Only the first summand in eqn (25) is function of $\Phi(z)$ and eqn (13) is equivalent to the minimization of the functional

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \text{trace}\,[(\mathbf{T}_1^*(e^{i\omega T_s}) + \widetilde{\Phi}^*(e^{i\omega T_s})\mathbf{T}_2^*(e^{i\omega T_s}))(\mathbf{T}_1(e^{i\omega T_s}) + \mathbf{T}_2(e^{i\omega T_s})\widetilde{\Phi}(e^{i\omega T_s}))]S_{d1}(\omega)d\omega =$$
$$= \frac{1}{2\pi}\text{trace}\,[(\mathbf{T}_1^*(e^{i\omega_0 T_s}) + \widetilde{\Phi}^*(e^{i\omega_0 T_s})\mathbf{T}_2^*(e^{i\omega T_s}))(\mathbf{T}_1(e^{i\omega_0 T_s}) + \mathbf{T}_2(e^{i\omega T_s})\widetilde{\Phi}(e^{i\omega_0 T_s}))]. \quad (26)$$

Similarly to Veremey and Knyazkin [4], let us choose such $\widetilde{\Phi}(e^{i\omega_0 T_s})$, that

$$\widetilde{\Phi}_0(e^{i\omega_0 T_s}) + \mathbf{T}_2^{-1}(e^{i\omega_0 T_s})\mathbf{T}_1(e^{i\omega_0 T_s}) = 0 \text{, or } \widetilde{\Phi}_0(e^{i\omega_0 T_s}) = -\mathbf{T}_2^{-1}(e^{i\omega_0 T_s})\mathbf{T}_1(e^{i\omega_0 T_s}). \quad (27)$$

After substitution $\widetilde{\Phi}_0(e^{i\omega_0 T_s})$ in eqns (22) and (23), obtain optimal dynamics for the frequency ω_0

$$\mathbf{F}_{\mathbf{x}_1}(e^{j\omega_0 T_s}) = \mathbf{H}_{1z}(e^{j\omega_0 T_s}) / A_z(e^{j\omega_0 T_s}) + \mathbf{B}_{1z}(e^{i\omega_0 T_s})\Theta^{-1}(e^{j\omega_0 T_s})\widetilde{\Phi}_0(e^{j\omega_0 T_s}),$$
$$\mathbf{F}_{\mathbf{u}_1}(e^{j\omega_0 T_s}) = A_z(e^{j\omega_0 T_s})\Theta^{-1}(e^{j\omega_0 T_s})\widetilde{\Phi}_0(e^{j\omega_0 T_s}). \quad (28)$$

Then let formulate the condition of optimality for the controller eqn (11): it provides the optimal dynamics of the closed-loop system eqns (10) and (11) for the frequency ω_0 if and only if its transfer matrix $\mathbf{W}(z)$ satisfies the following equation:

$$\mathbf{W}(e^{j\omega_0 T_s})\mathbf{F}_{\mathbf{x}_1}(e^{j\omega_0 T_s}) = \mathbf{F}_{\mathbf{u}_1}(e^{j\omega_0 T_s}) \text{, or } \mathbf{L}^{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\omega_0 \mathrm{T_s}})\mathbf{F}_{\mathbf{x}_1}(\mathrm{e}^{\mathrm{j}\omega_0 \mathrm{T_s}}) = \mathbf{F}_{\mathbf{u}_1}(\mathrm{e}^{\mathrm{j}\omega_0 \mathrm{T_s}}). \quad (29)$$

4 OBSERVER WITH MULTIPURPOSE STRUCTURE

Problem of polyharmonical disturbance suppression is solved, but it is necessary to provide eqn (9) and maximize the value J_1 simultaneously. These demands make impossible application of the approaches, suppressing any constant disturbance, such as PID-controller or speed control law [5], [7]. In accordance to eqn (7), $F_{rd}(1)$ and $\mathbf{F}_{r\mathbf{f}}(1)$ are functions of the matrix $\mathbf{L}^0 = \mathbf{L}(1)$ and the conditions $F_{rd}(1) = 0$, $\mathbf{F}_{r\mathbf{f}}(1) \neq 0$ can be rewritten as

$$\left(\mathbf{I} - \mathbf{A} + \mathbf{L}^0\mathbf{C}\right)^{-1}\mathbf{H} = \mathbf{e}_1, \left(\mathbf{I} - \mathbf{A} + \mathbf{L}^0\mathbf{C}\right)^{-1}\mathbf{E} = \mathbf{e}_2, \quad (30)$$

where $\mathbf{e}_1$, $\mathbf{e}_2$ are such vectors, that $\mathbf{C}_r\mathbf{e}_1 = 0$, $\mathbf{C}_r\mathbf{e}_2 \neq 0$. The matrix $\mathbf{L}^0$ can be received from the following system of linear equations:

$$\mathbf{L}^0\mathbf{C}\mathbf{e}_1 = \mathbf{H} + \mathbf{A}\mathbf{e}_1 - \mathbf{e}_1, \mathbf{L}^0\mathbf{C}\mathbf{e}_2 = \mathbf{E} + \mathbf{A}\mathbf{e}_2 - \mathbf{e}_2. \quad (31)$$

Choice of the vectors $\mathbf{e}_1$, $\mathbf{e}_2$ is not trivial and is described below in details.

The matrices $\Psi_0 = \mathbf{L}(e^{j\omega_0 T_s})$, $\mathbf{L}^0 = \mathbf{L}(1)$ can be received from eqns (29) and (31) and the final step of the problem solution is the synthesis of the transfer matrix $\mathbf{L}(z)$. One of the ways to construct it is repeated modal synthesis procedure [4], but there is a more convenient approach based on the synthesis of the fault detection observer in multipurpose structure, described, e.g. in Veremey [5], [6]:

$$\begin{aligned} &\hat{\mathbf{x}}[k+1] = \mathbf{A}\hat{\mathbf{x}}[k] + \mathbf{B}\delta[k] + \mathbf{v}[k], \\ &\mathbf{z}[k+1] = \mathbf{A}\mathbf{z}[k] - \mathbf{v}[k] + \mathbf{L}_{\mathbf{z}}(\mathbf{e}_{\mathbf{y}} - \mathbf{C}\mathbf{z}), \\ &\mathbf{v} = \mathbf{L}_0\mathbf{C}\mathbf{z} + \xi,\ \xi = \mathbf{K}(z)(\mathbf{e}_{\mathbf{y}} - \mathbf{C}\mathbf{z}), \end{aligned} \tag{32}$$

where $\mathbf{z}$ is a state space vector of the auxiliary observer, estimating the dynamic error $\mathbf{e}_{\mathbf{x}}$; $\mathbf{L}_0$, $\mathbf{L}_z$ are given matrices, such that the roots of the polynomials $\Delta_0 = \det(\mathbf{A} - \mathbf{L}_0\mathbf{C})$, $\Delta_{\mathbf{z}} = \det(\mathbf{A} - \mathbf{L}_{\mathbf{z}}\mathbf{C})$ are located in open unit disk, and $\mathbf{K}(z)$ represents a dynamical corrector, providing desired filtering features. Similarly to Veremey [6], we can demonstrate that characteristic polynomial of the closed-loop system eqns (5) and (32) is a product of the polynomials $\Delta_0(z)$, $\Delta_z(z)$, and characteristic polynomial $\Delta_{\mathbf{K}}(z)$ of the corrector, that makes possible to easily guarantee desired degree of stability. Eqn (32) can be transformed to the form

$$\begin{aligned} &\mathbf{v} = \mathbf{T_{11}}(z)\mathbf{e_y} + \mathbf{T_{12}}(z)\xi, \\ &\zeta = \mathbf{T_{21}}(z)\mathbf{e_y} + \mathbf{T_{22}}(z)\xi, \\ &\xi = \mathbf{K}(z)\zeta, \zeta = \mathbf{e_y} - \mathbf{C}\mathbf{z}, \end{aligned} \tag{33}$$

where

$$\mathbf{T}(z) = \begin{pmatrix} \mathbf{T}_{11}(z) & \mathbf{T}_{12}(z) \\ \mathbf{T}_{21}(z) & \mathbf{T}_{22}(z) \end{pmatrix} =$$

$$= \begin{pmatrix} \mathbf{L}_0\mathbf{C} \\ \hline -\mathbf{C} \end{pmatrix} \left(\mathbf{I}_n z - \left(\mathbf{A} - \mathbf{L}_0\mathbf{C} - \mathbf{L}_z\mathbf{C}\right)\right)^{-1} \left(\mathbf{L}_z \mid -\mathbf{I}_n\right) + \left(\begin{array}{c|c} 0 & \mathbf{I}_n \\ \hline \mathbf{I}_m & 0 \end{array}\right).$$

So, the transfer matrices $\mathbf{L}(z)$, $\mathbf{K}(z)$ can be expressed from eqn (33):

$$\mathbf{L}(z) = \mathbf{T}_{11}(z) + \mathbf{T}_{12}(z)\mathbf{K}(z)[\mathbf{I} - \mathbf{T}_{22}(z)\mathbf{K}(z)]^{-1}\mathbf{T}_{21}(z), \tag{34}$$

$$\begin{aligned} \mathbf{K}(z) = &\left[\mathbf{I} + \mathbf{T}_{12}^{-1}(z)\left[\mathbf{L}(z) - \mathbf{T}_{11}(z)\right]\mathbf{T}_{21}^{-1}(z)\mathbf{T}_{22}(z)\right]^{-1} \times \\ &\times \mathbf{T}_{12}^{-1}(z)\left[\mathbf{L}(z) - \mathbf{T}_{11}(z)\right]\mathbf{T}_{21}^{-1}(z). \end{aligned} \tag{35}$$

As a result, we can calculate $\mathbf{K}(e^{j\omega_0 T_s})$, $\mathbf{K}(1)$, providing the equalities $\mathbf{L}(y^{j\omega_0 T_s}) = \Psi_0$, $\mathbf{L}(1) = \mathbf{L}_0$ and the desired dynamics for zero and ω_0 frequencies respectively.

Note, that if roots z_i of the polynomial $\Delta_{\mathbf{K}}(z)$ meet the condition $|z_i| \le r$ then $\Delta_{\mathbf{K}}(z)$ can be parameterized $\Delta_{\mathbf{K}}(z) = \Delta^*(z, \gamma)$ in accordance to the discrete theorem of the polynomial roots' allocation [8]:

$$\Delta^*(z,\gamma)=\begin{cases}\tilde{\Delta}^*(z,\gamma), \text{if n divides 2},\\ (z+a_{d+1}(\gamma,\alpha_{st}))\tilde{\Delta}^*(z,\gamma), \text{if n does not divides 2},\end{cases}$$
$$\tilde{\Delta}^*(z,\gamma)=\prod_{i=1}^{p}\left(z^2+a_i^1(\gamma,\alpha_{st})z+a_i^0(\gamma,\alpha_{st})\right), p=[n/2], \tag{36}$$

where

$$a_i^1(\gamma,\alpha_{st})=-r(\exp(-\frac{\gamma_{i1}^2}{2}-\sqrt{\frac{\gamma_{i1}^2}{4}-\gamma_{i2}^2})+\exp(-\frac{\gamma_{i1}^2}{2}+\sqrt{\frac{\gamma_{i1}^2}{4}-\gamma_{i2}^2})),$$
$$a_i^0(\gamma,\alpha_{st})=r^2\exp(-\gamma_{i1}^2),\ a_{p+1}(\gamma,r)=r\exp(-\gamma_{d0}^2),$$
$$\gamma=\{\gamma_{11},\gamma_{12},\gamma_{21},\gamma_{22},\dots,\gamma_{p1},\gamma_{p2},\gamma_{p0}\}.$$

Note that nonzero coordinates of the vectors $\mathbf{e}_1$, $\mathbf{e}_2$ can be used as the parameters too:

$$\mathbf{e}_1=\mathbf{e}_1(\gamma_1),\ \mathbf{e}_2=\mathbf{e}_2(\gamma_2),$$

which means that the transfer matrix $\mathbf{L}(z)$ and, consequently, the functional J can be considered as function $J=J(\tilde{\gamma})$ of the parameter vector $\tilde{\gamma}=(\gamma,\gamma_1,\gamma_2)$ and maximized with application of any numerical method, e.g. Nelder–Mead algorithm.

5 NUMERICAL EXAMPLE

Let us illustrate the practical implementation of the proposed approach by the example of eqn (1), describing the horizontal motion of the marine ship with the constant speed and having the following parameters:

$$\mathbf{A}=\begin{pmatrix}0.9064 & 0.063 & 0\\ 0.0048 & 0.9283 & 0\\ 0 & 0.1 & 1\end{pmatrix},\ \mathbf{B}=\mathbf{E}=\begin{pmatrix}0.00196\\ 0.00160\\ 0\end{pmatrix},$$

$$\mathbf{H}=\begin{pmatrix}0.041\\ 0.00076\\ 0\end{pmatrix},\ \mathbf{C}=\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}, T_s=0.1\,.$$

The state vector $\mathbf{x}$ consists of drift angle β, yaw angular rate ω, and yaw φ. The external disturbance $d(t)$ can be presented by eqn (2) as

$$d[k]=\sin(\omega_0 kT_s)+0.1\sin(0.9\omega_0 kT_s)+0.1\sin(1.1\omega_0 kT_s),\ \omega_0=0.45.$$

Let us consider the observer, having the parameters $\mathbf{L}_z$, $\mathbf{L}_0$

$$\mathbf{L}_z=\begin{pmatrix}1.1529 & 0.3848\\ 3.2685 & 5.1390\\ 0.3854 & 1.5249\end{pmatrix},\ \mathbf{L}_0=\begin{pmatrix}0.3570 & 0.3494\\ 0.0977 & 0.2089\\ 0.3166 & 0.8619\end{pmatrix},$$

and the designed filter

$$\mathbf{K}(s) = \frac{1}{z^4 - 1.562z^3 + 0.6098z^2} \left(\begin{array}{l} 20403\, z^2 - 40713\, z + 20353 \\ 1173\, z^2 + -2367\, z + 1196 \\ -0.413\, z^2 + 118.9\, z - 117.3 \end{array} \right.$$

$$\left. \begin{array}{l} 16977\, z^2 - 33830\, z + 16888 \\ \ldots\ -3173\, z^2 + 6299\, z - 3132 \\ -1.6222z^2 - 3.2623z - 16433 \end{array} \right).$$

Fig. 1 represents frequency responses A_{rd} and A_{rf} of the transfer functions F_{rd} and $\mathbf{F_{rf}}$ (eqn (7)). Note that the response A_{rd} is close to 0 on zero frequency and in the area of ω_0, i.e. effect of the external disturbance $d(t)$ is successfully suppressed. Fault detection process is presented in Fig. 2: the constant part of the external disturbance intensifies at 50 s, the fault occurs at 150 s and is successfully detected.

6 CONCLUSION

A novel special approach in frequency domain to H_2-optimal discrete fault detection observers design is presented in this paper. This scheme is applicable to various marine vehicles, affected by external disturbance with isolatable central frequencies.

Solution of the H_2-optimization problem is not unique because of using of spectral approach, that allows to guarantee additional properties of the closed-loop system, e.g. integral action. Also, we used a dynamical correction of the basic observer with the multipurpose structure that simplifies resetting of the observer. Effectiveness of the proposed approach is demonstrated by the numerical example.

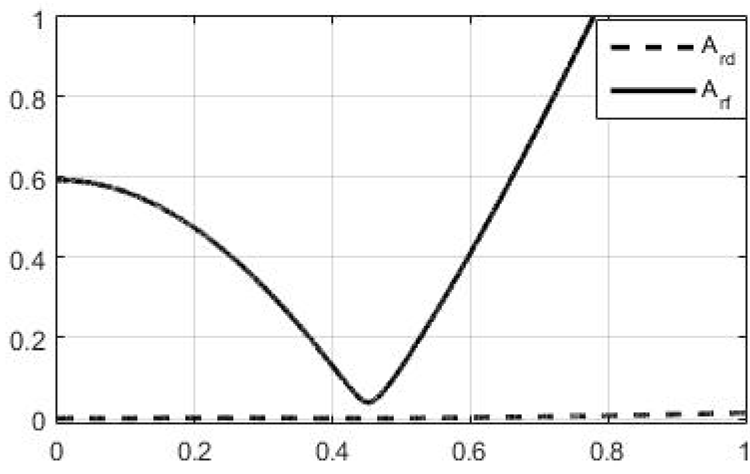

Figure 1: Frequency responses of the transfer functions F_{rd} and F_{rf}.

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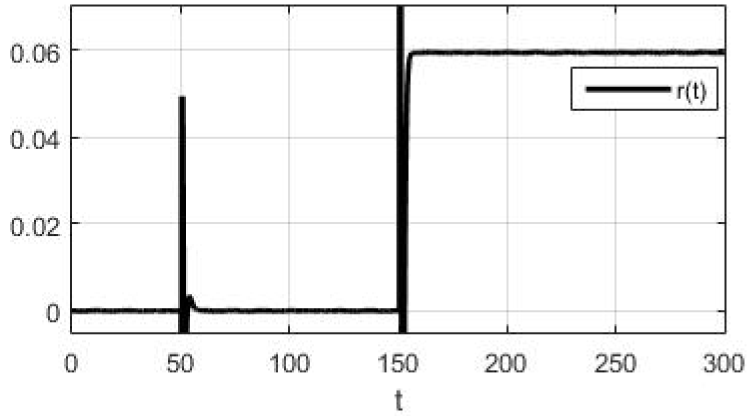

Figure 2: Fault detection process.

Sincerely, the described approach has also some negative features. Firstly, there are no universal conditions for integral action property. Secondly, duration of the optimization procedure can be unacceptable and significantly depends on initial values of parameters. Overcoming of these problems is the main direction of the future research.

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