

# DREM-BASED ONLINE IDENTIFICATION OF A SURFACE VESSEL DYNAMIC MODEL

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## ABSTRACT

The paper is devoted to the online identification of the nonlinear model of surface vessel dynamics. The mathematical formulation of the maritime ships is complicated due to the existence of nonlinear hydrodynamic forces and moments that are associated with vessel dynamics. For this reason, the coefficients of the model are not known, nor do they require clarification. The identification algorithm is based on the method of the dynamic regressor extension and mixing (DREM). On the first step using parameterisation, the regression model is obtained, where regressor and regressand depend on measurable signals: linear velocities in surge, the linear velocity in sway, the angular velocity in yaw and the rudder angle. At the second step, the new regression model is obtained using linear stable filters and delays. DREM allows replacing the regression model of the  $n$ th order with  $n$  first order regression models and estimate parameters separately. Finally, parameters are estimated by the standard gradient descent method. The efficiency of the proposed approach is demonstrated through a set of numerical simulations.

*Keywords: system identification, ship manoeuvring, 3 degrees of freedom, DREM.*

## 1 INTRODUCTION

The mathematical model of the maritime ship is complicated due to the nonlinear nature of hydrodynamic forces and moments that are associated with vessel dynamics, structural and parametric uncertainty, the presence of external disturbances. Moreover, some parameters are changing over time, for example, which are related to the loading of the vessel.

This is the reason why the system identification methods are playing an important role in the modelling of ship maneuvering motion. Usually, they base on free-running model tests or full-scale trials.

Various methods are used to identify the hydrodynamic coefficients of the surface vessel mathematical model: model reference method (Hayes [1]), extended Kalman filter method (Abkowitz [2], Herrero and Gonzalez [3]), recursive prediction error method (Zhou and Blanke [4]), least square method (Rhee et al. [5]), frequency domain identification method (Perez and Fossen [6]), neural network (Wang et al. [7]), etc.

In this paper, we propose an online identification algorithm based on dynamic extension and mixing and standard gradient descent methods. It allows estimating all parameters separately and provides global convergence of the estimation error to zero in the absence of noise.

## 2 MOTIVATION

### 2.1 Mathematical model

Consider the 3 DOF horizontal plane models for manoeuvring, which are based on the rigid-body kinetics:

$$M_{RB}\dot{v} + C_{RB}(v)v = \tau_{RB}(v, \delta), \quad (1)$$



where  $v = [u \ v \ r]^T$  is the generalized velocity, where  $u$  is the linear velocity in surge,  $v$  is the linear velocity in sway,  $r$  is the angular velocity in yaw,  $\delta$  is the rudder angel;  $M_{RB}$  is the rigid-body inertia matrix,  $C_{RB}(v)$  is a matrix of rigid-body Coriolis and centripetal forces and  $\tau_{RB}$  is a vector of the external forces and moments. A similar mathematical model of the vessel is considered in Sotnikova and Veremey [8], where the problem of dynamic positioning is solved.

Expanding the hydrodynamic forces and moments by 3rd-order truncated Taylor expansions about the steady state condition  $u = u_0$ , according to Abkowitz [2], the eqn (1) can be represented as:

$$\begin{pmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_g - Y_{\dot{r}} \\ 0 & mx_g - Y_{\dot{r}} & I_z - N_{\dot{r}} \end{pmatrix} \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} X \\ Y \\ N \end{pmatrix}, \quad (2)$$

where  $m$  is the mass of the ship,  $I_z$  is the moment of inertia about the vertical axes,  $x_g$  is the longitudinal coordinate of the ship's center of gravity,  $X$  and  $Y$  are the longitudinal and lateral hydrodynamic force components,  $N$  is the hydrodynamic yaw moment.

Replacing the hydrodynamic forces and moments in eqn (2) with 3rd-order truncated Taylor expansions, the functions  $X = X(u, v, r, \delta)$ ,  $Y = Y(u, v, r, \delta)$  and  $N = N(u, v, r, \delta)$  can be rewritten as :

$$X = X_u u + X_{uu} u^2 + X_{uuu} u^3 + X_{vv} v^2 + X_{rr} r^2 + X_{rv} r v + X_{\delta\delta} \delta^2 + X_{u\delta\delta} u \delta^2 + X_{v\delta} v \delta + X_{uv\delta} u v \delta, \quad (3)$$

$$Y = Y_v v + Y_r r + Y_{vvv} v^3 + Y_{vvr} v^2 r + Y_{vu} v u + Y_{ru} r u + Y_{\delta} \delta + Y_{\delta\delta\delta} \delta^3 + Y_{u\delta} u \delta + Y_{uu\delta} u^2 \delta + Y_{v\delta\delta} v \delta^2 + Y_{vv\delta} v^2 \delta + (Y_0 + Y_{0u} u + Y_{0uu} u^2), \quad (4)$$

$$N = N_v v + N_r r + N_{vvv} v^3 + N_{vvr} v^2 r + N_{vu} v u + N_{ru} r u + N_{\delta} \delta + N_{\delta\delta\delta} \delta^3 + N_{u\delta} u \delta + N_{uu\delta} u^2 \delta + N_{v\delta\delta} v \delta^2 + N_{vv\delta} v^2 \delta + (N_0 + N_{0u} u + N_{0uu} u^2). \quad (5)$$

The constant matrix  $M_{RB}$  is invertible, therefore eqn (2) can be rewritten in the explicit form:

$$\dot{u} = (m - X_{\dot{u}})^{-1} X, \quad (6)$$

$$\dot{v} = \frac{1}{\det M_{RB}^2} [(I_z - N_{\dot{r}}) Y - (mx_g - Y_{\dot{r}}) N], \quad (7)$$

$$\dot{r} = \frac{1}{\det M_{RB}^2} [-(mx_g - Y_{\dot{r}}) Y + (m - Y_{\dot{v}}) N], \quad (8)$$

where  $\det M_{RB}^2 = (m - Y_{\dot{v}})(I_z - N_{\dot{r}}) - (mx_g - Y_{\dot{r}})^2$ . In (6)–(8) we assume that all parameters of the model are unknown. The main objective is to construct estimates  $\hat{\theta}_i$  for unknown parameters vectors  $\theta_i$  of the described ship model, such that the norm of the estimation error converges to zero:

$$\lim_{t \rightarrow \infty} \|\tilde{\theta}_i(t)\| = 0,$$

where  $\|\cdot\|$  is the Euclidean norm,  $\tilde{\theta}_i(t) = \theta_i - \hat{\theta}_i(t)$ ,  $i = \overline{1,3}$ .

### 3 IDENTIFICATION

#### 3.1 Linear regression model

Let us rewrite the surface vessel dynamic model eqns (6)–(8) in a linear regression form:

$$y = \varphi^T \theta, \quad (9)$$

where  $y = (y_1 \ y_2 \ y_3)^T$  is the regressand,  $\varphi = (\varphi_{ij})$ ,  $i = \overline{1, n}$ ,  $j = \overline{1, 3}$  is the regressor,  $\theta = (\theta_1 \ \theta_2 \ \dots \ \theta_n)^T$  is the unknown parameters vector,  $n$  is the count of unknown parameters of the regression model.

The vector  $\theta$  of the unknown parameters has the following form:

$$\begin{aligned} \theta &= \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}_{40 \times 1}, \\ \theta_1 &= \frac{1}{m - X_u} \theta_X, \\ \theta_2 &= \frac{I_z - N_r}{\det M_{RB}^2} \theta_Y - \frac{m x_g - Y_r}{\det M_{RB}^2} \theta_N, \\ \theta_3 &= -\frac{m x_g - Y_r}{\det M_{RB}^2} \theta_Y + \frac{m - Y_v}{\det M_{RB}^2} \theta_N, \end{aligned} \quad (10)$$

where vectors  $\theta_X$ ,  $\theta_Y$  and  $\theta_N$  are unknown hydrodynamic parameters from eqns (3)–(5):

$$\begin{aligned} \theta_X &= (X_u \ X_{uu} \ X_{uuu} \ X_{vv} \ X_{rr} \ X_{rv} \ X_{\delta\delta} \ \dots \\ &\quad X_{u\delta\delta} \ X_{v\delta} \ X_{uv\delta})^T, \\ \theta_Y &= (Y_v \ Y_r \ Y_{vvv} \ Y_{vvr} \ Y_{vu} \ Y_{ru} \ Y_{\delta} \ \dots \\ &\quad Y_{\delta\delta\delta} \ Y_{u\delta} \ Y_{uu\delta} \ Y_{v\delta\delta} \ Y_{vv\delta} \ Y_0 \ Y_{0u} \ Y_{0uu})^T, \\ \theta_N &= (N_v \ N_r \ N_{vvv} \ N_{vvr} \ N_{vu} \ N_{ru} \ N_{\delta} \ \dots \\ &\quad N_{\delta\delta\delta} \ N_{u\delta} \ N_{uu\delta} \ N_{v\delta\delta} \ N_{vv\delta} \ N_0 \ N_{0u} \ N_{0uu})^T. \end{aligned}$$

From eqns (6)–(8) we can find the regressor matrix  $\varphi$  and the regressand  $y$ :

$$\varphi = \begin{pmatrix} \varphi_X & \mathbf{0}_{10 \times 1} & \mathbf{0}_{10 \times 1} \\ \mathbf{0}_{15 \times 1} & \varphi_Y & \mathbf{0}_{15 \times 1} \\ \mathbf{0}_{15 \times 1} & \mathbf{0}_{15 \times 1} & \varphi_N \end{pmatrix}_{40 \times 3}, \quad (11)$$

$$\begin{aligned} \varphi_X &= (u \ u^2 \ u^3 \ v^2 \ r^2 \ rv \ \delta^2 \ u\delta^2 \ v\delta \ uv\delta)^T, \\ \varphi_Y &= \varphi_N = (v \ r \ v^3 \ v^2r \ vu \ ru \ \delta \ \dots \\ &\quad \delta^3 \ u\delta \ u^2\delta \ v\delta^2 \ v^2\delta \ 1 \ u \ u^2)^T, \\ y &= \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{pmatrix}_{3 \times 1}. \end{aligned} \quad (12)$$

Notice, that the functions  $u(t)$ ,  $v(t)$ ,  $r(t)$  and  $\delta(t)$  are measurable signals, but the derivations  $\dot{u}(t)$ ,  $\dot{v}(t)$ ,  $\dot{r}(t)$  are unknown. This can be handled by special filtering. Let us consider the linear stable operator  $H_f(\cdot)$ , for instance, a simple exponentially stable LTI (Linear time-invariant) filter of the form:

$$H_f(p) = \frac{\alpha}{p + \alpha}, \quad (13)$$



where  $\alpha > 0$  is the adjustable parameter,  $p \equiv \frac{d}{dt}$  is the differential operator.

Applying operators  $H_f(\cdot)$  to the equation (9) yields:

$$\begin{aligned} y_f(t) &= \varphi_f^T(t)\theta, \\ y_f(t) &= [H_f(y)](t) = \frac{\alpha}{p+\alpha}y(t) = \frac{ap}{p+\alpha} \begin{pmatrix} u(t) \\ v(t) \\ r(t) \end{pmatrix}, \\ \varphi_f(t) &= [H_f(\varphi)](t) = \frac{\alpha}{p+\alpha}\varphi(t). \end{aligned} \quad (14)$$

Now, the components of the regressor matrix  $\varphi_f$  and the regressand  $y_f$  are measurable signals.

Note that the matrices  $\varphi$  and  $\varphi_f$  have a block-diagonal form. Therefore, the regression model (eqn (14)) can be divided into three independent regression models for estimating the parameters  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  respectively:

$$y_{fi}(t) = \varphi_{fi}^T(t)\theta_i, \quad (15)$$

where  $i = \overline{1,3}$ .

### 3.2 Dynamic regressor extension and mixing procedure

The DREM procedure generates 40 new, one-dimensional, independent regression models. Consider the estimation method for  $\theta_1$  (from the first equation of the system (eqn (15))). Parameters  $\theta_2$  and  $\theta_3$  are estimating similarly.

Let us introduce delay operators  $[H_{d_j}(\cdot)](t) = (\cdot)(t - d_j)$ , where  $d_j > 0$  is the delay value. Applying delay operators to the first regressor equation (15) gives:

$$\begin{aligned} y_{d_j}(t) &= \varphi_{d_j}^T(t)\theta_1, \\ y_{d_j}(t) &= [H_{d_j}(y_{f1})](t) = y_{f1}(t - d_j), \\ \varphi_{d_j}(t) &= [H_{d_j}(\varphi_x)](t) = \varphi_x(t - d_j). \end{aligned} \quad (16)$$

Using different delays  $d_j$ ,  $j = \overline{1, \dim\theta_1}$  we can construct the extended system of equations:

$$\begin{aligned} Y(t) &= \Phi(t)\theta_1, \\ Y(t) &= (y_{d_1}(t) \ y_{d_2}(t) \ \dots \ y_{d_{10}}(t))_{10 \times 1}^T, \\ \Phi(t) &= \begin{pmatrix} \varphi_{d_1}^T(t) \\ \varphi_{d_2}^T(t) \\ \vdots \\ \varphi_{d_{10}}^T(t) \end{pmatrix}_{10 \times 10}. \end{aligned} \quad (17)$$

Multiplying eqn (17) by the adjunct matrix of  $\Phi(t)$  gives:

$$\begin{aligned} \Psi(t) &= \Phi(t)\Theta, \\ \Theta &= \theta_1, \\ \Psi(t) &= adj\{\Phi(t)\}Y(t), \\ \Phi(t) &= adj\{\Phi(t)\}\Phi(t) = \det\{\Phi(t)\}. \end{aligned} \quad (18)$$



Note that the system eqn (18) contains ten independent scalar equations:

$$\Psi_i(t) = \phi(t)\theta_i, \quad (19)$$

where  $i = \overline{1,10}$ .

### 3.3 Parameters estimation

The estimates of the parameters  $\theta_i$  can be obtained using standart gradient descent method from the scalar regression model (eqn (19)):

$$\hat{\theta}_i(t) = \gamma_i \phi(t) (\Psi_i(t) - \phi(t)\hat{\theta}_i(t)), \quad (20)$$

where  $\hat{\theta}_i(t)$  is the estimate of the parameter  $\theta_i$ ,  $\gamma_i > 0$  is the adaptation gain,  $i = \overline{1,10}$ .

Consider the expression for the parameter estimation error  $\tilde{\theta}_i(t) = \hat{\theta}_i(t) - \theta_i$ ,  $i = \overline{1,10}$ :

$$\dot{\tilde{\theta}}_i(t) = -\gamma_i \phi^2(t)\tilde{\theta}_i(t). \quad (21)$$

The solution of the differential eqn (21) has the following form:

$$\tilde{\theta}_i(t) = \tilde{\theta}_i(t_0)e^{-\gamma_i \int_{t_0}^t \phi^2(s)ds}. \quad (22)$$

The zero equilibrium of the linear time-varying system (eqn (21)) is asymptotically stable if  $\phi(t) \notin \mathcal{L}_2$ , that is:

$$\lim_{t \rightarrow \infty} \int_{t_0}^t \phi^2(s)ds = \infty, \quad (23)$$

then  $\lim_{t \rightarrow \infty} \tilde{\theta}_i(t) = 0$ ,  $i = \overline{1,10}$ , in otherwords, the norm of the estimation errors converges to zero.

## 4 SIMULATIONS

In this section, we present simulation results that illustrate the efficiency of the proposed estimation algorithm. All simulations have been performed in MATLAB-Simulink.

A model of the Mariner class vessel  $L = 160.93$  m is taken as the simulations object. The principal dimensions of the ship are given in Table 1.

Table 1: Principal dimensions of the ship.

Parameter	Magnitude
Length ( $L$ )	160.93 m
Non-dimensional mass ( $m$ )	$798 \times 10^{-5}$
Non-dimensional inertia in yaw ( $I_z$ )	$39.2 \times 10^{-5}$
x-coordinate of centre of gravity ( $x_G$ )	-0.023 m

The rudder angle  $\delta$  is plotted in Fig. 1.

The hydrodynamic coefficients obtained by Chislett and Stroem-Tejsen (1965, Planar Motion Mechanism Tests and Full-Scale Steering and Maneuvering Predictions for a Mariner Class Vessel, Technical Report Hy-5, Hydro- and Aerodynamics Laboratory, Lyngby, Denmark), as given in Table 2, are used in the simulation.

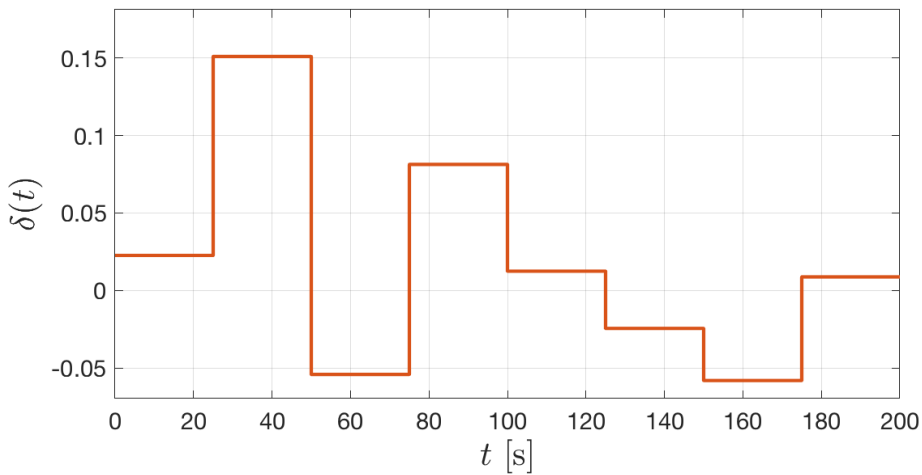
Figure 1: The rudder angle  $\delta(t)$ .

Table 2: Magnitudes of the hydrodynamic coefficients.

Accel coef	Magnitude ( $\times 10^{-5}$ )	X- coef	Magnitude ( $\times 10^{-5}$ )	Y- coef	Magnitude ( $\times 10^{-5}$ )	N- coef	Magnitude ( $\times 10^{-5}$ )
$X_{\ddot{u}}$	-42	$X_u$	-184	$Y_v$	-1160	$N_v$	-264
$Y_{\ddot{v}}$	-748	$X_{uu}$	-110	$Y_r$	-499	$N_r$	-166
$Y_{\ddot{r}}$	-9.354	$X_{uuu}$	-215	$Y_{vvv}$	-8078	$N_{vvv}$	1636
$N_{\ddot{v}}$	4.646	$X_{vv}$	-899	$Y_{vvr}$	15356	$N_{vvr}$	-5483
$N_{\ddot{r}}$	-43.8	$X_{rru}$	18	$Y_{vu}$	-1160	$N_{vu}$	-264
		$X_{\delta\delta}$	-95	$Y_{ru}$	-499	$N_{ru}$	-166
		$X_{u\delta\delta}$	-190	$Y_{\delta}$	278	$N_{\delta}$	-139
		$X_{rv}$	798	$Y_{\delta\delta\delta}$	-90	$N_{\delta\delta\delta}$	45
		$X_{v\delta}$	93	$Y_{u\delta}$	556	$N_{u\delta}$	-278
		$X_{uv\delta}$	93	$Y_{uu\delta}$	278	$N_{uu\delta}$	-139
				$Y_{v\delta\delta}$	-4	$N_{v\delta\delta}$	13
				$Y_{vv\delta}$	1190	$N_{vv\delta}$	-489
				$Y_0$	-4	$N_0$	3
				$Y_{0u}$	-8	$N_{0u}$	6
				$Y_{0uu}$	-4	$N_{0uu}$	3

The DREM parameters are the following:  $\alpha = 10$ ,  $d_i = 4i$ ,  $i = \overline{1,10}$  for estimation  $\theta_1$  and  $d_i = 4i$ ,  $i = \overline{1,15}$  for  $\theta_2$  and  $\theta_3$ . Plots for surge speed  $u(t)$ , sway speed  $v(t)$  and yaw rate  $r(t)$  are depicted in Fig. 2.

In Fig. 3 the plot for Euclidean norm of the parameter estimation errors  $\tilde{\theta}_i(t) = \theta_i - \hat{\theta}_i(t)$ ,  $i = \overline{1,3}$  is shown. After transition period the magnitude of the parameter estimation error is  $10^{-8}$ , while the order of the estimated parameters is  $10^{-3}$ .

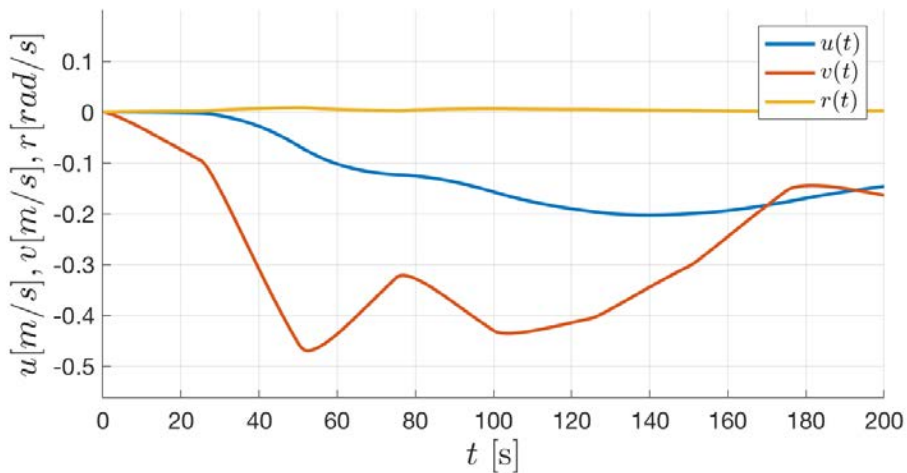


Figure 2: Surge speed  $u(t)$ , sway speed  $v(t)$  and yaw rate  $r(t)$ .

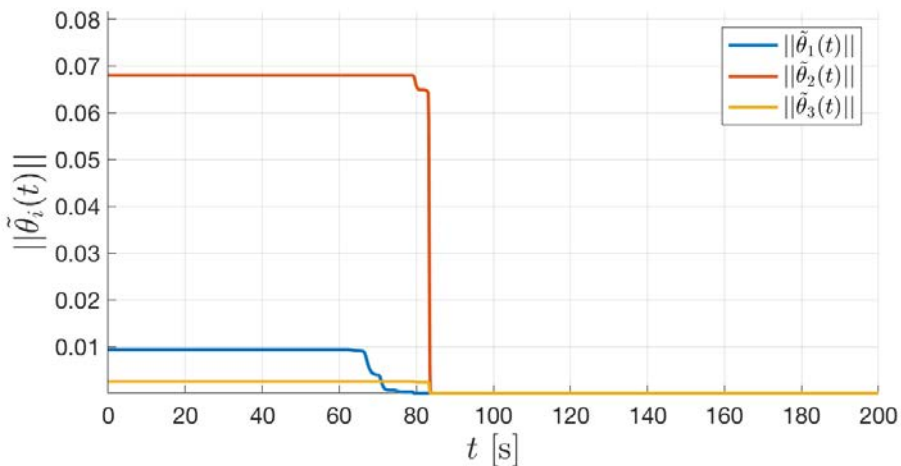


Figure 3: The norm of the parameter estimation errors  $\tilde{\theta}_i(t)$ .

## 5 CONCLUDING REMARKS AND FUTURE RESEARCH

In this paper we have introduced a new online parametric estimator for the 3 DOF horizontal plane models for manoeuvring, where the hydrodynamic forces and moments are replaced with 3rd-order truncated Taylor expansion.

The derivative of the generalized velocity is not measurable, and filtration technic has been applied to obtain a linear regression model with measurable regressor and regressand. Due to the high order of the obtained models, standard gradient approach cannot provide acceptable performance. Moreover, it requires hardly verifiable persistent excitation condition.

In this paper, we have proposed to apply the dynamic regressor extension and mixing (DREM) method to the constructed  $n$ th order regression model and to replace it with  $n$  the independent first order models. To estimate parameters, the standard gradient descent method is used. In scalar case to guarantee convergence of the estimation error to zero the new regressor should not be square integrable. The efficiency of the proposed approach is demonstrated through a set of numerical simulations.

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