Mechanistic model for four-phase sand/water/oil/gas stratified flow in horizontal pipes

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Abstract

Understanding the physics of multiphase flow as one of the most dominant flow regimes is posing many challenges to researchers in the field of fluid dynamics. In particular, in petroleum industries where production flow is always multiphase, pipeline engineers and operators have to have a clear understanding of the multiphase behaviour when transferring crude oil, associated gas, produced water and solid particles from wellheads to processing facilities. Many researchers have simplified the treatment of complex flows through considering them as the flows of two or three different phases. A comprehensive two-phase liquid/gas model developed by Xiao *et al.* in 1990, a two-phase solid/liquid model developed by Doron and Barnea in 1993 or a three-phase liquid/liquid/gas model by Taitel *et al.* in 1995 are just a few examples of these approaches. In this paper, to the best of the authors' knowledge, a mechanistic model has been developed to model the four-phase sand/water/oil/gas flow for a stratified flow regime in a horizontal pipe for the first time. The developed model takes into account some aspects of the three-layer model, which was introduced by Doron and Barnea in 1993 and the three-phase liquid/liquid/gas model of Taitel *et al*. This model considers the entire spectrum of phases comprising a stationary sand bed, a moving sand bed, water, and oil and gas layers, and the flow regime has been modelled by a set of 12 non-linear equations. The method for solving this system of non-linear equations is discussed, with an aim to calculate the phase holdups, phase velocities and pressure loss.
Keywords: mechanistic model, multiphase, four-phase, sand transport, stratified, settling velocity, suspending velocity.

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1 Introduction

The presence of different phases in the multiphase flow-line makes it extremely complicated to model. While other modelling techniques used to simulate single- and two-phase flow regimes have been developed in recent decades, understanding the physics governing the four-phase flow still poses great challenges to researchers. It is surprising to know that very little research has been conducted so far to explain and formulate the behaviour of four-phase liquid/liquid/gas/solid flow. This flow regime is of particular interest in petroleum industries because the entire flow regime from the reservoir to the processing facilities falls into this category.

Untreated reservoir production normally consists of oil, produced water, associated gas and solid, mainly in the form of sand particles. Sand particles in the pipeline should be kept moving to mitigate against the formation of a stationary sand bed at the pipe bottom. This stationary sand bed can jeopardise the integrity and performance of the pipeline by partially or completely blocking the pipe and is also a corrosion risk due to Microbially Induced Corrosion (MIC). Early investigations into liquid/sand flow were all based on visual observations in laboratories. Vocaldo and Charles [1] and Parazonka *et al.* [2] observed and classified several flow patterns in liquid/solid horizontal flow.

The classification terms that were used most commonly in such researches were "stationary bed", "moving bed", "heterogeneous suspension" and "pseudo-homogenous suspension". Transition criteria between these flow patterns was of particular interest, hence several correlation models were developed based on the experimental data to predict critical sand suspending velocity and critical sand deposition velocity, for example, Oroskar and Turian's [3] correlation for single-phase flow and Salama's [4] correlations for two-phase flow.

Theoretical models began to be developed and evolved through the work of other researchers; for example, the two-layer model of Wilson [5], Televantos *et al*. [6], Hsu *et al.* [7] and many others. Doron *et al*. [8] developed a two-layer model to predict flow patterns and pressure drop. Doron and Barnea [9] then modified their two-layer model to predict a stationary sand bed layer and introduced the first three-layer model for liquid/solid flow.

In the three-layer model, the sand bed is divided into two layers; the stationary sand bed and the moving sand bed. This assumption was confirmed by experimental results, showing that, while the upper layer of the sand bed is moving, the lower layer can be stationary [9]. The three-layer model has proved to be a sophisticated model for two-phase flow, capable of predicting geometrical properties for each layer, including liquid and sand holdups, phase velocities and pressure drop for the entire field. Results were in agreement with experimental data [9].

However, no model has ever been developed to consider the four-phase flow of liquid/liquid/gas/sand, which is dominant in petroleum industries. In this paper, for the first time (to best of the authors' knowledge) a four-phase oil/water/gas/sand flow model has been formulated using the mathematical framework used in Doron's papers (Doron *et al.* [8] and Doron and Barnea [9]).

2 Model description

A model has been developed for considering the stratified flow structure, consisting of five separate layers of gas, oil, water, moving sand bed and stationary sand bed. Figure 1 illustrates this setup.

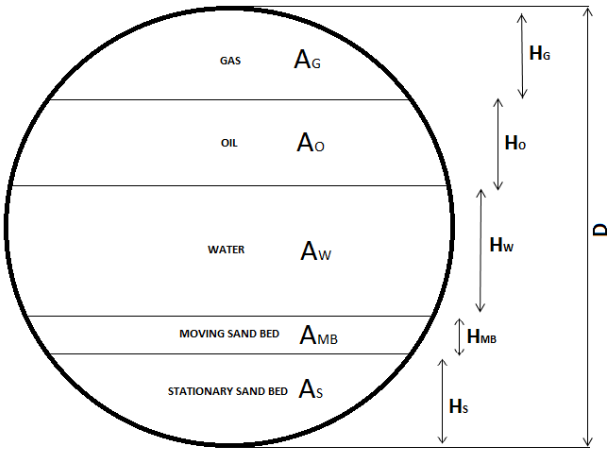

Figure 1: Geometrical illustration of four-phase model.

Water is heavier than oil and flows in the vicinity of the sand bed. If a moving layer of sand bed exists, it will be between the stationary sand bed layer and the water layer. Oil is always on top of the water layer and beneath the gas phase.

"H_i" represents the height of the layer and the subscript "i" is one of the following indexes: stationary sand bed or sand phase (S), moving sand bed layer (MB), water layer or water phase (W), oil layer or oil phase (O), gas layer or gas phase (G).

Each of these layers has interfaces with the pipe wall and with at least one other layer. The middle layers, i.e. moving sand bed, water and oil, each have interfaces with two layers.

To identify these interfaces, "S_i^k" represents the perimeter of interface between field "k" and "i", where field "k" is always on top of field "i", e.g. S_{MB}^W is the perimeter of the interface between the water layer and the moving sand bed layer. "S_i" is the perimeter of interface between the field "i" and the pipe wall. Subscripts "k" and "i" can be one of the five indexes for each layer, shown in figure 1.

"A_i" is the cross-sectional area for layer "i", and "A" is the pipe's cross-sectional area. All these geometrical parameters, including "A_i", "S_i^k" and "S_i", are used later to calculate the hydraulic diameter for each layer, are functions of "H_i", and can be described by the following equations:

$$A_s = \frac{1}{2}.\left(\frac{D}{2}\right)^2.\{2.\cos^{-1}\left[1-\frac{2.H_s}{D}\right]-\sin\left[2.\cos^{-1}\left[1-\frac{2.H_s}{D}\right]\right]\} \quad (1)$$

$$A_{MB} = \frac{1}{2}.\left(\frac{D}{2}\right)^2.\{2.\cos^{-1}\left[1-\frac{2.(H_s+H_{MB})}{D}\right] - \sin\left[2.\cos^{-1}\left[1-\frac{2.(H_s+H_{MB})}{D}\right]\right]\} - A_s \quad (2)$$

$$A_W = \frac{1}{2}.\left(\frac{D}{2}\right)^2.\{2.\cos^{-1}\left[1-\frac{2.(H_s+H_{MB}+H_W)}{D}\right] - \sin\left[2.\cos^{-1}\left[1-\frac{2.(H_s+H_{MB}+H_W)}{D}\right]\right]\} - (A_S+A_{MB}) \quad (3)$$

$$A_O = \frac{1}{2}.\left(\frac{D}{2}\right)^2.\{2.\cos^{-1}\left[1-\frac{2.(H_s+H_{MB}+H_W+H_O)}{D}\right] - \sin\left[2.\cos^{-1}\left[1-\frac{2.(H_s+H_{MB}+H_W+H_O)}{D}\right]\right]\} - (A_S+A_{MB}+A_W) \quad (4)$$

$$A_G = \frac{\pi.D^2}{4} - (A_S+A_{MB}+A_W+A_O) \quad (5)$$

$$S_S = D.\cos^{-1}\left[1-\frac{2.H_S}{D}\right] \quad (6)$$

$$S_{MB} = D.\left\{\cos^{-1}\left[1-\frac{2.(H_S+H_{MB})}{D}\right] - \cos^{-1}\left[1-\frac{2.H_S}{D}\right]\right\} \quad (7)$$

$$S_W = D.\left\{\cos^{-1}\left[1-\frac{2.(H_S+H_{MB}+H_W)}{D}\right] - \cos^{-1}\left[1-\frac{2.(H_S+H_{MB}}{D}\right]\right\} \quad (8)$$

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$$S_O = D.\left\{\cos^{-1}\left[1 - \frac{2.(H_S + H_{MB} + H_W + H_O)}{D}\right] - \cos^{-1}\left[1 - \frac{2.(H_S + H_{MB} + H_W}{D}\right]\right\} \quad (9)$$

$$S_G = \pi.D - (S_S + S_{MB} + S_W + S_O) \quad (10)$$

$$S_S^{MB} = D.\sin(\cos^{-1}\left[1 - \frac{2.H_s}{D}\right]) \quad (11)$$

$$S_{MB}^{W} = D.\sin(\cos^{-1}\left[1 - \frac{2.(H_s + H_{MB})}{D}\right]) \quad (12)$$

$$S_W^O = D.\sin(\cos^{-1}\left[1 - \frac{2.(H_s + H_{MB} + H_W)}{D}\right]) \quad (13)$$

$$S_O^G = D.\sin(\cos^{-1}\left[1 - \frac{2.(H_s + H_{MB} + H_W + H_O)}{D}\right]) \quad (14)$$

Apart from the stationary sand bed, other layers consist of moving phases; therefore each has a velocity, which is represented by U_i.

3 Model formulation

Since the first introduction of the "Mechanistic" approach by Taitel and Dukler [10], used to predict the behaviour of two-phase liquid/gas flow using physical phenomena instead of experimental correlation, this modelling approach has gained considerable attention among researchers.

One of the main advantages of their model is its simplicity. Taitel *et al.* [11] then used the same approach to develop a model for stratified three-phase liquid/liquid/gas flow. The two-layer model in Doron *et al.* [8] and the three-layer model developed by Doron and Barnea [9] used the same principles as Taitel and Dukler [10] to simplify the momentum conservation equations for each layer.

In this paper, for the first time, we are proposing a set of formulations using a combination of the Taitel *et al.* [11] approach towards liquid/liquid/gas and Doron and Barnea's [9] three-layer model for liquid/sand, to model the entire sand/water/oil/gas field, as detailed in figure 1.

Gas bubbles are assumed to be absent inside the oil or water layers. Oil and water are assumed to be fully separated, hence there is no oil droplet in the water layer and there is no water droplet in the oil layer.

It is further assumed that there is no sand particle in the gas phase [12]. Sand particles are considered to be water wetted and are therefore only transported by the water layer. Also, the entire system is assumed to be isothermal.

The conservation of mass and momentum equations for a four phase flow system are described below.

3.1 Mass conservation

The mass conservation equations for each phase have been developed as per the following:

Sand phase:

$$U_w . C_{S.W} . A_W + U_{MB} . C_{S.MB} . A_{MB} = U_{inlet} . C_{S.inlet} . A \quad (15)$$

Water phase:

$$U_w . (1 - C_{S.W}) . A_W + U_{MB} . (1 - C_{S.MB}) . A_{MB} = U_{inlet} . C_{W.inlet} . A \quad (16)$$

Oil phase:

$$U_O . A_O + U_G . C_{O.G} . A_G = U_{inlet} . C_{O.inlet} . A \quad (17)$$

Gas phase:

$$U_G . (1 - C_{O.G}) . A_G = U_{inlet} . C_{G.inlet} . A \quad (18)$$

In eqns (15) to (18), $C_{i.k}$ is the average volumetric concentration of phase "i" in layer "k", e.g. $C_{S.W}$ is sand concentration in water layer or $C_{O.G}$ is oil droplet volumetric concentration in gas layer. Index "inlet" refers to the pipe inlet conditions. $C_{S.MB}$ – sand particle concentration in the moving bed layer – is assumed to be 0.52 for cubic packing [13].

Using diffusion equations, the average sand concentration in the water layer $C_{S.W}$, in a direction perpendicular to the pipe axis, is:

$$C_{S.W} = \int_{(H_S+H_{MB})}^{(H_S+H_{MB}+H_W)} \{C_{S.MB}.\exp(\frac{\omega.[(H_S+H_{MB})-y]}{\varepsilon})\}\, dy \tag{19}$$

ε and ω are diffusion coefficient and terminal settling velocity respectively and are calculated using the method detailed in Doron *et al.* [8]. U_{MB} is obtained using Doron and Barnea [9] method based on a torque balance for a single sand particle.

$$U_{MB} = \sqrt{\frac{0.779.(\rho_S-\rho_W).g.d.\{C_{S.MB}.\frac{H_{MB}}{d}+(1-C_{S.MB})\}}{\rho_W.C_D}} \tag{20}$$

ρ_S and ρ_W are sand and water densities and d is diameter of single sand particle. C_D is the drag coefficient for single sand particle and is calculated using the method of Bird *et al.* [14]:

$$C_D = \begin{cases} 18.5\, Re_s^{-0.6} & for \quad 0.1 < Re_s < 500 \\ 0.44 & for \quad 500 < Re_s < 2 \times 10^5 \end{cases} \tag{21}$$

Re_s is the Reynolds number for single particle based on a single particle diameter and the terminal settling velocity of a single particle [8]. The oil droplet concentration in the gas phase "$C_{O.G}$" is calculated using Paras and Karabelas' [15] model:

$$C_{O.G} = \int_{(H_S+H_{MB}+H_W+H_O)}^{D} \{\alpha+\beta.\exp(-k\frac{2.y}{D})\}\, dy \tag{22}$$

3.2 Momentum conservation

To calculate the pressure drop "$\frac{dP}{dx}$", the momentum continuity equations are written for all the moving layers. "τ_i^k" is the shear stress between layers "k" and "i". "τ_i" is the shear stress between layer "i" and the pipe wall.

Gas layer:

$$A_G.\frac{dP}{dx} = -\tau_G.S_G - \tau_O^G.S_O^G \tag{23}$$

Oil layer:

$$A_O.\frac{dP}{dx} = -\tau_O.S_O - \tau_W^O.S_W^O + \tau_O^G.S_O^G \tag{24}$$

Water layer:

$$A_W.\frac{dP}{dx} = -\tau_W.S_W - \tau_{MB}^W.S_{MB}^W + \tau_W^O.S_W^O \tag{25}$$

The shear stresses between different layers and between the flowing layers and the pipe wall are shown in figure 2.

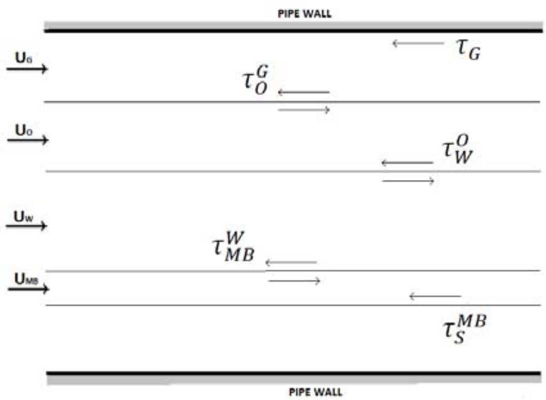

Figure 2: Shear stress between moving layers.

Shear stresses between the gas, oil or water layer and the pipe wall are:

$$\tau_i = \frac{1}{2}.\rho_i.f_i.U_i^2 \tag{26}$$

where "f_i" is the friction coefficient and is calculated using the following

$$f_i = \begin{cases} 16.Re_i^{-1} & Re_i \le 2000 \\ 0.046.Re_i^{-0.2} & Re_i > 2000 \end{cases} \tag{27}$$

The water, gas and oil Reynolds numbers in eqn (27) are:

$$Re_w = 4.U_W.A_W.\frac{[C_{S.W}.\rho_S + (1 - C_{S.W}).\rho_W]}{S_w.\mu_W} \tag{28}$$

$$Re_G = 4.U_G.A_G.\frac{[C_{o.G}.\rho_o + (1 - C_{O.G}).\rho_G]}{(S_G + S_O^G).[C_{o.G}.\mu_o + (1 - C_{O.G}).\mu_G]} \tag{29}$$

$$Re_O = 4.U_O.A_O.\frac{\rho_O}{S_O.\mu_O} \tag{30}$$

Sand particles in the water layer and oil droplets in the gas layer are assumed to be travelling with the same velocity as their carrier phases, i.e. water and gas, respectively. Hence, there will be no shear stress between suspended phases and carrier layers. The effect of the presence of sand particles in the water layer on mixture viscosity has also been neglected. Shear stress between gas and oil layer "τ_O^G" can be calculated as the following:

$$\tau_O^G = \frac{1}{2}.[C_{o.G}.\rho_o + (1 - C_{o.G}).\rho_G].(U_G - U_O).|U_G - U_O|.f_O^G \tag{31}$$

where "f_O^G"is the largest value between 0.014 and "f_G", as calculated by eqn (27). The same principle is used to calculate the shear stress between the oil and water layer "τ_W^O".

$$\tau_W^O = \frac{1}{2}.\rho_O.(U_O - U_W).|U_O - U_W|.f_W^O \tag{32}$$

"f_W^O" is the largest value between 0.014 and "f_O", as calculated by eqn (27). The shear stress between the water layer and the moving sand bed layer is

$$\tau_{MB}^W = \frac{1}{2}.[C_{S.W}.\rho_S + (1 - C_{S.W}).\rho_W].(U_W - U_{MB}).|U_W - U_{MB}|.f_{MB}^W \tag{33}$$

where "U_{MB}" will be calculated using eqn (20) and the friction coefficient from the equation below [6]:

$$\frac{1}{\sqrt{2.f_{MB}^W}} = -0.86.\ln[\frac{\{d/(4.\frac{A_W}{S_W + S_{MB}^W})\}}{3.7} + \frac{2.51}{Re_W.\sqrt{2.f_{MB}^W}}] \tag{34}$$

"Re_W" in eqn (34) is calculated using eqn (28), but "$\frac{A_W}{S_W}$" in eqn (28) will be replaced by "$\frac{A_W}{(S_W + S_{MB}^W)}$" to take into account the interface perimeter in hydraulic diameter calculation. The momentum continuity equation for the sand moving bed layer has a slightly different form than eqns. (23)–(25), due to the solid friction forces between the moving bed layer and the stationary bed.

$$A_{MB}.\frac{dP}{dx} = -\tau_S^{MB}.S_S^{MB} + \tau_{MB}^W.S_{MB}^W - \tau_{MB}.S_{MB} - F_{friction} \tag{35}$$

"$F_{friction}$" in eqn (34) is the summation of friction between the moving bed layer and the stationary layer, and the friction between sand particles in the moving bed layer and pipe wall, which is calculated using the formulation of Doron and Barnea [9] and Doron *et al.* [13]. The shear stress between the moving bed and the stationary bed "τ_S^{MB}" is

$$\tau_S^{MB} = \frac{1}{2}.\left[C_{S.MB}.\rho_S + (1 - C_{S.MB}).\rho_W\right].U_{MB}^2.f_S^{MB} \tag{36}$$

The friction coefficient in eqn (36) is calculated using an equation similar to eqn (34), the only difference being that the hydraulic diameter and Reynolds number in eqn (34) should be replaced by "$4.\frac{A_W}{S_W+S_{MB}^W}$" and "$4.U_{MB}.A_{MB}.\frac{[C_{S.MB}.\rho_S+(1-C_{S.MB}).\rho_W]}{(S_W+S_{MB}^W).\mu_W}$", respectively. The shear stress between the moving bed layer and the pipe wall is calculated similarly to eqns. (26) and (27).

4 Solving method

In all previous mechanistic models, including the three phase model of Taitel *et al.* [11] and the three layer model of Doron and Barnea [9], the entire flow is described by a set of nonlinear equations. The total number of these nonlinear equations varies, based on the developed model. For example, the Taitel *et al.* [11] model is represented by two nonlinear equations, while Doron and Barnea's [9] three layer model is made up of six nonlinear equations.

In all of these cases, the number of equations can be reduced even further by combining and rearranging the equations and then solving those numerically by estimating one of the variables as a priori and then employing an iterative trial and error method to solve other variables [8].

The present four-phase model is described by 12 unknowns, which are five holdups in the form of "H_i", four velocities in the form of "U_i", pressure drop "$\frac{dP}{dx}$", sand concentration in water "$C_{S.W}$" and oil droplet concentration in gas "$C_{O.G}$". These 12 unknowns are represented by the nonlinear eqns. (15)–(20), (22)–(25) and (35).

The arrangement and dependency of these 12 equations are in such a way that none of these unknowns can be identified as a priori. Therefore, the whole set of 12 equations should be solved simultaneously.

Solving these 12 nonlinear equations in the form of a system of equations has its own challenges. The possibility of having multiple roots for these nonlinear equations has been reported by some researchers [11, 16], which means solving this system of equations may not result in a unique set of results.

In order to have physically feasible results, they should satisfy a set of criteria. For example, the summation of all the heights should equate to the internal pipe diameter, and none of the heights should be negative. It is assumed that the water, oil and gas layers always exist.

However, the stationary sand bed may or may not be formed, depending on operating conditions. Sand particles may be all-moving or suspended in the water layer, which would result in a reduction of number of unknowns and equations. In this case, eqn (35) should be modified to remove the friction force between the moving bed and the stationary bed. If the water layer's velocity is greater than the hindered settling sand's velocity, as detailed in Doron *et al.* [8], then it can be assumed that all the sand particles are fully suspended in the water layer and consequently, the moving bed and the stationary bed disappear. As a result, eqns. (19) and (20) will be disregarded and "$C_{S.MB}$" is assumed to be equal to "$C_{S.inlet}$". When solving the equations, if it is determined that any or both of the stationary and moving bed are non-existent, then all the geometrical equations should be modified accordingly.

The moving sand bed layer can only exist if the water layer velocity is between "ω" and"U_{MB}", otherwise sand particles are considered to be fully suspended.

A computer code is being developed in MATLAB to solve this system of 12 nonlinear equations. The results will be shared with the research community as soon as the code is successfully compiled.

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