Comparison of drift-flux and multi-fluid approaches to modeling of multiphase flow in oil and gas wells

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Abstract

Multiphase flows in wellbores and pipes can be described within the framework of two major approaches, namely, the multi-fluid approach and the drift-flux approach. The fundamental difference between the approaches is in the formulation of the momentum conservation equations in the mathematical model. Within the multi-fluid model, a set of mass and momentum conservation equations is written for each phase in the continuum approximation, while the drift-flux model offers a single momentum equation for the mixture as a whole in terms of the volume-averaged velocity of the mixture. Both approaches require closure relations derived from and tuned with respect to laboratory data. In this work, we compare both approaches and apply them to solve the transient problem of terrain induced slugging relevant to the field of well testing and production and compare performance of the models.
Keywords: multiphase, flow, drift-flux, multi-fluid, oil, gas.

1 Introduction

In the petroleum industry, wellbores are drilled to deliver fluids initially sealed underground in reservoirs to the surface. Accurate prediction of the surface rates is important in designing the layout for the well testing job or the production plan for a hydrocarbon field. Hence, it is important to have tools for accurate numerical modeling of the transient wellbore flows.

Mathematical modeling of multiphase flows in pipes is an essential part in the simulation workflow for transport of hydrocarbons in wells and pipelines.

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A number of key phenomena observed in surface measurements of pressure and flow rate are attributed to the effects of multiphase flow in wellbores. These phenomena include, in particular, generation of long water slugs in near-horizontal wells, which may overwhelm surface equipment and even extinguish the flare, resulting in environmental issues.

We consider different approaches to modeling of a multi-phase wellbore flow, including a drift-flux model (see, for example, Shi *et al.* [1, 2]) and a well-established multi-fluid model (see Bendiksen *et al.* [3]). First, the general mathematical formulation is presented. Then, the numerical implementation of the drift-flux model is described. Finally, we present the results of numerical simulations for the case related to the problems of the terrain-induced slugging.

2 Mathematical models

Flows in long pipes are usually considered in the one-dimensional approximation. Flow variables, such as phase volume fractions, velocities, densities and pressure are treated as averaged over the pipe cross-section. Isothermal flows of two compressible fluids are governed by the system of mass and momentum conservation equations written in the differential form. Mathematical problem is formulated as an initial-boundary value problem for the system of first-order quasilinear equations.

2.1 Mass conservation

Both in the multi-fluid and drift-flux approaches the mass conservation equations for gas and liquid can be written in the following form:

$$\frac{\partial}{\partial t}(\alpha_g \rho_g) + \frac{\partial}{\partial x}(\alpha_g \rho_g v_g) = q_g, \tag{1}$$

$$\frac{\partial}{\partial t}(\alpha_l \rho_l) + \frac{\partial}{\partial x}(\alpha_l \rho_l v_l) = q_l. \tag{2}$$

Here, α is the volume fraction, ρ is the density, v is the velocity, q is the mass source, t is the time, x is the spacial coordinate along the pipe. Indices g and l indicate gas and liquid phases, respectively. The conservation of volume is expressed in terms of the volume fractions as $\alpha_g + \alpha_l = 1$. Mass exchange between the phases is neglected and the cross-section of the pipe is assumed to be constant. Since the flow is considered in an isothermal approximation, the densities of the phases are supposed to be given functions of pressure only $\rho_i = \rho_i(p)$, $i = g, l$. Continuity equations for each phase can be split into a set of equations for continuous and dispersed fields coupled by the mass transfer relationships (see, for example, Bendiksen *et al.* [3] and Bonizzi *et al.* [4]). For example, the model can describe continuous liquid phase and liquid droplets.

2.2 Momentum conservation

The fundamental difference between multi-fluid and drift-flux approaches is in the formulation of momentum conservation equations. Within the multi-fluid approach, a set of momentum conservation equations is written for each phase. The equations contain terms describing momentum exchange between the phases. Closure relations for these terms are required. Within the framework of the drift-flux formulation, momentum conservation equation is written for the volume averaged mixture velocity. To get the phase velocities, the additional closure relations should be provided.

2.2.1 Multi-fluid formulation

The momentum conservation equations for gas and liquid in the multi-fluid formulation can be written in the following form (see Bendiksen *et al.* [3]):

$$\begin{aligned}\frac{\partial}{\partial t}\left(\alpha_g \rho_g v_g\right) + \frac{\partial}{\partial x}\left(\alpha_g \rho_g v_g^2\right) &= -\alpha_g \frac{\partial p}{\partial x} + \alpha_g \rho_g g \cos\theta + \\ &+ \alpha_g \rho_g g \sin\theta \frac{\partial h_L}{\partial x} + \Phi_g + \Phi_{int} - P_I \frac{\partial \alpha_g}{\partial x},\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial}{\partial t}\left(\alpha_l \rho_l v_l\right) + \frac{\partial}{\partial x}\left(\alpha_l \rho_l v_l^2\right) &= -\alpha_l \frac{\partial p}{\partial x} + \alpha_l \rho_l g \cos\theta + \\ &+ \alpha_l \rho_l g \sin\theta \frac{\partial h_L}{\partial x} + \Phi_l - \Phi_{int} - P_I \frac{\partial \alpha_l}{\partial x}.\end{aligned} \tag{4}$$

Here, g is the gravitational constant, θ is the pipe inclination angle with respect to vertical, h_L is the liquid level in the pipe, Φ_i are the terms describing the wall friction ($i = g, l$), Φ_{int} is the term describing momentum exchange between phases. Terms involving $\cos\theta$ and $\sin\theta$ describe pressure difference due to the gravity with the latter term taking into account contribution due to the change of liquid level in a pipe. Interfacial pressure difference P_I is introduced to ensure the hyperbolicity of the system (see Osiptsov *et al.* [5]). The set of eqns. (1)–(4) supplemented with closure relations for friction losses, momentum exchange, and sum of volume fractions contains four equations for four unknown functions.

2.2.2 Drift-flux formulation

We adopt the following form of the mixture momentum conservation equation

$$\frac{\partial v_m}{\partial t} + v_m \frac{\partial v_m}{\partial x} = -\frac{1}{\rho_m}\frac{\partial p}{\partial x} + g\cos\theta + \frac{2 f v_m^2}{d}. \tag{5}$$

Here f is the friction factor, d is the pipe diameter and the mixture velocity and density are defined as $v_m = \alpha_l v_l + \alpha_g v_g$ and $\rho_m = \alpha_l \rho_l + \alpha_g \rho_g$, respectively. In this formulation the time derivative of mixture velocity is retained and the pressure losses due to gravity, friction, and acceleration are taken into account (see Hasan and Kabir [6]). The closure relation for friction factor is required. Note that eqn. (5) is postulated in the non-conservative form.

In the drift-flux formulation the gas velocity is given by

$$v_g = C_0 v_m + v_d, \tag{6}$$

where C_0 is the profile parameter that takes account of the non-uniform distribution of the gas phase and the velocity profile over the pipe cross-section and v_d is the drift velocity describing the local slip due to the density contrast between the phases (see Zuber and Findlay [7]). In the general case profile parameter and drift velocity are the functions of mixture velocity, gas volume fraction, and pressure. Combining eqn. (6) with the definition of the mixture velocity one can also express the liquid velocity as a function of mixture velocity, gas volume fraction, and pressure. Finally, one obtains the system of three eqns. (1), (2), and (5) with three unknown functions.

The drift-flux formula was originally developed for the dispersed bubble flows and later extended to cover all the values of gas volume fraction α_g ranging from zero to unity (see Ishii [8]). Moreover the model has been extended to cover the three-phase oil-water-gas flows that can be encountered in the petroleum industry (see Shi *et al.* [1, 2]).

An important feature of the drift-flux closure relations is that the phase velocities are allowed to have opposite signs. Hence, the counter-current flows occurring during gravity segregation in vertical pipes or in unsteady flow conditions in the pipeline can be simulated.

The drift-flux model is usually postulated, but in general it can be derived from the set of conservation laws for the phases written in the multi-fluid formulation in a certain asymptotic limit. This would provide the set of assumptions expressed in terms of governing dimensionless parameters, defining the domain of applicability for the model. The main assumption seems to be a non-inertial relative motion of phases (see Ishii [8]). The derivation of the drift-flux model goes beyond the scope of the present work, and will be addressed elsewhere.

Comparison of the approaches reveals that the drift-flux model has a number of advantages over the multi-fluid model. The drift-flux model contains a single momentum equation which requires less CPU time to calculate in contrast to the multi-fluid models that have to deal with two (gas and liquid) or three (gas, oil, and water) separate momentum equations. The drift-flux closures are smooth and differentiable that favors numerical stability and robustness. A set of closure relations is calibrated against laboratory data for vertical/inclined flows. The drift-flux model is free from singularities at small values of volume fractions. In the multi-fluid model, the disappearance of the phase may induce the growth of velocity to unrealistic values. As it was demonstrated earlier by Malekzadeh *et al.* [9] and will be shown below, the drift-flux model is capable of predicting terrain slugs. At the same time, the drift-flux model is not calibrated for horizontal and downward inclined flows, does not cover hydrodynamically induced slugs developing as a result of Kelvin-Helmholtz instability at the interface between the fluids, and is based on the assumption of a non-inertial interphase slip.

3 Numerical implementation

In a general compressible formulation, the problem of multi-phase flow in pipes is characterized by strong non-linear coupling between pressure and velocity fields. Solution of the full compressible three-phase problem is required. Such a solution could be obtained numerically using iterative schemes. Such schemes could be applied to obtain solutions for both drift-flux and multi-fluid models (see Osiptsov *et al.* [5]). In this section, the numerical implementation of an iterative numerical scheme for the drift-flux model is briefly described.

The system of equations for the drift-flux model described above can be solved using a semi-implicit numerical scheme based on the SIMPLE iterative method (Semi-Implicit Method for Pressure Linked Equations, see Patankar [10] and Ferziger and Perić [11]). The basis of this scheme is a representation of the pressure field p as a sum of guessed value p^* and correction p'. Consequently, all the pressure-dependent variables and velocity are represented as sums of guessed values (e.g. v_m^*) and corrections (e.g. v_m'). The implemented scheme can be briefly summarized as follows:

1. Guess the pressure field p^*. At the very first iteration the hydrostatic pressure distribution or the steady state solution can be used. At subsequent iterations, the solution from previous iteration is used.
2. Solve the mixture momentum equation (5) using guessed pressure field p^* to obtain guessed velocity field v_m^*.
3. Solve the pressure correction equation constructed from a combination of continuity equations (eqns. (1) and (2) in the two-phase case) to find p'.
4. Calculate the updated pressure field by summing up guessed value and correction, $p = p^* + p'$.
5. Calculate the velocity correction as $v_m' = -\triangle t \left(\partial p'/\partial x\right)/\rho_m$. This formula can be derived from the momentum equation (5). Then calculate the updated velocity field by summing up guessed value and correction, $v_m = v_m^* + v_m'$.
6. Calculate the phase velocities in the updated mixture velocity field using the drift-flux closure relations and advance volume fractions solving the mass conservation equations (eqns. (1) and (2) in the two-phase case).
7. Check if residuals of equations with the substituted updated solution are below the prescribed tolerance. If not, repeat the whole cycle treating the obtained updated pressure value as the guessed one.

The described solution procedure is implemented on the non-uniform staggered grid with pressures and other scalar variables being calculated at cell centers and velocities being calculated at cell faces. The solution allows the mixture velocity and the phase velocities to be positive or negative hence covering counter-current flows. When solving eqn. (5) the values of problem variables (e.g. mixture density) at the cell faces are determined using the upwind scheme with respect to the mixture velocity. When solving the pressure correction equation and mass conservation equations, the variables required at cell faces are determined using the upwind scheme with respect to the phase velocities.

4 Simulations

To evaluate performance of simulators the problem of terrain-induced slugging is selected. In undulating pipes and wellbores transporting multiple phases simultaneously, liquid can accumulate in lower parts of the system and block the passage of gas. This causes increase of gas pressure and subsequent blowout of liquid when the pressure reaches critical value. At the system outlet this is usually observed in form of large fluctuations of flowrates and pressure. In some cases, the liquid is delivered to the outlet in portions that can be referred to as terrain-induced slugs. For our study, we select the experimental data thoroughly reported by De Henau and Raithby [12]. The data is obtained for air-water flows. The simulation parameters that are not explicitly mentioned here could be found in this paper. The focus is on run 9 where distinct slugs were observed. The multi-fluid wellbore model is used *as is*, while the drift-flux model provides full access to the closure relations with a possibility to tune the coefficients.

4.1 Geometry, grid, and time steps

The experimental set up consists of four polyvinyl chloride (PVC) pipes of equal length of 3.84 m and diameter of 51.8 mm connected with flexible hoses of the same diameter and length of 0.314 m, a large PVC tank required for reproduction of terrain-induced slugging (see Taitel *et al.* [13] for details), a three-way injector with inlets for air and water and connection to the tank, and other equipment. In the numerical simulations, the PVC tank is modeled as a straight 53 m long pipe of diameter 51.8 mm upstream of the injection point. The described flowpath and the injection point are shown in fig. 1. The base computational grid is adopted from De Henau and Raithby [12] where authors suggest using $\triangle x = 0.239$ m for pipes and $\triangle x = 0.314$ m for the sections corresponding to the flexible hoses. Thus, the flowpath having total length $L = 62.8$ m contains total amount of 290 cells.

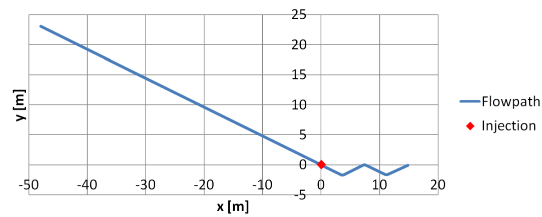

Figure 1: Geometry for the terrain-induced slugging problem.

The roughness of pipes used in experiments is not reported by the authors. An average value of roughness $4.25 \cdot 10^{-6}$ m over the range reported in The Engineering Toolbox [14] for PVC and plastic pipes is chosen. All the values taken from this resource throughout the paper are double checked against other resources not listed here.

4.2 Fluid properties

As reported by the authors, air and tap water were used in the experiment. The density of air fed into simulators was calculated in agreement with the ideal gas law

$$\rho = \frac{p}{RT},$$

where $R = 287.06$ J/kg/K is the specific gas constant for dry air, T is the temperature. The water density was calculated in agreement with the NIST Chemistry WebBook [15].

4.3 Heat transfer

The influence of temperature on the behavior of multi-phase flow is beyond the scope of this paper so the drift-flux simulation is isothermal. However, the multi-fluid simulation is carried out in both isothermal and non-isothermal formulation. The results not presented here suggested that temperature does not change significantly the parameters of interest summarized in tables 1 and 2 below.

4.4 Initial and boundary conditions

In the experiment the pipe was initially empty (filled with air only). So the initial conditions are specified as follows: air volume fraction $\alpha_g(x, 0) = 1$, air velocity $v_g(x, 0) = 0$ m/s, pressure $p(x, 0) = 98130$ Pa, temperature $T(x, 0) = 22.7$°C. The outlet of the pipe was kept at given atmospheric conditions. The air and water superficial velocities are reported at 101325 Pa and 21°C. Hence, the boundary conditions are summarized as follows: outlet pressure $p(L, t) = 98130$ Pa, outlet temperature $T(L, t) = 22.7$°C, inlet air mass flow rate $3.16 \cdot 10^{-4}$ kg/s, inlet water mass flow rate 0.27 kg/s.

4.5 Results and discussion

The results of simulations are shown in fig. 2 for experiment, a drift-flux simulation, and a multi-fluid simulation. As it was mentioned above, the drift-flux model is not calibrated for the downward inclined flows. In this work, for downward flows we used the same form of closure relations as for the upward flow with a possibility to tune the coefficients separately. The results presented in fig. 2 are obtained using the set of parameters described by Shi *et al.* [1] tuned to match approximately the slug frequency. Thorough investigation of the influence of other drift-flux parameters as well as the sensitivity study is left for future work.

As it could be seen from the picture, the multi-fluid simulator produced water slugs that are similar in shape and amplitude to the ones observed in experiment.

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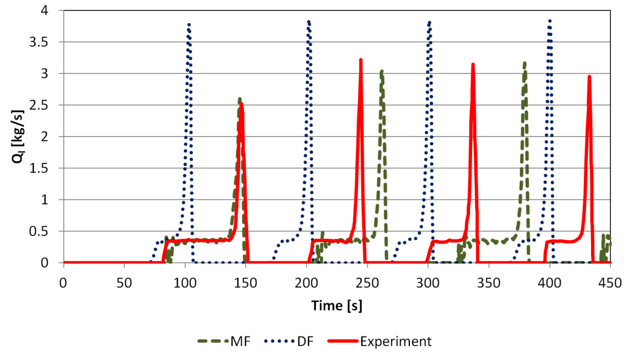

Figure 2: Mass flow rate, MF is the multi-fluid, DF is the drift-flux.

Note the remarkable agreement for the first slug produced by the multi-fluid simulator that is almost indistinguishably overlays the experimental data. The peak value (amplitude) of slugs is also in a very good agreement with the experiment. However, over the period of observation of 450 seconds the multi-fluid simulator produced only three slugs while the drift-flux simulator delivered four. This means that the period predicted by the multi-fluid simulator is higher than the period observed in the laboratory. These visual observations are reflected in table 1. Based on data in this table one can estimate the average error of simulation results with respect to the experiment. As can be seen from table 2, the multi-fluid average error is equal to 12.12% and is slightly higher than 9.49% of the drift-flux simulator. One can also compare pressure trends at two observation points P1 and P2 (located 3.72 m and 11.98 m along the pipe from inlet, respectively). This comparison is shown in fig. 3. Both the multi-fluid and the drift-flux simulations represent qualitatively the behavior observed in the experiment.

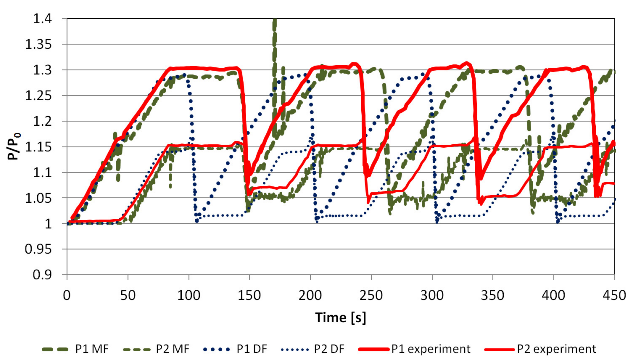

Figure 3: Pressure, MF is the multi-fluid, DF is the drift-flux.

Table 1: Analysis of the results for the pressure in the case of terrain-induced slugging.

	Slug 1	Slug 2	Slug 3	Slug 4	Average (slugs 2-4)	Total over 450 s
	Time of arrival [s]					
Experiment	81.84	201.76	298.55	395.94	–	–
Multi-fluid	84.06	207.51	324.54	446.01	–	–
Drift-flux	72.50	172.00	271.00	370.00	–	–
	Peak time [s]					
Experiment	146.32	244.56	336.92	432.59	–	–
Multi-fluid	145.03	262.05	379.53	496.01	–	–
Drift-flux	103.00	202.00	301.00	400.00	–	–
	Peak amplitude [kg/s]					
Experiment	2.52	3.22	3.14	2.96	3.11	–
Multi-fluid	2.60	3.05	3.17	3.18	3.13	–
Drift-flux	3.79	3.85	3.84	3.83	3.84	–
	Period [s]					
Experiment	–	98.24	92.37	95.67	95.42	–
Multi-fluid	–	117.02	117.48	116.49	117.00	–
Drift-flux	–	99.00	99.00	99.00	99.00	–
	Cumulative production [kg]					
Experiment	35.20	26.87	25.56	24.09	25.50	111.72
Multi-fluid	33.67	31.09	31.15	31.02	31.09	97.56
Drift-flux	27.32	26.76	26.78	26.79	26.78	107.64

Table 2: Average error of the results for the mass flow rate in the case of terrain-induced slugging: MF is the multi-fluid, DF is the drift-flux.

	Time of arrival of liquid	Peak amplitude	Period	Cumulative production per slug	Total production over 450 s	Average error [%]
Error MF [%]	2.72	0.73	22.61	21.89	12.67	**12.12**
Error DF [%]	11.41	23.64	3.75	4.99	3.64	**9.49**

A numerical simulation study is conducted for a multiphase flow in a well using different approaches to mathematical modeling, which include multi-fluid and drift-flux models. The drift-flux approach has a number of benefits, including a single momentum equation which requires less CPU time, smooth and differentiable closure relations, no singularities at small values of volume fractions, and capability of predicting terrain slugs. At the same time, the drift flux model is not calibrated for horizontal and inclined downward flows, does not cover hydrodynamically induced slugs developing as a result of Kelvin-Helmholtz instability at the interface between the fluids, and is based on the assumption of a non-inertial inter-phase slip.

Both approaches are applied to a model case of the terrain-induced slugging where experimental data are available. The multi-fluid model was used *as is* with no possibility to modify the closure relations. The first slug produced by the multi-fluid model is in remarkable agreement with the experimental data.

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Nevertheless, over the period of observation only three slugs appeared in the multi-fluid simulation results opposed to four slugs reported in the experiment. The slugs predicted by the drift-flux model are of different shape and have larger amplitude. However, the model was tuned to deliver four slugs as in the experiment. In the drift-flux results both liquid production per slug, the period between slugs, and total cumulative liquid production over the observation period are closer to the experiment than the multi-fluid predictions. The results suggest that both drift-flux and multi-fluid models can be further tuned for better match with the experimental data. In agreement with the results obtained earlier by Malekzadeh *et al.* [9] this simulation of the terrain-induced slugging case demonstrates the ability of the drift-flux model to simulate this type of highly transient phenomena.

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