An improved criterion to minimize FE mesh-dependency in concrete structures subjected to blast and impact loading

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Abstract

In the context of an increasing need for reliability and safety in concrete structures under blast and impact loading conditions, the behavior of concrete under high strain rate condition has been an important issue. Since concrete subjected to impact loading associated with high strain rate shows quite different material behavior from that in the static state, several material models are proposed and used to describe the high strain rate behavior under blast and impact loading. In the process of modelling, in advance, mesh dependency in the used finite element (FE) is the key problem because simulation results under high strain-rate condition are quite sensitive to applied FE mesh size. It means that the accuracy of simulation results may be deeply dependent on FE mesh size in simulations. This paper introduces an improved criterion which can minimize the mesh-dependency of simulation results on the basis of the fracture energy concept, and HJC (Holmquist Johnson Cook), CSC (Continuous Surface Cap) and K&C (Karagozian & Case) models are examined to trace their relative sensitivity to the used FE mesh size. To coincide with the purpose of the penetration test with a concrete plate under a projectile (bullet), the residual velocities of projectile after penetration are compared. The correlation studies between analytical results and the parametric studies associated with them show that the variation of residual velocity with the used FE mesh size is quite reduced by applying a unique failure strain value determined according to the proposed criterion.
Keywords: high strain rate concrete, penetration simulation, failure strain, mesh-dependency, fracture energy.

1   Introduction

Under hard impact, blast and impulsive load condition, concrete shows quite different behavior compared to that in static state. Many material properties of concrete such as strength, tangent modulus and critical strain are changed with strain rate. There are two main reasons for this unique behavior. First reason is inertia resistance effect. Concrete is a brittle material which fails due to internal microcrack creation and propagation. Decrease in the amount of microcrack at certain stress level reduces the stiffness of material. Under high strain-rate condition, there is not enough time to generate and propagate microcracks. This lack of microcracking decreases the amount of deformation along the loading direction. This means that stiffness of material in loading direction increases. Therefore, higher stress level than static condition is needed to propagate enough microcracks for failure in enhanced stiffness direction. Second reason is lateral inertia confinement effect. Lateral expansion caused by Poisson's ratio effect is limited during impact loading due to lack of time to expand. This limitation shows similar effects like lateral confining stress which increase strength of the material. In these reasons, strength and critical strain increases as strain-rate increases [1]. Therefore, several material models are proposed and used to describe the high strain rate behavior under blast and impact loading. In the process of modelling, mesh dependency in the used finite element (FE) is the key problem because simulation results under high strain-rate conditions are quite sensitive to applied FE mesh size. This paper suggests a criterion that minimizes mesh-dependency in applied FE mesh size on the basis of fracture energy concept. The typical high strain rate models (HJC [2], CSC [3], K&C [4]) are examined with penetration simulation to check relative sensitivity to the used FE mesh size. Furthermore, improvement in accuracy of simulation results is investigated by applying a unique failure strain value determined according to the proposed criterion.

2   Size effect on behavior of concrete

Since concrete is a brittle material which fails due to creation and propagation of internal microcrack in fracture process zone [5], concrete shows different failure behavior according to pattern of internal microcracking. Size of concrete specimen is the factor which makes difference in pattern of internal microcracking. As can be seen in Figure 1 [6], internal crack pattern changes according to size of specimens. And this difference in crack pattern makes the difference in strength and critical strain shown in Figure 2 [6]. The reason of this difference in crack pattern is that shear force cannot reach to bottom of specimen, as height of specimen get longer.

   Therefore, size of the concrete specimen should be considered in estimation of fracture energy. This size effect is also observed in element mesh size of FE analysis. When large finite elements are used, each element has a large effect on the structural stiffness. When a single element cracks, the stiffness of the entire structure is largely reduced to fails easily [7]. By focusing on this size effect on

failure behavior of concrete in FE mesh and specimen, criterion is proposed based on the fracture energy concept and numerical experiment for the size effect is conducted at the next chapter.
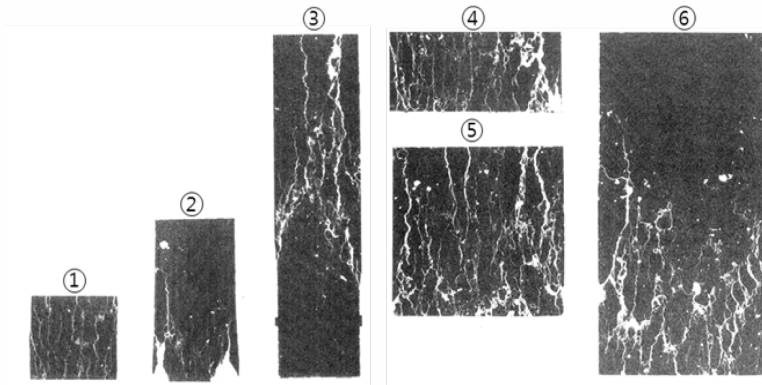

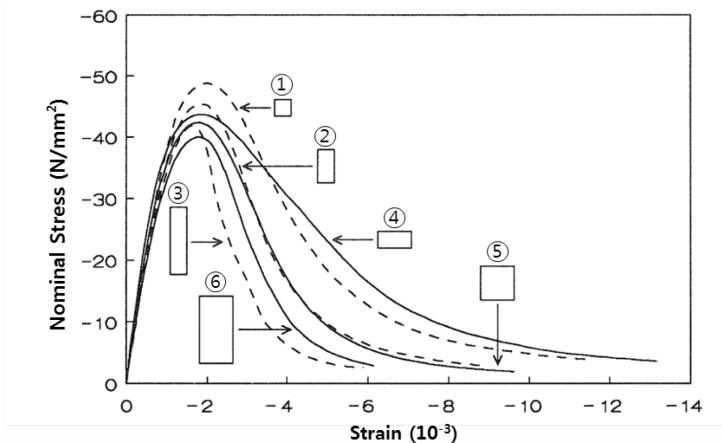Figure 1:   Crack patterns for different size of specimens.

Figure 2:   Stress-Strain curve for different size of specimens.

3   Introduction of a fracture energy based criterion

3.1 Estimation of fracture energy

As can be seen in Figure 3(a) [6], fracture energy is divided into continuum damage energy part and local fracture energy part. Continuum damage energy represents strain hardening portion which varies with size of specimen and local fracture energy represents strain softening portion which has constant value regardless of specimen size [7]. Since the main purpose of this paper is

modification of failure strain according to FE mesh size, local fracture energy which can convert size effect into difference of failure strain is used to propose a criterion. Local fracture energy part can be approximated by the area of a triangle in Figure 3(b) and be fomulated by adding $F(x)$ the size effect function which has length scale. As a result, eqn (1) is the fomulated equation for local fracture energy where ε_0 is the failure strain and ε_c is the critical strain.
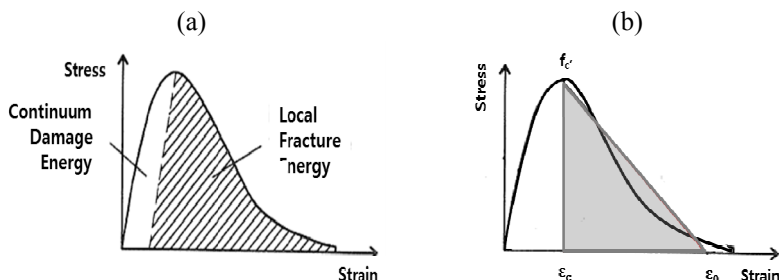

Figure 3:　　Fracture energy (a) Configuration, (b) Estimation.

$$G_f = \frac{1}{2}(\varepsilon_0 - \varepsilon_c)f_c'F(x) \tag{1}$$

3.2 Criterion proposal

The size effect function $F(x)$ consists of $F(b)$ and $F(h)$. $F(b)$ is the function of specimen width and $F(h)$ is the function of specimen length. First of all, $F(b)$ is estimated. Assuming crack pattern as an exponential function, crack pattern is formulated with $f(x)=\alpha e^{\beta x}$ [7]. When b is width of specimen and center of the specimen is x=0, boundary conditions are $f(0)=1$ and $f(b/2)=(b_0/b)^\gamma$ [9]. Using two boundary conditions, $\alpha=1$ and $\beta=(2/b)^\gamma \ln(b_0/b)$ are gained. $F(b)$ is calculated as eqn (2) by integrating $f(x)$ with b and inserting maximum length $b_0=6(mm)$ that uniform crack pattern occurs. Fracture energy is given by eqn (3) by subsituting $F(b)$ into eqn (1).

$$F(b) = \frac{2}{b_0}\int_0^{b/2} e^{-\left(\frac{2}{b}\right)\gamma\, ln\left(\frac{b}{b_0}\right)x}\,dx = \frac{\left(\left(\frac{b}{6}\right)^\gamma - 1\right)}{\gamma ln\left(\frac{b}{6}\right)} \tag{2}$$

$$G_f = \frac{1}{2}(\varepsilon_0 - \varepsilon_c)f_c'\frac{\left(\left(\frac{b}{6}\right)^\gamma - 1\right)}{\gamma\, ln\left(\frac{b}{6}\right)}F(h) \tag{3}$$

Regression analysis has been carried out to fomulate $F(h)$ with experiment data of failure strain value according to specimen size. In this procedure, $\gamma = 1$ is assumed. As can be seen in Table 1 [6, 8], failure strain is rarely affected by the width of specimen b. Subsequently, $F(b)$ is set to the constant value $F(50)$. $F(h)$ is govern by eqn (4), where critical strain $\varepsilon_c = 0.0135$ that is normal value

for 48(Mpa) concrete and fracture energy $G_f=8.5$(Nmm/mm^2) given by experiment [6]. Figure 4 shows result of regression analysis with the data from Table 1.

$$F(h) = \frac{2G_f}{(\varepsilon_0 - \varepsilon_c)f_c'F(50)} \qquad (4)$$

Table 1:   Measured failure strains according to size of specimens.

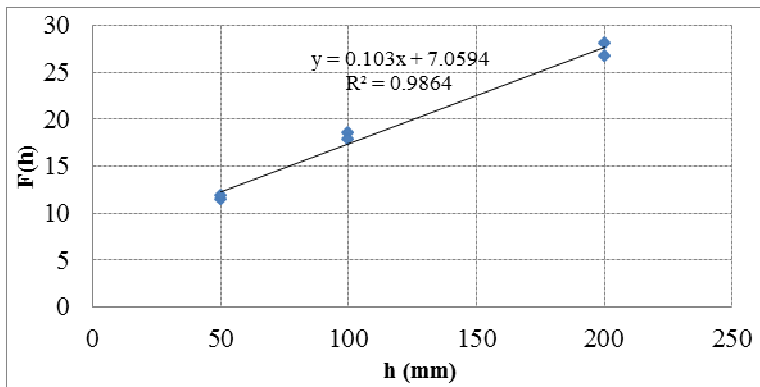| Height of specimen (mm)	Width of specimen (mm)	Failure strain (ε_0)
50	50	0.0103
50	100	0.01
100	100	0.0069
100	50	0.0071
200	50	0.0052
200	100	0.005

Figure 4:   Regression analysis for *F(h)*.

The trend line follows *F(h)* function. Finally *F(h)* is governed by eqn (5) and criterion is proposed by eqn (6).

$$F(h) = 0.0103h + 7.06 \qquad (5)$$

$$\varepsilon_0 = \frac{2G_f \gamma \ln\left(\frac{b}{b_0}\right)}{f_c'\left(\left(\frac{b}{b_0}\right)^\gamma - 1\right)(0.103h + 7.06)} + \varepsilon_c \qquad (6)$$

Unique failure strain for each mesh size is proposed by this criterion eqn (6) and applied to FE analysis under high strain rate condition. *b* is defined as mesh width size and *h* is defined as mesh height to apply for FE analysis. The optimized value of $\gamma = 4.9$, which shows good agreement with being applied to mesh, is assumed.

4 Analytical approach

4.1 Penetration simulation

In this section, penetration simulation is performed for HJC, CSC, K&C models using LS-DYNA commercial FE analysis program. Projectile radius is 0.0127 (mm) and depth is 144 (mm). Plate size is 305 (mm) x 305 (mm) x 178 (mm). Projectile is placed on plate like figure 5 and half modelling is used for saving time. Initial velocity of projectile is 400m/s [10]. Projectile mesh size is fixed to $4.87(mm^2)$ and plate mesh size is variable. Residual velocity of projectile after penetration is checked to find out mesh-dependency. Furthermore, after applying failure strain of criterion on concrete plate, residual velocity of projectile is rechecked to confirm improvement in mesh-dependency.
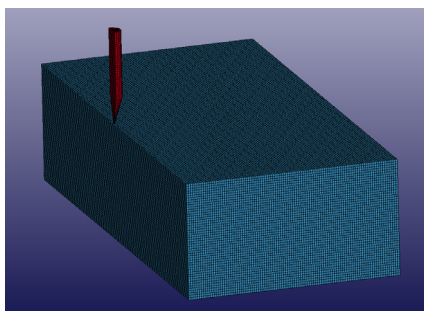

Figure 5: Configuration of penetration simulation.

4.2 Simulation results

Each model has 3 sets of cases which has different height of mesh. For case1, plate is divided into 5 pieces along the height direction, for case 2, 10 pieces, and for case 3, 25 pieces. All cases have 6 different width size of mesh. Residual velocity is measured according to mesh width size for each case. Figure 6, 7 and 8 shows the results of simulation for high strain-rate models, where h is height of mesh.

Black lines are case 1, grey lines are case 2, light grey lines are case3 and dashed lines represent the results of simulation that failure strain of criterion is applied to. Failure strain caculated by criterion is described in Table 2, where h is height of mesh and b is width of mesh. As can be seen in these figures, simulation results before applying criterion shows a significant mesh-dependency. HJC model has 201(m/s) difference in simulation results, CSC model has 198(m/s) and K&C model has 239(m/s) difference in simulation results according to mesh size. Reliablity of simulation results cannot be assured with these values. When failure strain determined by the criterion is applied to concrete plate in FE anlalysis, variation of simulation results are greatly decreases. As can be seen dashed lines in figure 6, 7, 8, difference in residual velocity is reduced. After criterion is applied to concrete plate, HJC model has

121(m/s) difference in simulation results, CSC model has 131(m/s) and K&C model has 100(m/s) difference in simulation results according to mesh size. In conclusion, mesh dependency in penetration simulation accompanied with high strain rate is improved by using the criterion proposed in this paper.
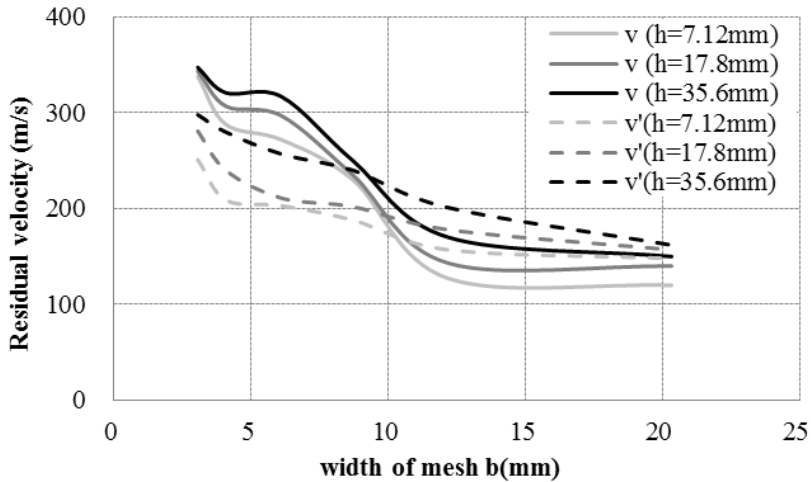

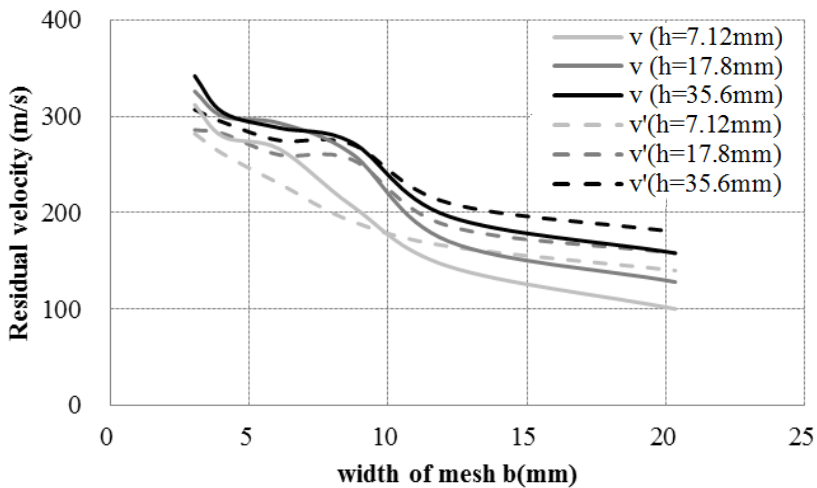Figure 6:   Residual velocity versus width of mesh (HJC model).

Figure 7:   Residual velocity versus width of mesh (CSC model).

Table 2:   Failure strain calculated by criterion according to size of mesh.

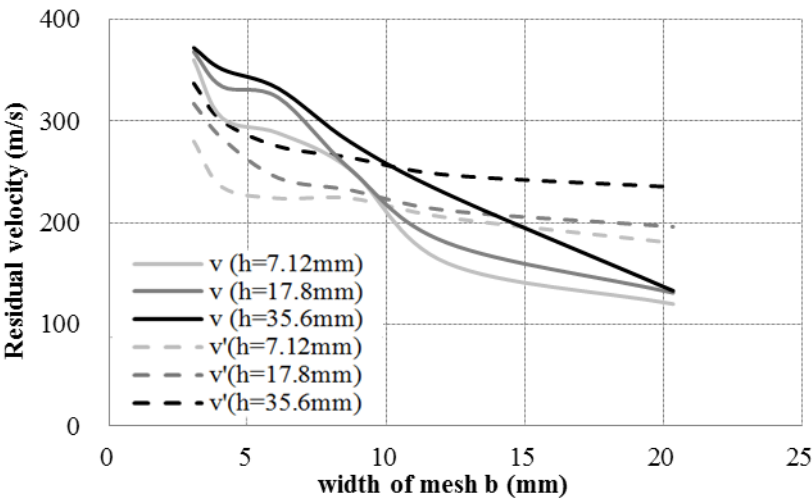| b (mm)	Case 1 (h=35.6mm)	Case 2 (h=17.8mm)	Case 3 (h=7.12mm)
3.05	0.0354	0.0414	0.0460
4.07	0.0354	0.0414	0.0460
6.1	0.034	0.0397	0.0445
8.71	0.0133	0.0154	0.0171
12.2	0.00512	0.00578	0.00634
20.3	0.00187	0.00196	0.00203

Figure 8:   Residual velocity versus width of mesh (K&C model).

5   Conclusion

In this study, the mesh-dependency is investigated in the penetration simulations with high strain-rate models (HJC model, CSC model and K&C model). This study proposes a criterion based on the concept of fracture energy to minimize the mesh-dependency. The analytic approaches for penetration simulation of steel projectile into concrete plate is performed according to the FE mesh size of concrete plate. Simulation results show that the variation of residual velocity with the used FE mesh size is reduced by applying a unique failure strain value determined according to the proposed criterion. In conclusion, the mesh-dependency in FE analysis of concrete structure subjected to blast and impact loading can be minimized with the criterion proposed in this paper.

Acknowledgements

References

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