The design of a foldable triangulated scissor grid for single-curvature surfaces

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Abstract

Deployable scissor structures can repeatedly transform between a compact and an expanded state, leading to a broad range of architectural applications. During the past few decades, a myriad of different configurations and shapes have been proposed. However, many of them show problems either in the deployment process, during which additional stresses and deformations are encountered, or in the in-plane stability of the structure once erected, requiring extra bracing components to solve. One of us has developed a configuration, based solely on translational scissor units, that allows combining the benefits of a rigid triangulated grid and a stress-free deployment process for single-curvature geometries. We have created a novel general design method for generating this type of foldable scissor structure and have explored its full geometrical potential, as is presented in this paper. The benefits and special characteristics of this type of structure are also discussed. The proposed configuration allows generating a large variety of single-curvature shapes useful for creating mobile self-supporting shelters. Additionally, a scale model has displayed promising kinematical and structural behaviour. Therefore, a full-scale prototype will be developed in the near future.

Keywords: foldable scissor structure, geometric design method, deployable shelter, triangular cells, translational scissor unit, double-layer grid.

1 Introduction

Deployable scissor structures are mechanisms that are able to rapidly and reversibly transform from a stowed to an expanded configuration. Once deployed, constraints are added to lock the mechanism and obtain a load-bearing

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structure. When suited with a cover such as a membrane, they are particularly fit for lightweight, mobile and temporary applications or for adding transformable components to static buildings. Applications include tents and stages for temporary or travelling events, shelters for disaster relief and retractable roof structures for sports stadia. Aside from these architectural applications, they are also implemented in the aerospace industry, e.g. as solar arrays (Gantes [1]).

A scissor structure consists of a planar or three-dimensional linkage of scissor units. Each scissor unit is formed by two rods interconnected by a revolute joint, called the intermediate hinge point. Depending on the location of this point on the rods and the shape of the rods, three different basic unit types can be distinguished: the translational, polar and angulated unit. Throughout the years, a wide range of possible configurations in many different shapes has been proposed for scissor structures using these basic units (see Escrig and Valcarcel [2], Hanaor and Levy [3] and De Temmerman [4]). The majority of these configurations display one or two of the following three properties which benefit the scissor grid: curvature, triangulation and foldability. The first property, curvature, is often introduced into a scissor structure to enhance its structural performance, or simply to fulfil functional or aesthetical requirements. The second property, a triangulated double-layer grid in which each cell consists of three scissor units, results in geometrical rigidity. No additional bracing components are therefore required. The last property, foldability, refers to a grid that is kinematically compatible. It ensures a smooth and reliable deployment process, in contrast to incompatible grids which display additional stresses in the structure and deformations of the struts throughout different stages of deployment. Although the work of Zeigler [5], Rosenfeld *et al.* [6] and Gantes [1] on snap-through or clicking scissor mechanisms has shown that these stresses and deformations can theoretically be turned into an advantage, in practice they remain difficult to control.

A scissor grid combining these three beneficial properties is very desirable. Unfortunately, Langbecker [7] proved mathematically that a foldable triangulated scissor grid is only possible for two configurations: planar surfaces based on translational scissor units or spherical surfaces composed from angulated scissor units. Generating a non-planar or non-spherical surface requires the triangular cells to be composed out of more than three scissor units, which introduces extra hinges and thus strongly reduces the rigidity. However, De Temmerman [4] has refuted Langbecker's statement: he demonstrated that it is also possible to generate foldable triangulated single-curvature scissor grids based on translational scissor units. This new configuration was obtained by allowing angular deformations in the grid. The following sections of this paper cover a continuation of the work presented in De Temmerman [4] with a new and simplified geometric design method and a full overview of the geometrical potential and corresponding architectural possibilities (fig. 1).

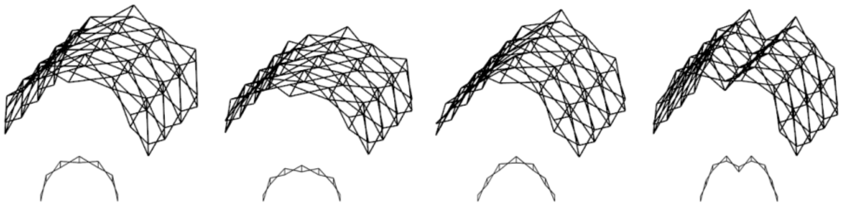


Figure 1: Example shapes for scissor grids based on the proposed concept. From left to right: circular, elliptical, parabolic and arbitrary curvature.

2 Geometric design method

This section covers the proposed design method. First, the geometry and kinematics of its basic building block, the translational scissor unit, is given, followed by the design method for a simple planar linkage. This method has previously been presented in De Temmerman [4] and is briefly summarised in this paper, as it contains concepts required to understand the following subsection. Finally, the novel design method for a triangulated foldable scissor grid for single-curvature surfaces is explained.

2.1 The translational scissor unit

A translational scissor unit consists of two straight rods interconnected by a revolute joint at the intermediate hinge point. The imaginary lines connecting the upper and lower ends of the rods, called the unit lines (fig. 2), are parallel throughout all stages of the deployment. Geometrically this means that the rods and the unit lines always form two similar triangles or, in case of a symmetrical unit, two congruent triangles. All translational units have a single degree-of-freedom, characterised by the deployment angle θ. As θ changes, the unit deploys. Each value for θ corresponds with a certain value for the structural thickness t, which is measured between the lower and upper end points of the rods (fig. 2). A translational unit with two rods of equal length describes a planar geometry and is therefore referred to as a plane-translational unit (fig. 2a). Curvature can be introduced by using units with unequal bar lengths, called curved-translational units (fig. 2b and c).

Translational units can be linked at their ends to form planar or three-dimensional configurations. The most simple scissor linkage is the well-known lazy-tong mechanism (fig. 3a). It consists of a linear chain of symmetrical plane-translational units. An example of a curved two-dimensional linkage is shown in fig. 3b.

A common and effective way to achieve a three-dimensional scissor grid for a myriad of singly and doubly curved shapes makes use of translational units and is described in Zanardo [8]. Here, two planar scissor linkages are copied along each other to form a grid with quadrangular cells (fig. 4). In this configuration, the structural thickness t has to be identical throughout the structure. Due to its quadrangular cells, the grid requires bracing to solve issues concerning its in-plane stability.

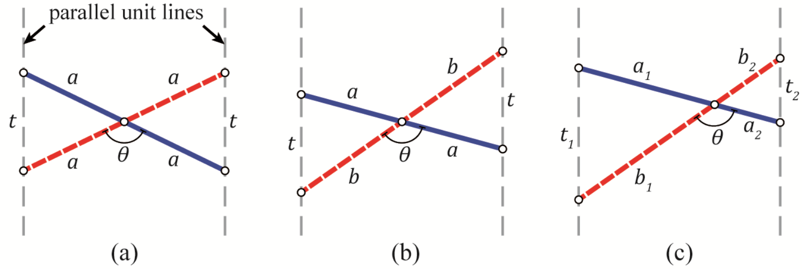


Figure 2: (a) A symmetrical plane-translational unit; (b) a symmetrical curved-translational unit; (c) a non-symmetrical curved-translational unit.

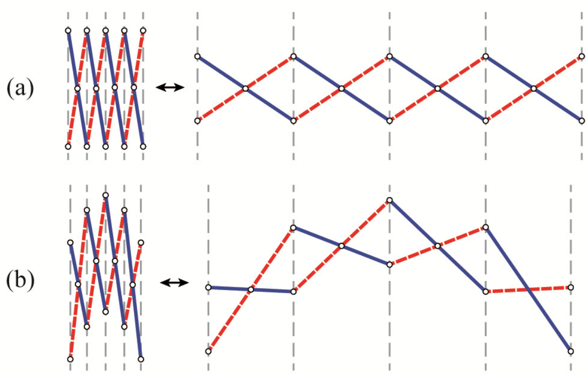


Figure 3: (a) A linkage of plane translational units (lazy-tong mechanism); (b) a linkage of curved translational units.

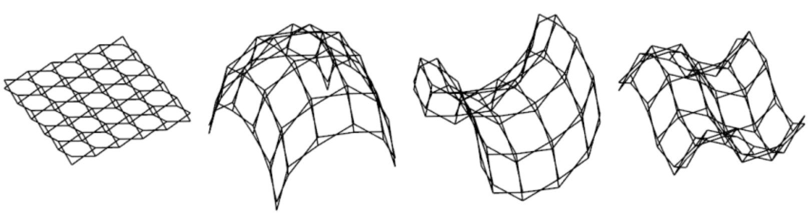


Figure 4: Foldable scissor grids with quadrangular cells based on translational scissor units.

2.2 Design of a planar linkage of translational units

Escrig [9] formulated the deployability equation which applies for any linkage of two translational scissor units (fig. 5):

$$k'_i + l'_i = k_{i+1} + l_{i+1} \tag{1}$$

This equation ensures that a linkage can be folded to its most compact state, which occurs when the rods of the scissor units overlap in the theoretical line model ($\theta = 0$ in fig. 2). As a result the entire linkage is reduced to a single line. It is not a necessary, nor a sufficient condition to obtain a working scissor mechanism. Nevertheless, since scissor structures are mainly implemented for their ability to expand, it is only logical to opt for a maximal expansion and thus to apply this condition.

Eqn (1) can be geometrically represented by an ellipse since the sum of the distances of any point of an ellipse to the foci of that ellipse is always constant. Consequently, the intermediate hinge points of any scissor unit linked at the same two nodes must all lie on a single ellipse for which the foci coincide with these nodes (fig. 5). The distance between the foci of this ellipse therefore equals the structural thickness t. Since they form a geometrical representation of the deployability equation, ellipses can be used as a tool to geometrically design a scissor linkage. For reasons of simplicity – and because it is usually the case – this geometric design method is now demonstrated for a linkage with a constant structural thickness t. As a result, all scissor units of this linkage will be symmetrical (fig. 2a, b) and all ellipses representing the deployability equation will be identical. Consider the curve c shown in fig. 6. This curve can be populated with translational scissor units by applying the following steps:

Step 1 Draw an ellipse centred at a point P_0 of this curve. The foci of this ellipse give the location of the upper and lower start points of the rods of the first scissor unit. The distance between its foci sets the structural thickness t and its major axis sets the direction of the unit lines (fig. 6a).

Step 2 Draw an ellipse that is double the size of the first ellipse and that is centred at the same point. The intersection point of this double ellipse with the curve c gives the location of the centre point P_1 of the next single ellipse. The foci of this single ellipse determine the location of the upper and lower end points of the first scissor unit (fig. 6b).

Step 3 Draw the first scissor unit by crosswise connecting the start and end points found in the previous two steps (fig. 6c).

Step 4 Using the intersection point found in step 3 and its corresponding single ellipse, repeat steps 2 to 4 to each time find the next scissor unit until the linkage is completed (fig. 6d).

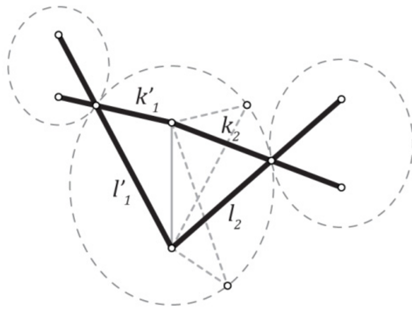


Figure 5: Geometrical representation of the deployability equation using ellipses.

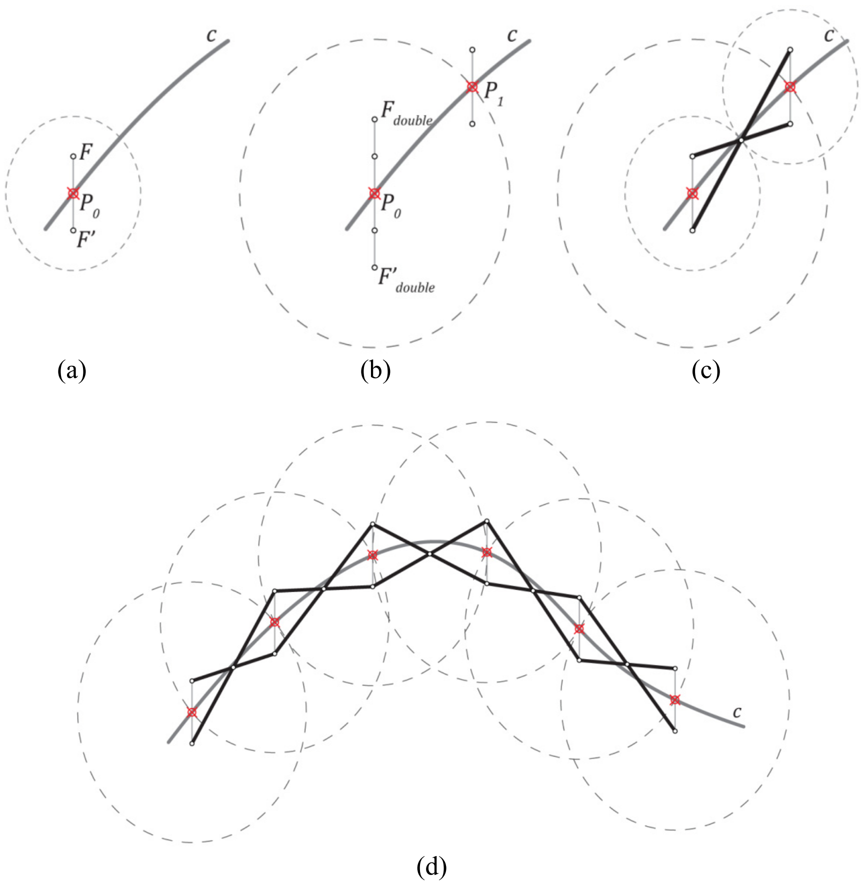


Figure 6: Geometric design method for a planar linkage of translational scissor units based on a curve *c*.

2.3 Design of a foldable and triangulated single-curvature scissor grid

A single cell of the proposed triangulated scissor grid consists of three translational units (fig. 7a). Langbecker [7] presented proof that such a unit can only be foldable if it consists solely of plane-translational units, which can only generate planar scissor grids. However, this proof was based on a triangular cell with fixed angles. As was first demonstrated in De Temmerman [4] a grid that does allow angular distortions is in fact foldable for single curvature surfaces. Fig. 7 demonstrated how the projected triangle of such a cell changes from isosceles to equilateral as it is folded.

In the triangulated grid, three main directions can be distinguished (fig. 8). In case of a flat surface, all three directions consist of only plane-translational units (fig. 8a). When single curvature is added, direction A still consists of plane-translational units, while directions B and C become curved-translational units (fig. 8b). In both cases, the structural thickness t is equal throughout the structure. To design this scissor grid, it is sufficient to design only a single strip of the grid (fig. 9). The rest of the grid can be generated by mirroring and copying this strip. The nodes of the strip are located on two parallel planes with a distance d between them. This distance d can be written as a function of the structural thickness t and the rod lengths L_A of the scissor units of direction A:

$$d = \frac{1}{2}\sqrt{{L_A}^2 - t^2} \tag{2}$$

Since the strip is still a three-dimensional geometry, its design is of a more complex nature than that of a planar scissor linkage: compared to section 2.2, the ellipses become ellipsoids of revolution and the base curve is extruded to form a surface. However, by shifting the design of a three-dimensional grid to its two-dimensional projection, this complexity can be largely reduced. This procedure is demonstrated in the following steps which allow to generate a foldable triangulated scissor grid based on a planar curve c.

Step 1 Define the desired structural thickness t and the rod lengths L_A of the scissor units of direction A. The distance d can be calculated using eqn (2).

Step 2 Draw a set of identical ellipses that are centred on this curve and each intersect the curve at the centre points of their neighbouring ellipses, similar to section 2.2. Their minor radius must equal $\sqrt{3}\mathrm{d}$ and their major radius must equal $\sqrt{3}\mathrm{L_A}/2$ (fig. 10a).

Step 3 For each ellipse, determine the points at a distance $\mathrm{t}/2$ from its centre along the positive and negative direction of its major axis. The resulting points give the locations of the upper and lower nodes of the scissor grid projected on one of the planes shown in fig. 9b. The projected scissor units can be obtained by crosswise interconnecting these points (fig. 10b).

Step 4 To translate the two-dimensional projection into the three-dimensional strip, project every other lower and upper point pair onto a parallel plane located at a distance d from the curve plane. Crosswise connecting this new set of end points gives the scissor units of direction B and C (fig. 10c).

Step 5 Mirror these scissor units along the projection plane and copy alongside each other a desired number of times. Complete the triangular cells by crosswise interconnecting the end points in direction A (fig. 10d).

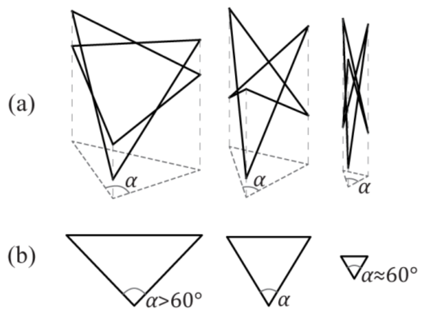


Figure 7: The angles of a non-planar triangular cell composed of translational scissor units change as it deploys: (a) isometric view; (b) top view.

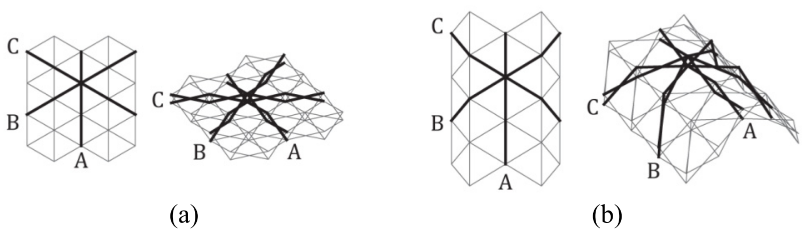


Figure 8: Indication of the main directions in a planar (a) and a curved (b) triangulated scissor grid.

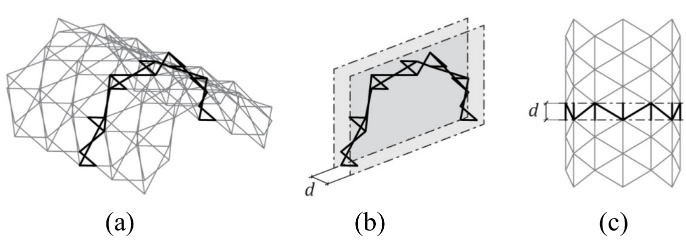


Figure 9: Strip of the scissor grid which, when mirrored and copied, generates the entire scissor grid: (a) the strip in the structure; (b) the strip located between two parallel planes; (c) top view of the strip in the structure.

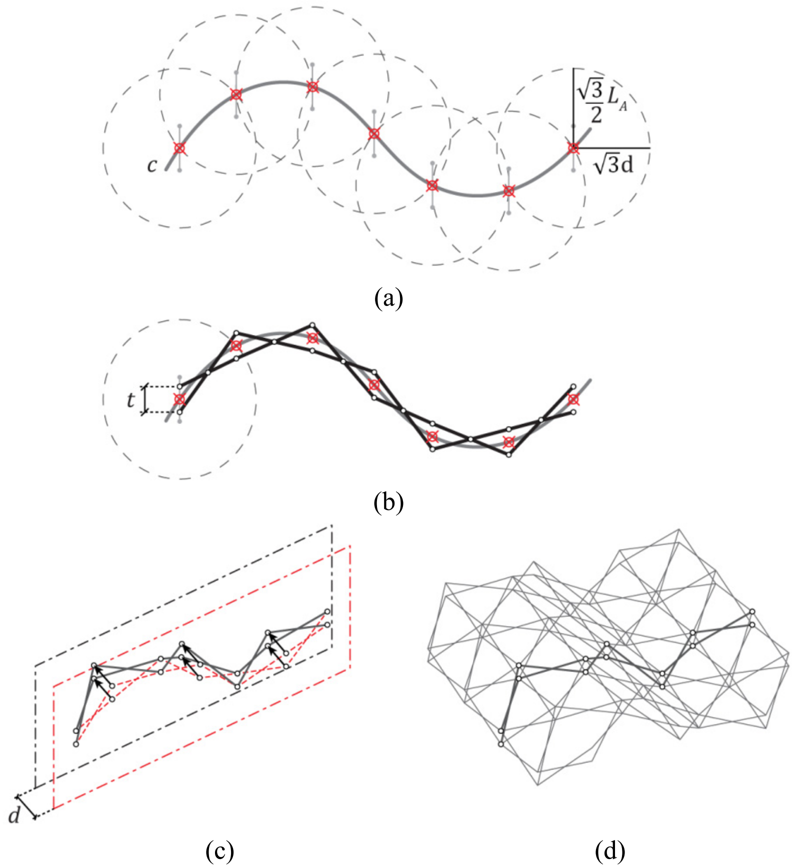


Figure 10: Geometric design method for a foldable triangulated scissor grid consisting of translational scissor units and based on a planar curve c.

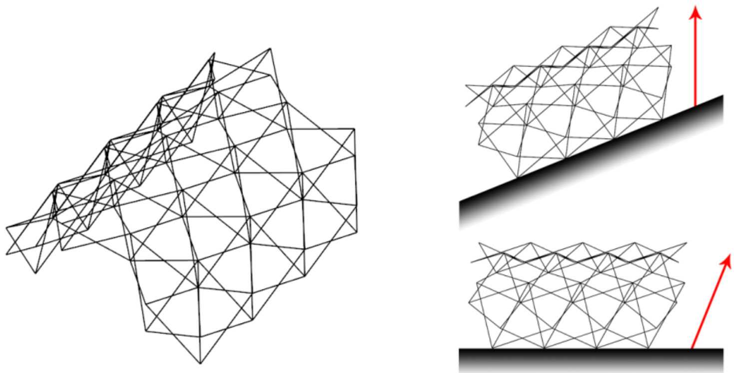
3 Variations of the concept

The triangulated scissor grid as described above does not cover the full geometric potential of foldable triangulated scissor grids based on translational scissor units. Two variations exist, leading to altered geometries.

In a first variant, the identical plane-translational units in direction A of the grid (fig. 8) are replaced by identical curved-translational units. The units in direction B are no longer a mirrored version of the units in direction C, thus increasing the amount of unique components required. The resulting grid still describes the same single-curvature geometry, only it is now slanted (fig. 11). This might make them better fit for a similarly slanted terrain, but less fit for any other terrain (fig. 11b).

The second variant maintains the plane-translational units in direction A, but linearly reduces the structural thickness in direction B and C. All units in directions B and C have the same proportions, but diminish in size. The units of each row in direction A shrink proportionally. As a result, directions B and C describe straight lines, while the scissor grid now curves in direction A and in doing so it describes a conical surface. This cone is however truncated, as the rod lengths of the scissor units approach zero as they near the top. Consequently, a tent based on this configuration will have an oculus at its peak (fig. 12).

These variants require alterations in the design method described in the previous section. However, since they are less likely to be applied in the built environment due to a reduced flexibility or due to a limited added value compared to the added complexity, their design method is not covered in this paper.



(a) (b)

Figure 11: A foldable triangulated scissor grid populated only by curved-translational units: (a) isometric view; (b) side view of the structure on a slanted and flat terrain; the arrow marks the unit line direction.

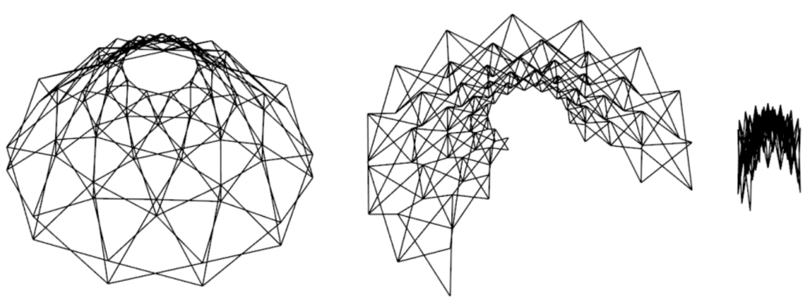


Figure 12: A foldable triangulated scissor grid with varying structural thickness describing a conical surface: line model in three stages of the deployment.

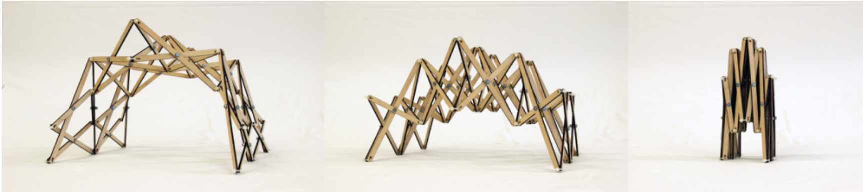


Figure 13: Scale model of a foldable scissor structure based on the proposed concept: three stages of deployment.

4 From theoretical model to application

The design method presented in section 2.3 generates theoretical zero-thickness line models of a foldable triangulated scissor grid. To translate this theoretical model into a buildable one, the lines have to be converted to rods with a discrete thickness and the nodes have to be converted to joints which take up a certain space and volume. Additionally, the joints have to provide the correct rotational degrees of freedom to the rods. It is in this latter fact that the proposed configuration is more complex than its counterparts consisting of for example quadrangular cells: not only do the joints have to allow a rotational movement of the rods of the scissor units in their respective support planes – as is the case for any scissor mechanism –, it also has to enable the angular distortions of the grid. This will require a special joint, for which a first proposal was given in De Temmerman [4]. Nevertheless, the disadvantage that this increased level of complexity in the joint design brings seems to be small compared to the advantages that characterise this configuration. Indeed, a first scale model has clearly demonstrated a highly rigid grid combined with a smooth stress-free deployment process (fig. 13).

Next to being buildable, the model also needs to be applicable in the built environment in order for it to be useful. Fortunately, single-curvature geometries lend themselves very well to a variety of covers. A well-known shape is the barrel vault, well suited for generating self-supporting scissor structures which can serve as mobile and rapidly erectable shelters. The curvature can freely be chosen by the designer depending on possible constraints as well as functional, structural and aesthetical parameters and it can for example be circular, elliptic or parabolic (fig. 1). Other shapes are possible as well, such as overhanging canopies. These will require external supports and are thus more likely to be implemented as transformable components fixed to static constructions.

5 Conclusions and future work

Curvature, triangulation and foldability are properties that have proven to be very beneficial for scissor grids. The configuration discussed within this paper combines these three properties for a large variety of single-curvature grids. This scissor grid can for example take the shape of a barrel vault, which forms the basis for a mobile self-supporting shelter. By basing the design method of the three-dimensional scissor grid on its two-dimensional projection, it becomes similar to the design method of a planar scissor linkage, thus significantly reducing its complexity. Slight alterations to the proposed configuration have uncovered two variants, further increasing the geometrical possibilities. However, the practical use of these variants seems to be limited.

The envisioned architectural applications of the proposed configuration combined with the promising structural and kinematical behaviour of the scale model has awoken our interest in this type of scissor grid. Therefore we will further investigate it, aiming to develop a full scale prototype of a rapidly and easily deployable self-supporting shelter. In this future research the greatest focus will lie on the design of the joints, as this will largely determine the success or failure of the proposed concept on a larger scale. Other critical design decisions will include the materialisation of the components, the design and attachment of a membrane cover and the anchorage of the erected structure.

Acknowledgement

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