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Numerical analysis of steady-state radiative-conductive heat transfer in selected plastic

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Abstract

Heat transfer by simultaneous conduction and radiation in an optical thick and scattering medium is considered. Consideration is given to a cylindrical coordinate system consisting of two infinite, isothermal, parallel plates separated by a finite distance. The space between the plates is filled with a selected plastic. The thermal conductivity λ of it consists of two components: the phonon λ_c = const and the radiative λ_r thermal conductivity. The Rosseland's diffusion model of radiation - conduction heat transfer to determine the extinction coefficient E is applied. Dependence of thermal conductivity λ and extinction coefficient E on sample thickness l has been taken into consideration. The problem is formulated in terms of a nonlinear differential equation and the numerical calculations of steady-state temperature field in the block polymerized polyamide 6 are obtained.

1 Introduction

Heat transfer in high-molecular polymers is of a great importance for their usage in engineering practice. Polymer substances of a high-density and of a compact structure are of the investigators interest for their usage as materials of construction of heat exchangers. Thermal efficiency of polymer heat exchangers can be comparable to the ones made of metals.
The radiation contribution in the heat transfer in the material is the main cause of discrepancies between measured and theoretically calculated values of (for instance) temperature or heat flux density inside the plates. The research method of heat transfer in polymer substances of a high - density and of a compact structures can not be simplified to take only the heat conduction into consideration. The radiation heat transfer, particular at lower temperatures, is as a rule, not taken into account. The radiation contribution is not predominant in

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the polymers of a high - density and of a compact structures but it is important, however due to its coupling with the conduction. Determination of this contribution in heat transfer is very important in engineering practice.

One of the research directions of the coupled heat transfer with its radiation component is taking into account a whole set of thermophysical parameters such as: the absorption coefficient, the scattering coefficient and the scattering phase-function. Another approach is based on replacement of these components by the thermal conductivity λ only. The thermal conductivity λ can be divided into two components, that means: the phonon λ_c and the radiative λ_r thermal conductivity. Numerical calculations based on above defined thermal conductivity λ are closer to reality, that means, that the calculations can be directly compared with the experimental results of temperature distributions inside the medium. Parabolic equation describing the heat transfer is easy to solve numerically. Calculations carried out in such a case can be useful to explain the following mechanisms of heat transfer concerning the radiative heat conductivity. Such an approach for description of the phenomenon (coupled radiative-conductive heat transfer) seems to be much simpler and more precisely than the integral equations in which most of parameters still become unknown for the specified material. The proposed research is - in our opinion - of great importance for engineering practice. It is noteworthy that the authors of this work have experimentally stated the dependence of radiative thermal conductivity λ_r on a location inside the medium [1]. Both the dependence of radiation intensity on a sample's thickness and on its temperature distributions and on the boundary conditions is given in professional literature for other materials [3].

In order to provide some insight into the problem of heat transfer by simultaneous conduction and radiation in a high-molecular plastic, consideration was given to a cylindrical coordinate system consisting of two isothermal, parallel plates separated by a finite distance. The space between the plates was assumed to be filled with a thermal radiation absorbing, emitting and scattering medium: block - polymerized polyamide 6. An experimental dependence of thermal conductivity of polyamide 6 on temperature difference between the so called „hot" and „cold" surface of a sample and on its thickness has been taken into consideration. The solution of this problem is describe in the following.

2 Description of the problem and theoretical model

The purpose of this investigation was to provide simple insight and to demonstrate an approach to the situation of heat transfer problems by simultaneous conduction and radiation which can be used for an explanation of the anomaly of a temperature distribution in the specimens being under steady - state heat transfer conditions.

Let us consider the polyamide 6 sample . The heat flux is proportional to the thermal conductivity and to the local temperature gradient. The effects of short and long range interactions in the sample of high thickness are strictly

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connected with radiation. As an example, studies of the radiative absorption by using a spectrophotometer are very often carried out for a very thin samples (hundredths of millimetre) and for that reason the influence of the short range and the long range interactions within the sample of complex structure of high thickness materials, are neglected. In real case, the polyamide 6 is much more transparent to thermal radiation in comparison to its absorption investigations [1].

A schematic diagram of a physical system is shown in Fig.3 and Fig.1 and may represent, for example, one layer of an insulating material (Fig.3) or a sample in guarded hot plate apparatus (Fig.1) [1, 2].

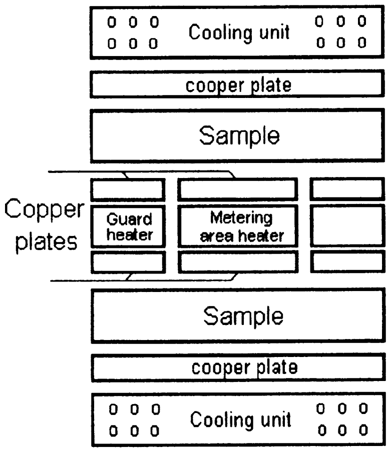

Figure 1: Measurement pile of a guarded hot plate apparatus for studies of thermal conductivity of plastic.

A guarded hot plate apparatus was applied for studies of the thermal conductivity and coupled radiation - conduction heat transfer [1]. Two identical specimens of 200 mm in diameter were put symmetrically one on each side of an electrical heater. Two cooling plates were the outer parts of the measurement pile. The basic electric heater was surrounded by a guard of a ring one.

In polyamide 6 samples the temperature slip ΔT_s, (which can be understood as follows: $\Delta T_s = T(z{=}0) - T_c$, where $T(z{=}0)$ denotes the temperature of the surface of the specimen from the cooler's side and T_c denotes the temperature of the cooler), at the specimen - cooler boundary has been stated experimentally when the difference between the heater temperature T_h and the cooler temperature T_c was greater than $\Delta T > 20$ K. The obtained temperature distributions were of straight lines shape inside the specimens for all temperature differences ΔT. The measured temperature slip ΔT_s took place only from the coolers side [1,2]. The results of the measured temperature slip ΔT_s as a function of the specimen thickness and the temperature difference ΔT for polyamide 6 are shown in Fig.2. The temperature differences, under a constant value of heater's temperature $T_h = 65$ °C were examined. Further calculations and considerations have been limited only into the case $\Delta T = 65$K. The thermal conductivity was derived from the relationship

$$\lambda = \frac{q_{ex}}{\Delta T_{real}} \tag{1}$$

where $\Delta T_{real} = T_h - (T_c - \Delta T_s)$; q_{ex} - experimental heat flux density
Approximation of the polyamide 6 thermal conductivity vs. a sample thickness for $\Delta T = T_h - T_c = 65$K is shown in Fig.3.

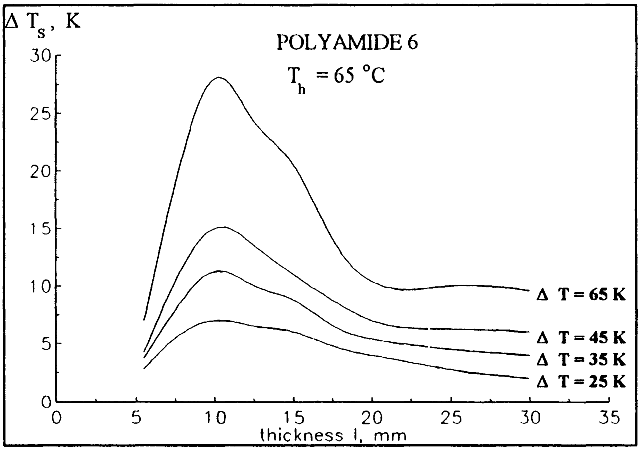

Figure 2: Dependence of the measured temperature slip ΔT_s near the cooler for the polyamide 6 sample on its thickness l for various temperature difference values ΔT between the heater T_h and the cooler T_c .

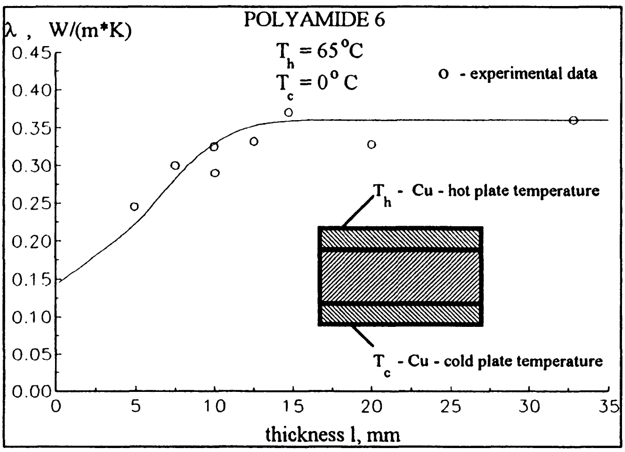

Figure 3: Approximation of polyamide 6 thermal conductivity experimental data vs. a specimen thickness l and a schematic diagram of physical system ($\Delta T = T_h - T_c = 65$ K).

2.1 Problem formulation

Let us consider the plane-parallel plate of cylindrical form having thickness l and diameter $\phi >> l$ in which conductive-radiative heat exchange is considered. The Rosseland's diffusion model of radiation-conduction heat transfer has been applied. The thermal conductivity λ, shown in Fig.3, consists of two components, the phonon λ_c and the radiative λ_r thermal conductivity (the last component also depends on the location z inside the medium [1, 2])

$$\lambda = \lambda_c + \lambda_r \tag{2}$$

The following mathematical model of steady-state heat transfer in polyamide 6 sample has been examined:

$$div(\vec{q}_c + \vec{q}_r) = 0 \tag{3}$$

with

$$\vec{q}_c = -\lambda_c \vec{grad}T(r,z) \tag{4}$$

$$\vec{q}_r = -\frac{16\sigma n^2}{3E(z)} T^3 \vec{grad}T(r,z) \tag{5}$$

$$T_{Cu} = \begin{cases} T_1, & z = 0 \\ T_2, & z = l \end{cases} \tag{6}$$

where

$E(z)$ - the extinction coefficient;
σ - Stefan-Boltzmann constant;
T_{Cu} - Cu - plate temperature;

and with boundary conditions

$$T\big|_{z=0} = T_1, \qquad T\big|_{z=l} = T_2 \tag{7}$$

$$\lim_{r\to 0}\frac{\partial T}{\partial r} = 0, \qquad T\big|_{r=l} = \frac{T_2 - T_1}{l} z + T_1 \tag{8}$$

The linear temperature distribution at $r=l$ (Eq.8) has been confirmed experimentally. To determine the radiative thermal conductivity λ_r one can assume that the phonon thermal conductivity λ_c is practically constant and equals to $\lim_{l\to 0}\lambda(l) = 0.151$ $Wm^{-1}K^{-1}$ (for the polyamide 6 sample). Therefore, knowing the experimentally obtained thermal conductivity dependence $\lambda = \lambda(l)$, (see Fig.3), the radiative thermal conductivity $\lambda_r(l)$ is calculated from relationship

$$\lambda_r(l) = \lambda(l) - \lambda_c \tag{9}$$

and is shown in Fig. 4.

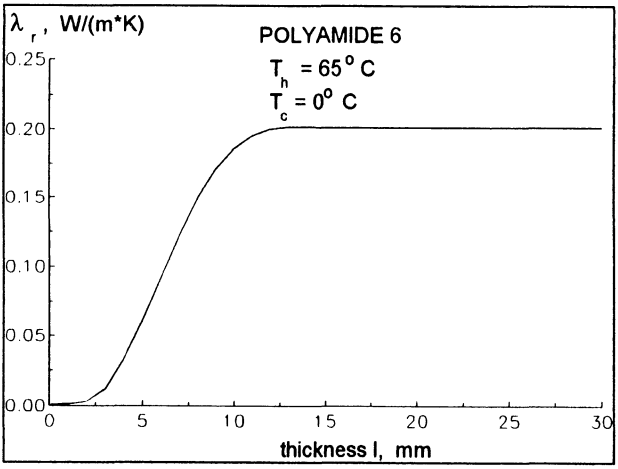

Figure 4: Dependence of radiative thermal conductivity λ_r on sample's thickness l under constant temperature difference $\Delta T = 65$ K for polyamide 6.

Assumption of additive approximation of the heat flux density and the Rosseland's diffusion model permit to express the heat flux density in a form:

$$q = -\left[\lambda_c + \frac{16\sigma n^2}{3E(z)} T^3\right]\frac{dT}{dz} \tag{10}$$

Therefore, the radiative thermal conductivity λ_r can be given as:

$$\lambda_r(z) = \frac{16\sigma n^2}{3E(z)} T^3 \tag{11}$$

According to the Rosseland's diffusion model the radiative thermal conductivity is equal to

$$\lambda_r(l) = \frac{4\sigma\, n^2\,(T_h^4 - T_c^4)}{3E(T_h - T_c)} \tag{12}$$

where the temperature (T^3) can be rewritten in a form

$$T^3 = \frac{1}{4}(T_h + T_c)(T_h^2 + T_c^2) \tag{13}$$

which leads us to a final expression for $E(l)$ in a form

$$E(l) = \frac{4\sigma n^2\,(T_h + T_c)(T_h^2 + T_c^2)}{3\lambda_r(l)} \tag{14}$$

which is shown in Fig.5

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Initial value of $E(l \to 0) = 30000\ \mathrm{m}^{-1}$ has been determined experimentally by using Perkin-Elmer spectrophotometer 1725X [1]. The refraction coefficient n for polyamide 6 sample was assumed to be n = 1.54 [1].

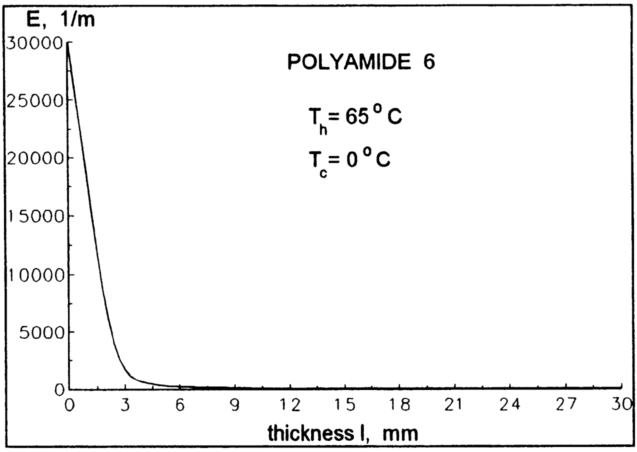

Figure 5: Dependence of extinction coefficient *E(l)* on polyamide 6 sample's thickness and under condition $\Delta T = 65$ K

2.2 Method of solution

The steady-state heat transfer problem for polyamide 6 sample has been carried out numerically by means of a finite difference scheme.
A two-dimensional grid

$$\begin{aligned} &\omega_h\{(r_i, z_j):\ r_i = ih_r,\ h_r = R/IK,\ i = \overline{0, IK}; \\ &z_j = jh_z,\ h_z = l/JK,\ j = \overline{0, JK}\} \end{aligned} \tag{15}$$

has been generated and the method of local one-dimensional approximation with respect to coefficients' iterations :

$$\begin{aligned} \Lambda_r[\lambda \mathrm{u}^n] &= 0 \\ \Lambda_z[\lambda \mathrm{u}^n] &= 0 \end{aligned} \tag{16}$$

$n = 0,1,..$ (number of iteration)

where

$$\Lambda_r[\lambda \mathbf{u}] = \frac{2}{(r^2_{i+1/2} - r^2_{i-1/2})}\left[\lambda_{i+1/2}\left(\frac{\mathbf{u}_{i+1} - \mathbf{u}_i}{h_r}\right) - \lambda_{i-1/2}\left(\frac{\mathbf{u}_i - \mathbf{u}_{i-1}}{h_r}\right)\right] \tag{17}$$

$(i = 1,2,..,IK\text{-}1)$

$$\Lambda_z[\lambda \mathbf{u}] = \frac{1}{h_z}\left[\lambda_{j+1/2}\left(\frac{\mathbf{u}_{j+1} - \mathbf{u}_j}{h_z}\right) - \lambda_{j-1/2}\left(\frac{\mathbf{u}_j - \mathbf{u}_{j-1}}{h_z}\right)\right] \tag{18}$$

$(j = 1,2,..,JK\text{-}1)$

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$$\begin{aligned} \lambda_{i\pm1} &= 0.5(\lambda(u_{i\pm1,j}) + \lambda(u_{i,j})) \\ r_{i\pm1} &= 0.5(ih_r + (i \pm 1)h_r) \end{aligned} \tag{19}$$

has been applied.

The extinction coefficient's $E(z)$ values are changing considerably within a few millimetres of the sample thickness l (see Fig.5), so the following approximation has been assumed

$$E(z) = \begin{cases} A[1+\exp(B-Cz)]^{(-1/D)}, & 0 < z \leq 4 \\ A = 40825.951,\ B = -1.3236141,\ C = -1.2001553,\ D = 0.77396241 & \\ A - B\exp(-Cz^D), & 4 < z \leq 30 \\ A = 451.37261,\ B = 352.45942,\ C = 3148.3907,\ D = -4.9312582 & \end{cases} \tag{20}$$

2.3 Temperature slip determination and the results

It has been stated experimentally that the temperature slip ΔT_s at the border specimen-cooler (z=0) takes place for the polyamide 6 sample and its value is a function of the sample thickness l. Exemplary solution of the boundary problem (3)÷(9) for the 30 mm of polyamide 6 sample (ΔT = 65 K) presented in Fig.6. In Fig.6 we can see the temperature distribution obtained numerically in which the curve 1 presents the case of pure conduction ($\lambda = \lambda_c$) and the curve 2 presents the case of radiation-conduction coupled heat transfer ($\lambda = \lambda_c + \lambda_r$).

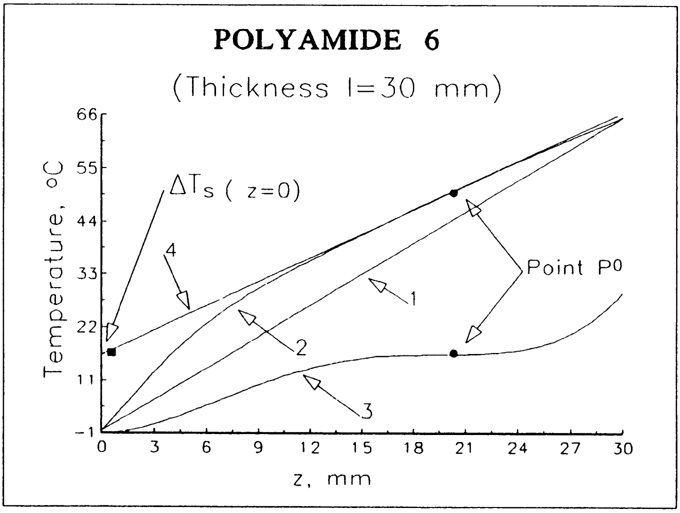

Figure 6: Calculated temperature distribution in the polyamide 6 sample.
1-pure conduction, 2-radiation-conduction, 3-first derivative of curve 2 at point (z=0), 4-extrapolation of linear section of curve 2.

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Extrapolation of the linear section of curve 2 at point (z=0) is treated as the temperature slip ΔT_s, and this value is verified experimentally. To do this, the calculated values (curve 2) have been best fitted with polynomial of the 4-th degree

$$T(z) \approx \sum_{i=0}^{4} a_i z^i \tag{21}$$

with coefficients

$$a_0 = -0.51807039,\ a_1 = 4.9718166,\ a_2 = -0.2499142,$$
$$a_3 = 8.42234 \cdot 10^{-3},\ a_4 = 1.066170 \cdot 10^{-4}$$

(standard error $S = 0.1461$, correlation coefficient $r = 0.99996$)

Then, deriving its first derivative $T'(z=0)$, curve 3 has been obtained. Point P^0 is the point where the tangent to the curve $T(z)$ (curve 2) best fits to the linear section of the considered temperature distribution $T(z)$. Repeating such a procedure for other sample's thickness, the maximum of the temperature slip is observed for the thickness l =10 mm (see Fig.7) and the calculated value $(\Delta T_s)_{cal}$ =27.4 °C is in a good agreement with the experimental one $(\Delta T_s)_{ex}$ = 28 °C .

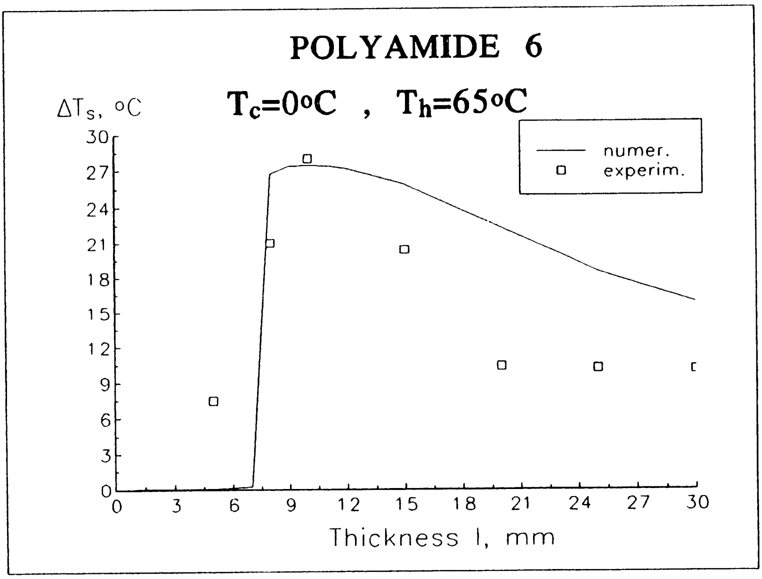

Figure 7: Calculated and experimentally measured temperature slip ΔT_s for polyamide 6 sample.

Discrepancies between the measured and the calculated values of the temperature slip ΔT_s (for thickness l greater than 12 mm) may result from the

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errors of determining the extinction coefficient E. According to the numerical calculations, thermal emissivity has a very small influence on the temperature distribution inside the sample (only for a small sample's thickness l < 2 mm).

3 Conclusions

- The method proposed in this paper assuming a constant thermal conductivity λ_c may be applied for many plastics of compact structure and high density.
- Determination of the temperature slip as a result of making tangent to a curve of the temperature distribution inside the specimen and its extrapolation to the boundary point (z=0) is, of course, a way to avoid difficulties connected with the solution of the integral equation describing the problem.
- The results of experimental data of the thermal conductivity, which best characterize real working conditions of insulating materials, are used to carry out numerical calculations of coupled radiative-conductive heat transfer.
- Knowledge of the temperature slip ΔT_s may be used to match an optimum thickness of insulating material.
- The proposed method makes use of a real extinction coefficient E which describes effects of the short and long-range interactions in a sample of a few centimetre thickness.

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