Viscous dissipation and temperature dependent viscosity effects in simultaneously developing flows in flat microchannels with convective boundary conditions

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Abstract

The effects of viscous dissipation and temperature dependent viscosity in simultaneously developing laminar flows of liquids in straight microchannels of arbitrary but constant cross-section are studied with reference to convective thermal boundary conditions. Viscosity is assumed to vary linearly with temperature, in order to allow a parametric investigation, while the other fluid properties are held constant. A finite element procedure, based on a projection algorithm, is employed for the step-by-step solution of the parabolized momentum and energy equations. Axial distributions of the local overall Nusselt number and of the apparent Fanning friction factor in flat microchannels are presented with reference to both heating and cooling conditions for two different values of the Biot number. Examples of temperature profiles at different axial locations are also shown.

Keywords: laminar forced convection, microchannels, entrance region, temperature dependent viscosity, viscous dissipation, convective boundary conditions.

1 Introduction

In several heat transfer problems concerning microchannel flows, the thermal resistance of the channel wall must be taken into account in order to obtain accurate solutions. Moreover, in many situations of practical interest, fluid velocity and temperature fields develop simultaneously, resulting in overlapping hydrodynamic and thermal entrance regions. This occurs when fluid heating or cooling begins at the microchannel inlet, where also the velocity boundary layer

starts developing. In such a situation, entrance effects on fluid flow and forced convection heat transfer cannot be neglected if, as it happens very often in laminar flows, the total length of the microchannel is comparable with that of the entrance region. Temperature dependence of fluid properties can also play an important role in the development of velocity and temperature fields. If, as it is assumed in this paper, the fluid is a liquid, viscosity is the property which exhibits the most relevant variations with respect to temperature. Therefore, the main effects of temperature dependent fluid properties can be retained even if only viscosity is allowed to vary with temperature, while the other properties are assumed constant. Finally, viscous dissipation effects cannot often be neglected in ducts with very small hydraulic diameters, like microchannels, even for ordinary liquids, characterised by moderate values of viscosity. It is worth noting that, with temperature dependent viscosity, viscous dissipation affects both temperature and velocity distributions along the microchannel. Even if all these aspects have already been considered in the past, to the authors' knowledge no systematic studies are reported in the literature taking into account the combination of wall thermal resistance, entrance, temperature dependent viscosity and viscous dissipation effects. Only recently, the authors have presented a couple of papers where the influences of the last three factors are analyzed, but the wall thermal resistance is neglected [1, 2]. It must be noticed, however, that a simplified way to account for it and, possibly, also for the external convection resistance, is represented by the specification of convective boundary conditions at the wall, with an appropriately defined heat transfer coefficient, when solving the energy equation. This approach is adopted in the present paper which, thus, represents an extension of the previous work.

In the past decades, many authors have investigated, both analytically and numerically, simultaneously developing flows in straight ducts of constant cross-section. Comprehensive reviews of these theoretical studies, referring to ducts of different cross-sectional geometries, can be found in [3, 4]. However, since a basic assumption made in almost all such studies is that fluid properties are constant, the corresponding solutions are adequate only for problems involving small temperature differences. In fact, experimental results for problems involving large temperature differences substantially deviate from constant property solutions [3, 5].

In this paper, we present the results of a parametric study on the simultaneously developing laminar flow of a liquid in flat microchannels. The effects of wall thermal resistance, temperature dependent viscosity and viscous dissipation on heat transfer and pressure drop are investigated, while the other liquid properties are considered constant. A finite element procedure [6], based on a SIMPLE-like algorithm [7], is employed for the step-by-step solution of the parabolized momentum and energy equations [7, 8] in a two-dimensional domain corresponding to the cross-section of the duct. Due to the high value of the ratio between the total length and the hydraulic diameter in microchannels, such an approach is very advantageous with respect to that based on the steady-state solution of the set of governing elliptic equations in a three-dimensional domain corresponding to the whole microchannel. The above procedure has already been used, disregarding viscous dissipation effects, in the simulation of simultaneously developing flows of liquids

with temperature dependent viscosity in straight macro- and microchannels [9, 10] and, on the additional assumptions of non-negligible viscous dissipation effects, in the study of thermally and simultaneously developing liquid flows in microchannels when prescribed constant temperature boundary conditions are applied to the rigid walls [1, 2]. Here the same procedure is used to study a similar problem with reference to flows in flat microchannels with convective thermal boundary conditions. In order to allow a parametric investigation, viscosity is assumed to vary linearly with temperature in the range considered.

2 Mathematical model

When the effects of axial diffusion can be neglected and there is no recirculation in the longitudinal direction, steady-state flow and heat transfer in straight microchannels of constant cross-section are governed by the continuity and the parabolized Navier-Stokes and energy equations. With reference to incompressible fluids with temperature dependent thermophysical properties, in the hypotheses of negligible body forces and significant effects due to viscous dissipation, these equations can be written in the following form

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \quad (1)$$

$$\rho u \frac{\partial u}{\partial x} = \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left(\mu \frac{\partial u}{\partial z} \right) - \rho v \frac{\partial u}{\partial y} - \rho w \frac{\partial u}{\partial z} - \frac{d\bar{p}}{dx} \quad (2)$$

$$\begin{aligned} \rho u \frac{\partial v}{\partial x} = & \frac{\partial}{\partial y} \left[2\mu \frac{\partial v}{\partial y} - \frac{2}{3} \left(\mu \frac{\partial v}{\partial y} + \mu \frac{\partial w}{\partial z} \right) \right] \\ & + \frac{\partial}{\partial z} \left(\mu \frac{\partial w}{\partial y} + \mu \frac{\partial v}{\partial z} \right) - \rho v \frac{\partial v}{\partial y} - \rho w \frac{\partial v}{\partial z} - \frac{\partial p}{\partial y} \end{aligned} \quad (3)$$

$$\begin{aligned} \rho u \frac{\partial w}{\partial x} = & \frac{\partial}{\partial z} \left[2\mu \frac{\partial w}{\partial z} - \frac{2}{3} \left(\mu \frac{\partial v}{\partial y} + \mu \frac{\partial w}{\partial z} \right) \right] \\ & + \frac{\partial}{\partial y} \left(\mu \frac{\partial w}{\partial y} + \mu \frac{\partial v}{\partial z} \right) - \rho v \frac{\partial w}{\partial y} - \rho w \frac{\partial w}{\partial z} - \frac{\partial p}{\partial z} \end{aligned} \quad (4)$$

$$\rho c u \frac{\partial t}{\partial x} = \frac{\partial}{\partial y} \left(k \frac{\partial t}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial t}{\partial z} \right) - \rho c v \frac{\partial t}{\partial y} - \rho c w \frac{\partial t}{\partial z} + \mu \Phi_v \quad (5)$$

where

$$\begin{aligned} \Phi_v = & 2 \left[\left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 \\ & + \left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 - \frac{2}{3} \left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)^2 \end{aligned} \quad (6)$$

According to the assumption of parabolic flow, all the derivatives in the axial direction are neglected in the diffusive terms of eqns. (2) to (6) [8]. In the above equations, x and y, z are the axial and the transverse coordinates, respectively, while u and v, w represent the axial and the transverse components of velocity. Finally, t is the temperature, p is the deviation from the hydrostatic pressure, $\bar{p}$ is its average value over the cross-section, while ρ, μ, c and k represent density, dynamic viscosity, specific heat and thermal conductivity of the fluid, respectively.

The solution domain can be bounded by rigid walls or by symmetry axes. On rigid boundaries the usual no-slip conditions, that is, $u = v = w = 0$, are imposed together with the convective boundary condition $q'' = -k\partial t/\partial n = h_a(t - t_a)$, where q'' is the local wall heat flux, n is the outward normal to the boundary, h_a is the external convective heat transfer coefficient and t_a is the ambient fluid temperature. Instead, symmetry conditions are $\partial u/\partial y = \partial w/\partial y = 0, v = 0$ and $\partial t/\partial y = 0$ on boundaries perpendicular to the y axis and $\partial u/\partial z = \partial v/\partial z = 0, w = 0$ and $\partial t/\partial z = 0$ on boundaries perpendicular to the z axis.

The model equations are solved using a finite element procedure which represents an extended version of one previously developed for the analysis of the forced convection of constant property fluids in the entrance region of straight ducts [6]. The added new features mainly consist in the possibility of taking into account the effects of temperature dependent properties and of viscous dissipation. The adopted procedure is based on a segregated approach which implies the sequential solution of momentum and energy equations on a two-dimensional domain corresponding to the cross-section of the channel. A marching method is then used to move forward in the axial direction. The pressure-velocity coupling is dealt with using an improved projection algorithm already employed by one of the authors (C.N.) for the solution of the Navier-Stokes equations in their elliptic form [11].

3 Numerical results

As stated above, the laminar forced convection in the entrance region of flat microchannels with convective thermal boundary conditions is studied. The hypotheses made here are that viscous dissipation effects are not negligible and that liquid heating/cooling begins at the microchannel inlet, where also the velocity boundary layer starts developing. Therefore, at the entrance of the microchannel, uniform values of the velocity u_e and of the temperature t_e can be specified as the appropriate inlet conditions. At the microchannel wall the convective boundary condition $q''_w = h_a(t_w - t_a)$ is applied, where q''_w and t_w are the local wall heat flux and temperature, respectively.

The dynamic viscosity is assumed to vary linearly in the temperature range considered and μ_e and μ_a are its values at t_e and t_a , respectively. The temperature dependence of the dynamic viscosity in the range between t_e and t_a is represented by the ratio $\mu_e/\mu_a = \text{Pr}_e/\text{Pr}_a = \text{Re}_a/\text{Re}_e$, where $\text{Pr}_e = \mu_e c/k$ and $\text{Pr}_a = \mu_a c/k$ are local Prandtl numbers, while $\text{Re}_e = \rho u_e D_h/\mu_e$ and $\text{Re}_a = \rho u_e D_h/\mu_a$ are local Reynolds numbers based on the hydraulic diameter D_h . Note that, while the values of local Reynolds and Prandtl numbers depend on temper-

ature, the local Péclet number $Pe = RePr = Re_e Pr_e = Re_a Pr_a$ always has the same value. Since the viscosity of liquids decreases with increasing temperature, $Pr_e/Pr_a > 1$ corresponds to fluid heating ($t_e < t_a$) and $Pr_e/Pr_a < 1$ to fluid cooling ($t_e > t_a$), while $Pr_e/Pr_a = 1$ refers to isothermal flows ($t_e = t_a$) or to constant viscosity fluids. Moreover, the reference Brinkman number $Br_m = \mu_m u_e^2 / [k(t_e - t_a)]$, evaluated at the reference fluid temperature $t_m = (t_e + t_a)/2$, is negative for fluid heating and positive for fluid cooling. In all the computations, the same values $Re_m = \rho u_e D_h / \mu_m = 500$ and $Pr_m = \mu_m c / k = 5$ of the Reynolds and Prandtl numbers have been assumed. Therefore, for the values of the ratio $Pr_e/Pr_a = 1/2, 2/3, 1, 3/2$ and 2 considered here, the local Reynolds number varies between 375 and 750 in the temperatures range between t_e and t_a . In addition to $Br_m = 0$, corresponding to negligible viscous dissipation, two reasonable non-zero values of the reference Brinkman number have been chosen, namely, $Br_m = \pm 0.01$ and ± 0.1 . Finally, two values of the Biot number $Bi = h_a D_h / k = 5$ and 20 have been selected as representative of situations where convective boundary conditions should be specified.

In the following, numerical results concerning axial distributions of $f_{app} Re_m$ and of the local overall Nusselt number $Nu_o = U D_h / k$ are presented. In these expressions f_{app} is the apparent Fanning friction factor defined as [3]

$$f_{app} = \frac{(\bar{p}_e - \bar{p}) D_h}{2 \rho u_e^2 x} \quad (7)$$

while

$$U = \frac{q_w''}{t_b - t_a} \quad (8)$$

is the local overall heat transfer coefficient, where t_b is the bulk temperature [3, 4]. The local overall heat transfer coefficient U can be expressed as [3, 4]

$$U = \left(\frac{1}{h} + \frac{1}{h_a} \right)^{-1} \quad (9)$$

with the local convective heat transfer coefficient h defined as

$$h = \frac{q_w''}{t_b - t_w} \quad (10)$$

On the basis of eqn. (9), the local overall Nusselt number Nu_o can be expressed as

$$Nu_o = \left(\frac{1}{Nu} + \frac{1}{Bi} \right)^{-1} \quad (11)$$

where $Nu = h D_h / k$ is the local Nusselt number.

The computational domain has been defined taking into account existing symmetries. Therefore, the cross-section of the flat channel considered here, having the half-spacing between the plates equal to a , corresponds to a rectangle of unit base and height a , which has been discretised by means of 9-node Lagrangian

Figure 1: Axial distributions of the local overall Nusselt number $(Nu_o)_c$ for simultaneously developing flows of constant property fluids with $Pr_m = 5$ in flat microchannels with different Br_m : (a) $Bi = 5$ and (b) $Bi = 20$.

parabolic elements. A total of 50 elements, whose sizes gradually increase with increasing distance from the walls, and 303 nodal points have been used in the domain discretisation. Of course, it has first been verified that this discretisation is fine enough to give mesh-independent results. In all the computations, the axial step has gradually been increased from the starting value $\Delta x/D_h = 0.0001$ to the maximum value $\Delta x/D_h = 0.05$.

The procedure outlined in the previous section and employed for the numerical simulations had already been validated, on the assumptions of constant property fluid and negligible viscous dissipation, by comparing heat transfer and pressure drop results with existing literature data for laminar simultaneously developing flows in straight channels, both three-dimensional and axial-symmetric [6, 9, 10]. In order to assess the accuracy of the present computations, additional validation tests have been carried out. Asymptotic values of the Nusselt number $(Nu_\infty)_c = h_\infty D_h/k$ and fully developed values of the Poiseuille number $(fRe)_c$ for a constant property fluid are compared here with available literature data for flat microchannels. The computed values $(Nu_\infty)_c = 7.54075$ and 17.50000 for $Br_m = 0$ and $Br_m \neq 0$, respectively, are in excellent agreement with the corresponding literature values $(Nu_\infty)_c = 7.54070$ and 17.5 [3]. Moreover, the computed fully developed value of the Poiseuille number $(fRe)_c = 24.0000$ coincides with the corresponding literature value $(fRe)_c = 24$ [3].

The influence of viscous dissipation on the local overall Nusselt number is illustrated in fig. 1, where numerical results concerning axial distributions of $(Nu_o)_c$ for constant property flows ($Pr_e/Pr_a = 1$) with different Biot and Brinkman numbers are presented. As expected, for a given value of the Biot number Bi the same asymptotic value of $(Nu_o)_c$ is reached for fully developed conditions with any value of Br_m , while Br_m strongly affects the Nusselt number in the intermediate range of $X^* = x/(D_h Pe)$. The influence of Br_m is also significant for low values of X^* , where $(Nu_o)_c$ values are lower than those corresponding to $Br_m = 0$ if $Br_m < 0$ (heating), and higher if $Br_m > 0$ (cooling). It is also apparent in fig. 1

Figure 2: Axial distributions of the ratio $Nu_o/(Nu_o)_c$ for flat microchannels with $Pr_m = 5$ and different Br_m : (a) $Bi = 5$ and (b) $Bi = 20$.

that entrance effects on the local overall Nusselt number $(Nu_o)_c$ are marginal for low values of Bi , that is, when the wall thermal resistance prevails.

The influence of temperature dependent viscosity on the local overall Nusselt number is shown in fig. 2, where axial distributions of the ratio $Nu_o/(Nu_o)_c$ for flat microchannels with different Biot and Brinkman numbers are presented. As can be noticed, the ratio Pr_e/Pr_a significantly affects the Nusselt number as long as the flow develops, while its influence is rather small when fully developed conditions are reached. It must be observed that the very high values of the ratio $Nu_o/(Nu_o)_c$ found at intermediate X^* for heating ($Pr_e/Pr_a > 1$) are not very significant, since they are simply due to a moderate axial shifting of curves representing axial distributions of Nu with respect to the constant property ones reported in fig. 1. By comparing axial distributions of $Nu_o/(Nu_o)_c$ for different Brinkman numbers Br_m and the same Biot number Bi , it can be seen that, in simultaneously developing flows, the influence of temperature dependent viscosity is more evident than that of viscous dissipation. Moreover, for a given $|Br_m|$, liquid heating ($Br_m < 0$ and $Pr_e/Pr_a > 1$) and liquid cooling ($Br_m > 0$ and $Pr_e/Pr_a < 1$) lead to the same asymptotic value of the ratio $Nu_o/(Nu_o)_c$ provided that the corresponding values of the ratio Pr_e/Pr_a are reciprocal to each other, i.e., the liquid exhibits the same variation of viscosity in the temperature range between t_e and t_a . On the basis of eqn. (10), this implies that fully developed profiles of the absolute values of the dimensionless temperature $T'_\infty = (t_\infty - t_w)/(t_b - t_w)$ are the same for both heating and cooling. This also implies that fully-developed axial velocity profiles are equal for liquid heating and liquid cooling, so that the term Φ_v , defined in eqn. (6), has the same distribution in both cases.

The combined effects of temperature dependent viscosity and viscous dissipation on pressure drop are illustrated with reference to axial distributions of $f_{app}Re_m$ for flat microchannels with different Biot and Brinkman numbers. To facilitate the analysis of the results, the axial distribution of $(f_{app}Re_m)_c$ for simultaneously developing flows of constant property fluids in flat channels is presented in fig. 3, while axial distributions of the ratio $f_{app}Re_m/(f_{app}Re_m)_c$ for

Figure 3: Axial distribution of $(f_{app} Re_m)_c$ for simultaneously developing flows of constant property fluids in flat ducts: numerical results (solid line) and literature data (squares).

different Biot and Brinkman numbers are reported in fig. 4. The dimensionless axial coordinate appearing in figs. 3 and 4 is defined as $X^+ = x/(D_h Re_m)$. Available literature data from [3] are also reported in fig. 3 for comparison. As can be seen, near the entrance the values of $f_{app} Re_m/(f_{app} Re_m)_c$ are more influenced by temperature dependence of viscosity than by viscous dissipation, while both effects increase and become comparable when fully developed conditions are approached. As expected, fully developed values of $f_{app} Re_m/(f_{app} Re_m)_c$ are larger than 1 for cooling and smaller than 1 for heating, while the opposite occurs near the entrance. Curves for heating and cooling cross each other at axial positions that get closer to the entrance as the Biot number increases. By inspection of figs. 2 and 4, it is evident that the influence of temperature dependent viscosity is greater on pressure drop than on heat transfer rate.

The differences between the local values of $Nu_o/(Nu_o)_c$ and $f_{app} Re_m/(f_{app} Re_m)_c$ found for different values of the ratio Pr_e/Pr_a can be explained taking into

Figure 4: Axial distributions of the ratio $f_{app} Re_m/(f_{app} Re_m)_c$ for flat micro-channels with $Pr_m = 5$ and different Br_m : (a) $Bi = 5$ and (b) $Bi = 20$.

Figure 5: Transverse profiles of dimensionless temperature T at different axial locations for flat microchannels with $Pr_m = 5$, $|Br_m| = 0.1$ and $Bi = 5$: (a) fluid heating and (b) fluid cooling.

account the temperature distributions over the cross-sections, which, in turn, affect the velocity profiles. As an example, to show the effects of temperature dependent viscosity, transverse profiles of the dimensionless temperature $T = (t - t_a)/(t_e - t_a)$ at selected axial locations are compared in fig. 5 for flows with a temperature dependent viscosity fluid ($Pr_e/Pr_a = 2$ for heating and $1/2$ for cooling) and a constant property fluid in the case of $Bi = 5$ and $|Br_m| = 0.1$. The dimensionless transverse coordinate appearing in fig. 5 is defined as $Z = z/D_h = z/(4a)$. It is worth noting that in the case of fluid heating, temperature profiles for intermediate axial positions exhibit a point of inflection because of viscous dissipation effects, while this does not occur in the case of fluid cooling. Moreover, it is apparent that the effect of temperature dependent viscosity increases with the distance from the entrance and is more relevant for fluid heating ($Br_m = -0.1$) than for fluid cooling ($Br_m = 0.1$).

4 Conclusions

The effects of viscous dissipation and temperature dependent viscosity in simultaneously developing laminar flows of liquids in flat microchannels have been studied. Reference has been made to convective thermal boundary conditions at the wall of the microchannels. In order to allow a parametric investigation, viscosity has been assumed to vary linearly with temperature, while the other fluid properties have been held constant. Numerical results confirm that, in the laminar forced convection in the entrance region of straight microchannels, both temperature dependence of viscosity and viscous dissipation effects cannot be neglected in a wide range of operative conditions.

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