30 YEARS' EXPERIENCE ON THE OPTIMIZATION OF CABLE-STAYED BRIDGES

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ABSTRACT

Cable-stayed bridges optimization consists of finding the stiffness and mass arrangement of the load-bearing members (deck, towers and cable-stays) and the cable forces distribution, aiming to minimize cost and to achieve an adequate structural behaviour under static and dynamic loading. The first works on this topic were reported over 40 years ago but it is attracting growing interest with more than half of the publications in the last decade. The main goal of this paper is to share the perspective of this research groups 30 years' experience in this domain. This paper starts with an overview of the optimum design of cable-stayed bridges followed by a presentation of previous research works by the authors. Current research and future developments envisaged are also referred to. The first works consisted of the optimization of steel bridges considering three-dimensional modelling, box-girder decks, seismic action and uncertainty-based optimization. The optimum design of concrete bridges and the simultaneous optimization of structure and control devices in steel footbridges subjected to pedestrian-induced vibrations and steel bridges under seismic action followed. The optimization of bridges with complex geometries and other cable-supported concrete bridges, like extradosed bridges and under-deck cable-stayed bridges, are subjects of recent research. The optimization of long-span and multi-span bridges, including novel cable arrangements, and the optimum design considering robustness are of major relevance in future developments. Although with limited scope at present, given the problem size, it is expected an increasing use of metaheuristic algorithms, artificial neural networks and surrogate models.
Keywords: cable-stayed bridges, optimization, cable forces, optimum design, sizing design variables, shape design variables.

1 INTRODUCTION

Cable-stayed bridges are widely used all over the world for medium-to-long spans. Their popularity is due to its structural and construction efficiency but also to economic and aesthetic advantages, due to their elegant and transparent appearance.

Modern cable-stayed bridges feature multiple inclined cable-stays providing the deck with a continuous support and a natural prestressing. This allows spanning long distances with slender decks (main span-to-depth ratio of about 250). The cable-stays transfer the loads to the towers that, acting in compression, transmit these loads to the foundations. The cable forces distribution and the stiffness of the load-bearing members (deck, towers and cable-stays) determine the behaviour of these highly redundant structures. There is a wide range of structural solutions for cable-stayed bridges, especially, for small and medium spans. However, the symmetrical typology with three spans and two towers has been commonly adopted for medium-to-long spans. The towers are usually made of concrete, with the deck being of concrete, steel or steel–concrete composite. The historical background, a detailed review of the main features and a comprehensive analysis of the structural behaviour of cable-stayed bridges can be found in several references [1]–[5].

Cable-stayed bridges design is a complex problem that comprises defining the structural system, finding the members cross-sectional sizes, computing the cables forces distribution, geometrical non-linear effects and erection stages. The time-dependent behaviour of concrete and the dynamic actions add more complexity to this design problem. Therefore,

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optimization techniques are especially appropriate for solving this large and complex design problem seeking cost minimization and structurally efficient solutions considering both, static and dynamic actions.

A convex optimization problem features a convex objective function and a convex feasible domain. The feasible domain is convex if all the inequality constraints are concave and the equality constraints are linear. A convex optimization problem has only one minimum, and the Karush–Kuhn–Tucker (KKT) conditions are sufficient to establish it. The domain in structural optimization is in general nonconvex. However, convex optimization is relevant because a sequence of approximate problems is used in explicit optimization [6].

Convex optimization and nonconvex optimization strategies can be used to iteratively modify the design variables seeking a design improvement. Convex optimization strategies need the derivatives of the objective function and all the design constraints with respect to the design variables. This information (sensitivity analysis) is used to define the search direction along which the current design variables are modified seeking an optimum solution. These approaches converge in polynomial time to a local (not necessarily global) optimum solution. Nonconvex optimization strategies do not use sensitivity analysis and consist of different procedures, such as random search, branch-and-bound or metaheuristic algorithms (genetic algorithms, evolutionary algorithms, particle swarm optimization, simulated annealing). Although they are easier to implement, they feature an exponential convergence time with respect to the number of design variables to find the optimum solution. They usually finish with the best solution found so far, not even guaranteeing to be a local optimum unless the solution satisfies the KKT optimality conditions.

The optimization problem of cable-stayed bridges may be nonconvex and the feasible domain may be non-connected when considering dynamic loading. The cable-stayed bridge optimization usually features a large number of design variables and design constraints, which are nonlinear and conflicting. This leads to a complex design space and a computationally costly problem. Considering this, an efficient convex optimization technique associated with multiple starting points was adopted preferably to a metaheuristic optimization technique. Finding the active set of constraints in gradient-based nonlinear programming algorithms may pose a considerable difficulty in solving problems with hundreds or thousands of constraints. A procedure was envisaged considering all constraints according to their relative merit. There are several approaches for this constraint aggregation, such as, the Kreisselmeier–Steinhauser function [7] or the least p-norm [8].

In previous works, the optimization of cable-stayed bridges was formulated as a multi-objective problem which is solved by the minimization of a convex scalar function obtained via an entropy-based approach. This function aggregates all the design objectives and creates a convex approximation close to the boundaries of the original nonconvex domain. This approach avoids the complicated procedure of finding the active set of constraints in a problem with a significant number of constraints. All the constraints are included in the scalar function with different probabilities of becoming active. As iterations proceed, there is decreasing uncertainty about which of the constraints are more important to find the optimum. This procedure reduces the cost objective by improving its value with respect to the previous iterations and simultaneously keeps all the constraints within limits. The minimization of the scalar function is combined with a multi-start strategy to obtain optimal local solutions and the best is selected as the optimum design.

A recent literature survey by Martins et al. [9], comprising a detailed review of 90 articles, revealed that the first works on this topic were presented over 40 years ago. However, it can be stated that represents a topic of growing interest with more than half of the articles published in the last decade. With 30 years' experience and 26 articles published and indexed in the Web of Science and Scopus electronic databases, this paper aims to share the perspective of our research group. Future developments are expected, therefore, it also aims at contributing to draw the attention of future researchers in this topic. These publications were analysed and a detailed review was conducted to identify the main characteristics, the principal conclusions and contributions of each.

In Section 2, the formulation of the optimum design problem of cable-stayed bridges is described. Section 3 presents a detailed review of previous articles by the authors. The most recent works are also pointed out. Finally, Section 4 presents some concluding remarks, the current trends and possible future developments within this research topic.

2 OPTIMUM DESIGN OF CABLE-STAYED BRIDGES

2.1 Overview

In the optimum design of cable-stayed bridges, the objective function is usually formulated based on criteria related to structural efficiency and/or economy, aiming to minimize total cost, total strain energy or some norm of the deck vertical displacements and towers horizontal displacements. Regardless of the objective function chosen, safety and service criteria should be met to achieve a feasible design. The problem can be posed as a single objective optimization to minimize the cost of the structure while satisfying displacement and stress constraints imposed according to the design codes.

In previous works by the authors, the optimum design problem of cable-stayed bridges is formulated as a multi-objective optimization problem from which a Pareto optimal solution vector is found. This means that no other feasible vector exists that could decrease one objective without increasing at least another one.

2.2 Structural analysis

The finite element method was used for assessing the structural response under static and dynamic loadings. A linear elastic analysis is commonly adopted and various loading cases should be considered. These should refer to the complete bridge under permanent load and live load placed to obtain the most unfavourable effects.

The balanced cantilever method is usually considered the reference erection method for these bridges. In this method, the geometry of the structure, the displacements and stresses change during the construction process. Therefore, the analysis should consider the construction stages aiming to evaluate the corresponding displacements and stresses that should be included as constraints in the optimization problem.

In these bridges, there are three main sources of geometric nonlinearities: the nonlinear axial force–elongation relationship for the inclined cable stays due to the sag caused by their own weight; the nonlinear axial force and bending moment–deformation relationships for the towers and the deck under combined bending and axial forces; and the geometry change caused by large displacements. A second-order elastic analysis can be adopted to account for the geometric nonlinear effects. To consider the cable sag, the cables can be modelled as truss elements with an equivalent modulus of elasticity given by Ernst formulation. Two approaches were used to include the geometrical nonlinearities in the deck and towers, namely, the equivalent lateral force method and computing the stiffness matrix of the elements in the deck and towers with the elastic ($\underline{K}_E$) and geometric ($\underline{K}_G$) contributions. Step-by-step integration of the dynamic equilibrium equation and modal/spectral approaches were utilized to access the structural response under dynamic actions.

2.3 Design variables

The design variables, $\underline{x}$, can be grouped in three sets: sizing, shape and mechanical. The first set concerns the cross-sectional dimensions of towers, deck and cable-stays. The second set pertains the bridge geometry (tower height, lateral and central span lengths, cable anchor positions). Both sets directly influence the mass, stiffness and cost of the structure. The third set represents the cable-stays prestressing forces, which do not have direct impact in cost, but play a fundamental role in controlling the behaviour of cable-stayed bridges.

The cables forces optimization problem usually features one or two design variables per each cable-stay depending on if the cable force, the cable area or both are considered as design variables. Consequently, in modern cable-stayed bridges featuring a large number of cable-stays, this problem easily presents more than 20 design variables. An extra 10 to 20 design variables are added by considering sizing and shape design variables. This issue may justify the relatively small numbers of publications (18.4% of the cable forces optimization works and 14.7% of the optimum design works) using metaheuristic algorithms [9]. Some strategies were proposed to reduce the number of design variables by variable linking thus reducing the computational effort.

2.4 Design objectives

The first design objective concerns the cost minimization and can be expressed as

$$g_1(\underline{x}) = \frac{C}{C_0} - 1 \le 0\,, \tag{1}$$

where C is the current cost of the structure and C_0 is a reference cost, that represents the initial cost of each analysis and optimization cycle. This approach makes the cost always one of the main objectives for the optimization algorithm.

A second set of objectives refer to limiting the deck vertical displacements and the towers horizontal displacements, aiming at the desired deck profile at the end of construction and the minimization of the tower bending deflections

$$g_2(\underline{x}) = \frac{|\delta|}{\delta_0} - 1 \le 0\,, \tag{2}$$

where δ and δ_0 are the displacement value and the limit value for the displacement under control, respectively.

The stress objectives for the deck and towers members are defined based on the provisions of the relevant design codes according to the structural material used. In general, these goals can be expressed by

$$g_3(\underline{x}) = \frac{\sigma}{\sigma_{allow}} - 1 \le 0\,, \tag{3}$$

where σ and σ_{allow} are the acting stress and the corresponding allowable stress, respectively. For concrete members, different values of the allowable stress should be considered for service conditions and for strength verification. For service conditions, appropriate allowable values should be considered according to the different concrete strength in tension and in compression. For strength verification, the allowable value should represent the structural concrete member's resistance, including reinforcement, evaluated according to acting internal forces, such as, bending and axial force, shear force or biaxial bending and axial force.

Another set of objectives refers to the stresses in the cable-stays which can be written as

$$g_4\left(\underline{x}\right) = \frac{\sigma}{k \cdot f_{pk}} - 1 \le 0 , \tag{4}$$

$$g_5\left(\underline{x}\right) = 1 - \frac{\sigma}{0.1 f_{pk}} \le 0 , \tag{5}$$

where σ and f_{pk} are the acting stress in the stays and the characteristic value of the prestressing steel tensile strength, respectively. The value of k in eqn (4) is usually equal to 0.55 during construction, 0.45 or 0.50 for service conditions and 0.74 for strength verification. Eqn (5) concerns to a lower limit for tension in the stays to ensure their structural efficiency.

More than 1,000 design goals can be easily expected when solving this optimization problem due to the structural discretization, the number of loadings and erection stages that need to be considered.

2.5 Objective function

A multi-objective optimization problem aims to minimize the set of all design objectives over the design variables. This can be expressed as a *minimax* problem, which is discontinuous and non-differentiable and consequently it is difficult to solve numerically. By using the Shannon/Jaynes Maximum Entropy Principle and Cauchy's arithmetic–geometric mean inequality [10] this problem is solved by replacing the *minimax* problem into the minimization of an unconstrained convex scalar function given by

$$\min F\left(\underline{x}\right) = \min \frac{1}{\rho} \ln \left[\sum_{j=1}^{M} e^{\rho\left(g_j\left(\underline{x}\right)\right)} \right], \tag{6}$$

where $\underline{x}$ is the vector of N design variables, M is the number of design objectives and ρ is a control parameter which must not be decreased through the analysis and optimization process. This function is both continuous and differentiable and, thus, considerably easy to solve. This entropy-based optimization approach creates an inside convex approximation of the original nonconvex domain. The convex approximation parameter ρ is increased throughout the iterative procedure to make the convex underestimates closer to the original nonconvex domain.

Generally, in multi-objective optimization problems the designer assigns weighting factors to each goal according to its relevance. In this formulation, an entropy allocated factor is assigned to each goal, being modified in each iteration. The unbiased entropy-based factor assigned to each goal is the exponential of the goal (modified by the changes in the design variables) multiplied by the parameter ρ.

This algorithm requires a sequence of positive values of ρ increasing towards infinity. The aggregation parameter, ρ, is a user-defined parameter and can be computed by different schemes, such as, adaptive approach [11], [12] or the value which makes the objective function, eqn (6), stationary with respect to ρ [10]. The use of a constant parameter is also possible, but a value not large enough may lead to a conservative design. From a practical viewpoint values in the range 100–2,000 lead to the same results.

The design objectives, $g_j(\underline{x})$, do not have an explicit algebraic form, therefore, the problem is solved by an explicit approximation given by taking Taylor series expansion of all the objectives, around the current design variable vector, truncated after the linear term.

2.6 Sensitivity analysis

The sensitivity analysis is a fundamental aspect when using gradient-based optimization algorithms from which depends on the evolution of the analysis and optimization process and its accuracy. The sensitivity analysis provides the gradients of the objective function and all the design goals with respect to the design variables. From the various methods to perform the sensitivity analysis, the discrete direct method was used with both, analytical and semi-analytical derivatives. This is justified by the computational efficiency, the availability of the source code, the accuracy and that the number of design goals is far larger than the number of design variables.

The sensitivities of displacements with respect to the design variables are obtained by differentiating the static equilibrium equation. The stress sensitivities are calculated from the chain derivation of the finite element stress–displacement relation.

The nonlinear structural response (including geometrical nonlinearities and dynamic actions) to changes in the design variables is approximated by the first-order Taylor series approximation. To ensure the accuracy of this linear approximation in each analysis-and-optimization cycle, bound constraints with move limits should be imposed on the variation of the current value of each design variable. The values of the allowable variations can be selected depending on the nonlinearity of the goal functions.

3 PREVIOUS AND CURRENT RESEARCH

3.1 Optimum design of steel bridges

Our first work on the optimization of steel bridges cast as a multi-objective problem with goals of minimum cost and stresses [13] used the two-dimensional analysis of a three-span symmetrical bridge including erection stages. The cable anchor positions on the deck and towers and the cross-sectional sizes of the structural members were considered as design variables. The results emphasised the relevance of considering the erection stresses and the cable anchor positions as design variables aiming at cost reduction.

Aiming at a more computationally efficient analysis-and-optimization procedure, Negrão and Simões [14], [15] focused on the development and implementation of an analytical sensitivity analysis and optimization for cable-stayed bridges design. The optimum design was posed as a multi-objective optimization with goals of minimum cost of material, stresses and displacements. Cable anchor positions on the deck and towers and cross-sectional sizes of the structural members were considered as design variables.

The optimization of steel bridges considering box-girder followed [16]–[18]. This solution for the deck is very effective for long span bridges, due to its high torsional stiffness and streamlined profile, favouring a good aerodynamic behaviour. Additional design variables were considered, namely, shape and sizing design variables of the box-girder, cross-sectional sizes of cable-stays and towers, cables prestressing forces and shape design variables (cables anchor positions in the deck and towers, towers height, lateral and central span lengths). The multi-criteria approach for the optimum design problem considered goals of minimum cost, maximum stresses, minimum stresses in stays and deflections under dead load condition.

The seismic action is of major concern when designing cable-stayed bridges built on earthquake prone areas. Thus, the optimum design of steel bridges under seismic action was also investigated [19] considering both, modal response spectrum and time-history approaches. The analytical sensitivity analysis was carried out for both approaches providing the structural response to seismic action with respect to changes in the design variables. The cables areas and the cross-sectional dimensions of the deck and towers were considered as design variables. It is worth referring that both approaches provided adequate solutions for optimization of steel bridges under seismic vibrations. However, each method poses specific problems associated with either the code implementation or required runtime. Generally, the modal/spectral approach should be preferred, but the basic assumptions of this method make it inadequate for non-linear problems. Given the high computational cost of the time-history approach, in an analysis-and-optimization procedure, it should be reserved for non-linear problems.

The optimum design of steel bridges subjected to seismic action and the effect of control devices [20] started a number of works on this subject. A time-history approach was adopted for accessing the structural response under different earthquake recordings. The multi-objective optimization considered design goals of cost, and different evaluation criteria related to internal forces and displacements in selected locations. A total of 37 design variables comprising shape, sizing and control were considered. The results shown that the integrated structure-and-control optimization provided minimum cost solutions with improved dynamic performance.

The simultaneous structural-control optimization problem of steel bridges under seismic action was addressed considering three-dimensional modelling and erection stages [21]. The modal/spectral approach was used to evaluate the structural response under seismic action defined according to Eurocode 8 provisions. More than 50 design variables were considered, namely, cross-sectional sizes of deck and towers, cables cross-sectional areas and prestressing forces, bridge geometry (cables anchor positions, spans lengths tower geometry), and stiffness and damping of the deck–tower connection. Different numbers of cables were considered in the multi-start strategy combined with the convex optimization algorithm. A numerical example with three cases concerning the deck-tower connection was analysed: free, fixed and with viscous dampers. The later presented the most cost-effective solution.

3.2 Optimum design of concrete bridges

The calculation of the cable forces distribution is a unique feature of cable-stayed bridges. These forces are crucial to control the construction process and, thus, achieving the desired geometry and stress distribution for the complete bridge. Many researchers focused on the cable forces optimization problem. The time-dependent behaviour of concrete adds difficulty to this problem which was specifically addressed in a previous work [22]. The problem was posed as a multi-objective optimization with goals of stresses, and minimum deck vertical displacements and minimum towers horizontal displacements. To adequately evaluate the bridge behaviour during construction and at completion, the structural analysis included the construction sequence, the load history and the time-dependent effects of creep, shrinkage and ageing of concrete. The time-dependent effects were evaluated according to Eurocode 2 formulations. The creep model was based on linear viscoelasticity and takes into account ageing effects. Shrinkage strains are time-dependent but stress independent. The creep function was approximated by a Dirichlet series to evaluate the concrete creep deformations under variables stress. The cable-sag effect was considered by modelling the cables as bar elements with equivalent modulus of elasticity given by Ernst formulation. A numerical example with a total of 32 design variables corresponding to two sets (installation and adjustment) forces was considered. A computer program developed in MATLAB environment was used to perform the structural analysis, sensitivity analysis and optimization. The second-order effects in the deck and towers were latter included [23], [24] through a second order elastic analysis by the equivalent lateral force method. The results shown the relevance of considering the construction stages, the time-dependent effects and the geometrical nonlinearities for adequately predict the bridge behaviour and compute the cable prestressing forces.

The construction stages, the concrete time-dependent effects and the geometrical nonlinearities were also considered in the optimum design of concrete bridges with different deck cross-sections [25] and including deck prestressing [26]. The optimum design was formulated as a multi-criteria problem with objectives of minimum cost, minimum displacements, stresses under service conditions and strength criteria. Three options were analysed for the deck cross-section: beam-and-slab, single-cell box and tri-cell box. Rectangular hollow sections were considered for the towers. A total of 68 design variables were considered corresponding to the cables areas and prestressing forces, and the cross-sectional sizes of the deck and towers. In the optimum design of concrete bridges, special attention should be paid when computing the concrete members' resistance that should include the steel reinforcement. Moreover, an additional difficulty arises when computing the sensitivities of the strength design goals due to the fact that the resistance of each concrete member depends on the correspondent cross-sectional design variables. This was solved by calculating the sensitivities of these design objectives in normalized form.

More recently, the optimization of concrete bridges under seismic action was studied [27]. The computer program previously developed in MATLAB environment was improved to allow three-dimensional modelling, static and dynamic analysis. The finite element method was used for the structural analysis considering static loading (dead load and road traffic live load), the time-dependent effects and the geometrical nonlinearities. The modal superposition method was used to access the structural response under seismic action defined according to Eurocode 8 elastic response spectrum. The complete quadratic combination (CQC) was used for modal combination due to the modal coupling that is present in the dynamic response of these bridges. The multi-criteria optimum design considered design objectives of cost, deflections, natural frequencies and stresses for both, serviceability and ultimate limit states defined according to Eurocode 2 provisions. The design variables were the cable-stays areas and prestressing forces, the deck and towers cross-sections. The modal analysis requires obtaining the stiffness matrix in the dead-load permanent state. Thus, the second-order-effects in the deck and towers were considered by computing the stiffness matrix with the elastic and geometric contributions. The seismic action governs the design of these structures, being especially demanding for the strength verification of the structural members, mainly the towers. The optimum seismic design of concrete cable-stayed bridges should be further investigated with a special focus on the geometry and seismic design of towers and the arrangement of the cable suspension system. Moreover, the spatial variability of the seismic ground motion, the soil–structure interaction, the use of passive and active control devices and the simultaneous optimization of the structure and control devices should be considered in future developments.

3.3 Optimum design of steel footbridges

This topic has been a subject of major research with a focus on the simultaneous optimization of structure and control devices. The first work on this subject was published in 2010 [28] consists of a two-dimensional model of a two span cable-stayed footbridge subjected to a crowd of joggers during a running event and using one active tendon.

The least cost solution of a passive and active non-symmetric cable-stayed footbridge was considered in Ferreira and Simões [29]. The active bridge was controlled using a single active tendon and for the passive bridge, a passive damper was used in the vertical deck–tower connection. The finite element method was used for the two-dimensional analysis of the bridge under both, static and dynamic loadings. A time-history procedure was used to evaluate the displacements and accelerations of the bridge subjected to a running event. A total of 29 design variables were considered, corresponding to shape, sizing, mechanical and control design parameters. The multi-objective optimization reduced simultaneously cost, accelerations and displacements while satisfying allowable stresses. Numerical results shown that both passive and active optimum designs are efficient, with different geometry, mass distribution and cost (22% higher in the passive design).

Further research comprised the optimum design of a two-span non-symmetric cable-stayed footbridge, subjected to a running event and considering four different control techniques [30], namely, no control, viscous dampers (VD), and VD with passive or semi-active tuned-mass dampers (TMDs). A total of 27 design variables were considered in the multi-objective optimization including static and dynamic design goals. Static and dynamic design criteria were considered in the multi-objective optimization problem. The results revealed that the solution with VD is the most cost effective, because the damper adds an important control action to the most relevant modes. This was achieved due to the simultaneous optimization of tower position. Furthermore, the optimum structural solution (cross-sectional sizes and geometry) for the different cases is rather different depending on the control devices, meaning that the integrated structural-and-control optimization is a design advantage.

The computational model was further improved for the three-dimensional optimum design of steel footbridges [31] with a focus on the control of the synchronous lateral excitation, “lock-in”, that occurs in long-span footbridges. The optimum design was formulated as a multi-objective problem with goals of minimum cost, static and dynamic design criteria. A total of 43 sizing, geometry and control design variables were considered. VD were adopted as control devices. An analytical sensitivity analysis was used, including a formulation for finding the pedestrian “lock-in” sensitivities. The algorithm provides minimum cost solutions which simultaneously satisfy the ultimate and service limit considered, such as, stresses throughout the structure, buckling of the structure and the members, displacements, accelerations and the dynamic stability of the structure when subject to synchronous lateral excitation. The algorithm was able to shift the natural frequencies to fall outside the critical ranges for the vertical acceleration and for the “lock-in” phenomena. To satisfy the vertical acceleration and the “lock-in” criteria in the remaining modes, the algorithm changed the tower position and the cables anchor positions, and modified the stiffness and damping properties of the deck-tower connection. These properties are important because if a stiff tower-deck connection is used the structural frequencies will increase, but no damping will be added. On the other hand, with a free connection the frequencies will decrease but, also, no damping will be added. The connection properties are also important for the static design criteria, in particular to control the deck displacements.

3.4 Optimum design under uncertainty

This topic was firstly addressed considering the Two-Phase Method for fuzzy optimization of steel cable-stayed bridges [32]. This method is based on the Fuzzy Set Theory and corresponds to a non-probabilistic description of uncertainty. In the first phase, the fuzzy solution is obtained by using the Level Cuts Method and in the second phase the crisp solution, which maximizes the membership function of fuzzy decision-making, is found by using the Bound Search Method. Fuzzy goals were generated involving Young's modulus, stress limits and loading. Shape and sizing design variables were considered and their values were assumed to be deterministic. The optimization problem was posed as the minimization of bridge cost, stresses and displacements. This approach provided solutions featuring high design levels and materials savings when compared to the deterministic design.

The reliability-based optimum design of steel cable-stayed bridges [33] and glulam cable-stayed footbridges [34] was also studied. The first-order second moment method (FORM) was used to compute the reliability indices associated with various limit state functions. Second-order bounds of the probability of failure were considered to evaluate the structural system probability of failure. The bound intervals obtained show that most of the nearly 1000 limit states considered are highly correlated. A discrete reliability analytical sensitivity analysis was derived and used in the optimization algorithm. The multi-objective optimum design problem was formulated as the minimisation of stresses, displacements, reliability and bridge cost. Shape, sizing and mechanical design variables were considered. Material properties and loadings were considered as random variables. Optimum solutions featuring cost reduction and reduced probability of failure were obtained. Although, in these examples, the probability of failure refers to critical stresses throughout the structure, induced by loadings, other failure modes or criteria could be used as well, such as the excessive deflection or cable under-stressing. The advanced simulation method combined with the response surface method will be proposed in future research.

3.5 Recent research

The optimization of other types of concrete cable-supported bridges, such as, extradosed bridges [35] and under-deck cable-stayed bridges [36] was a focus of recent research. The computational tool previously developed for the optimization of concrete cable-stayed bridges was generalized to solve these problems. The optimum design of both structures aims to find an adequate balance between the stiffness of the main girder and the cables suspension effect, depending on the cross-sectional dimensions and the prestressing forces. Besides the cross-sectional areas and prestressing forces of the extradosed or under-deck cables, these problems require considering the cross-sectional areas and prestressing forces of the internal tendons in the deck. The time-dependent effects of concrete and prestressing steel should be included. Numerical examples comprising a symmetrical extradosed concrete bridge with a total length of 330 m and a main span of 150 m were analysed. Static loading (dead load and road traffic live load) and seismic action were considered in the optimum design considering 40 design variables and more than 1,600 design objectives. For these examples, the seismic action governs the design when medium or high intensity seismic action is considered. There is a wide range of structural solutions for these bridge types, therefore, requiring additional research. Tower geometry, towers–deck–piers connection, deck cross-section design and the number of cables influence the structural behaviour of extradosed bridges. The use of passive and active control devices and the simultaneous optimization of the structure and control devices should be considered in future developments. Further studies concerning under-deck cable-stayed bridges and the optimization of tied-arch bridges are expected in future developments.

The optimum design of bridges with complex geometries was another topic of current research. The computer program previously used for the three-dimensional optimum design of steel footbridges [31] was improved for the optimum design of curved bridges with VD as control devices [37]. Numerical examples comprising different bridge lengths (180, 220, 260, 300 and 340 m) were analysed. A total of 59 design variables were considered, including, bridge geometry (tower shape, number of cables and their location), cross-sectional sizes, properties of control devices and cables prestressing. The optimization problem was formulated with objectives of least bridge cost, static and dynamic design criteria. From the results it could be stated that the design is governed by dynamic comfort requirements in particular the horizontal and vertical accelerations and the synchronous lateral instability, "lock-in". The algorithm was able to control the relevant modes, either by increasing/decreasing the frequencies to fall outside the critical range or adding damping to meet the dynamic design criteria. The tower location (main span-to-total length ratio) is one of the design variables that substantially influences the static response and the dynamic control of both, horizontal and vertical modes of vibration. The optimum solutions feature "A"-shaped towers with fan cable layout. This is relevant in curved cable-stayed bridges to reduce the tower torsion due to the different cable forces in both sides of the tower. The results revealed that the static and dynamic design coupling becomes more important for curved bridges design as the cable tensioning problem becomes more complex and the vibration modes exhibit simultaneous vertical, horizontal and torsional responses.

The algorithm was further improved for studying cable-stayed footbridges with "S"-shaped deck and external pylons [38]. A special attention was paid to generating initial feasible designs to start the optimization process. Given that dampers cannot be located at the pylon–deck connection because it does not exist, the structure was controlled by passive and semi-active tuned mass dampers. A time-history analysis was adopted to consider the non-linear dynamic behaviour of the semi-active device. Two optimal solutions were sought using semi-active (S1) and passive (S2) dampers. The results shown that both solutions feature different geometries and cross-sectional dimensions. The pylon height is markedly higher for structure S2 because this improves the bridge dynamic behaviour. The results also revealed that the pylon acts like a dynamic leverage arm magnifying the vertical response of the deck. As previously stated, the simultaneous structural-and-control optimization provides superior results than optimizing the structure and control devices separately.

The most recent article concerns the optimum seismic design of a 350 m curved steel road bridge [39]. A time-history approach was used to access the dynamic response considering three seismic events. The spatial variability of the seismic ground motion and erection stages were considered. The deck–tower connection is a key aspect for the seismic response. The longitudinal displacement between deck and tower was released, the stiffness and damping of the transverse and vertical supports were considered as design variables. Linear viscous dampers were adopted as control devices. This dynamic problem is highly nonlinear, therefore, the design space features multiple non-connected domains. Two cases were considered: seismic action scaled to 30% (Case 1) and full seismic intensity (Case 2). The results revealed that the optimum design of Case 2 features 21.5% more cost than the optimum design of Case 1 and 43% more cost than a similar straight bridge. Moreover, the complexity of the curved bridge makes more difficult to find feasible starting solutions.

4 CONCLUSIONS AND FUTURE DEVELOPMENTS

The optimization of long-span bridges and multi-span bridges with novel cable arrangements, such as, crossing-cables [40] are drawing the attention of some researchers. The cable forces optimization problem is still a subject of interest for several researchers which are presenting novel approaches for solving this problem, mainly using metaheuristic algorithms [41], [42]. Therefore, new developments are expected in these topics.

Recently, it can be noticed an increasing use of metaheuristic algorithms, artificial neural networks and surrogate models [43] in the optimization field. The application of these techniques in the optimum design of cable-stayed bridges will be the focus of upcoming research.

The rapid increase in the computational resources available will contribute to the application of soft computing strategies in this domain. These are easier to implement and do not require a deep understanding of optimization concepts and sensitivity analysis techniques. Moreover, the increasing computational capacity will allow including more demanding problems, such as, response to wind and earthquakes, in the optimum design of these bridges. The shape and sizing optimization of bridge decks considering wind effects has been a subject of major research in recent years [44], [45]. This is a complex and relevant topic in the design of long-span bridges and, thus, further developments are expected.

The dynamic behaviour is of utmost importance in the design of these structures and the use of control devices plays a key role improving the structural response under dynamic actions. Therefore, the simultaneous optimization of structure and control devices will continue as a subject of major relevance in forthcoming research. The reliability-based optimum design and the robust design including, for example, cable losses scenarios, were not sufficiently addressed in previous research and will represent topics of interest for future developments.

REFERENCES

[1] Troitsky, M.S., *Cable-Stayed Bridges: Theory and Design*, Crosby Lockwood Staples: London, 1977.

[2] Podolny, W. & Scalzi, J.B., *Construction and Design of Cable-STAYED bridges*, 2nd ed., Wiley: New York, 1986.

[3] Walther, R. (ed.), *Cable Stayed Bridges*, 2nd ed., Telford: London, 1999.

[4] Gimsing, N.J. & Georgakis, C.T., *Cable Supported Bridges: Concept and Design*, 3rd ed., Wiley: Hoboken, NJ, 2012.

[5] Svensson, H., *Cable-Stayed Bridges: 40 Years Of experience Worldwide*, 1st ed., Ernst, Wiley-Blackwell: Berlin, 2012.

[6] Haftka, R.T. & Gürdal, Z., *Elements of Structural Optimization*, 3rd revised and expanded ed., Kluwer Academic Publishers, 1992.

[7] Kreisselmeier, G. & Steinhauser, R., Systematic control design by optimizing a vector performance index. *IFAC Proceedings Volumes*, **12**(7), pp. 113–117, 1979.

[8] Charalambous, C., Nonlinear least *p*th optimization and nonlinear programming. *Mathematical Programming*, **12**(1), pp. 195–225, 1977.

[9] Martins, A.M.B., Simões, L.M.C. & Negrão, J.H.J.O., Optimization of cable-stayed bridges: A literature survey. *Advances in Engineering Software*, **149**, 102829, 2020.

[10] Simões, L.M.C. & Templeman, A.B., Entropy-based synthesis of pretensioned cable net structures. *Engineering Optimization*, **15**(2), pp. 121–140, 1989.

[11] Poon, N.M.K. & Martins, J.R.R.A., An adaptive approach to constraint aggregation using adjoint sensitivity analysis. *Struct. Multidiscipl. Optim.*, **34**(1), pp. 61–73, 2007.

[12] Zhang, K.-S., Han, Z.-H., Gao, Z.-J. & Wang, Y., Constraint aggregation for large number of constraints in wing surrogate-based optimization. *Struct. Multidiscipl. Optim.*, **59**(2), pp. 421–438, 2019.

[13] Simões, L.M.C. & Negrão, J.H.O., Sizing and geometry optimization of cable-stayed bridges. *Computers and Structures*, **52**(2), pp. 309–321, 1994.

[14] Negrão, J.H.O. & Simões, L.M.C., Three dimensional nonlinear optimization of cable-stayed bridges. *Advances in Structural Optimization*, eds B.H.V. Topping & M. Papadrakakis, Civil-Comp Press: Edinburgh, UK, pp. 203–213, 1994.

[15] Negrão, J.H.O. & Simões, L.M.C., Optimization of cable-stayed bridges with three-dimensional modelling. *Computers and Structures*, **64**(1–4), pp. 741–758, 1997.

[16] Simões, L.M.C. & Negrão, J.H.O., Optimization of cable-stayed bridges with box-girder decks. *Computer Aided Optimum Design of Structures V*, eds S. Hernandez & C.A. Brebbia, Wessex Institute of Technology: Southampton, UK, pp. 21–32, 1997.

[17] Negrão, J. & Simões, L, Shape and sizing optimization of box-girder decks of cable-stayed bridges. *Computer Aided Optimum Design of Structures VI*, eds S. Hernandez, A.J. Kassab & C.A. Brebbia, Wessex Institute of Technology: Southampton, UK, pp. 323–332, 1999.

[18] Simões, L.M. & Negrão, J.H.J., Optimization of cable-stayed bridges with box-girder decks. *Advances in Engineering Software*, **31**(6), pp. 417–423, 2000.

[19] Simões, L.M.C. & Negrão, J.H.J.O., Optimization of cable-stayed bridges subjected to earthquakes with non-linear behaviour. *Engineering Optimization*, **31**(4), pp. 457–478, 1999.

[20] Ferreira, F.L.S. & Simões, L.M.C., Optimum design of a controlled cable stayed bridge subject to earthquakes. *Structural and Multidisciplinary Optimization*, **44**(4), pp. 517–528, 2011.

[21] Ferreira, F. & Simões, L., Synthesis of three dimensional controlled cable-stayed bridges subject to seismic loading. *Computers and Structures*, **226**, 2020.

[22] Martins, A.M.B., Simões, L.M.C. & Negrão, J.H.J.O., Cable stretching force optimization of concrete cable-stayed bridges including construction stages and time-dependent effects. *Structural and Multidisciplinary Optimization*, **51**(3), pp. 757–772, 2015.

[23] Martins, A.M.B., Simões, L.M.C. & Negrão, J.H.J.O., Optimization of cable forces for concrete cable-stayed bridges. *Proceedings of the 14th International Conference on Civil and Structural and Environmental Engineering Computing*, eds B.H.V. Topping & P. Iványi, Civil-Comp Press: Stirlingshire, UK, Paper 227, 2013.

[24] Martins, A.M.B., Simões, L.M.C. & Negrão, J.H.J.O., Optimization of cable forces on concrete cable-stayed bridges including geometrical nonlinearities. *Computers and Structures*, **155**, pp. 18–27, 2015.

[25] Martins, A.M.B., Simões, L.M.C. & Negrão, J.H.J.O., Optimum design of concrete cable-stayed bridges. *Engineering Optimization*, **48**(5), pp. 772–791, 2016.

[26] Martins, A.M.B., Simões, L.M.C. & Negrão, J.H.J.O., Optimum design of concrete cable-stayed bridges with prestressed decks. *International Journal for Computational Methods in Engineering Science and Mechanics*, **17**(5–6), pp. 339–349, 2016.

[27] Martins, A.M.B., Simões, L.M.C. & Negrão, J.H.J.O., Optimization of concrete cable-stayed bridges under seismic action. *Computers and Structures*, **222**, pp. 36–47, 2019.

[28] Ferreira, F.L.S. & Simões, L.M.C., Optimum design of active and passive cable stayed footbridges. *Proceedings of the 10th International Conference on Computational Structures Technology*, eds B.H.V. Topping, J.M. Adam, F.J. Pallarés, R. Bru & M.L. Romero, Civil-Comp Press: Stirlingshire, UK, Paper 167, 2010.

[29] Ferreira, F. & Simões, L., Optimum cost design of controlled cable stayed footbridges. *Computers and Structures*, **106–107**, pp. 135–143, 2012.

[30] Ferreira, F. & Simões, L., Optimum design of a controlled cable-stayed footbridge subject to a running event using semiactive and passive mass dampers. *Journal of Performance of Constructed Facilities*, **33**(3), 2019.

[31] Ferreira, F. & Simões, L., Optimum design of a cable-stayed steel footbridge with three dimensional modelling and control devices. *Engineering Structures*, **180**, pp. 510–523, 2019.
[32] Simões, L.M.C. & Negrão, J.H., Optimum design of cable-stayed bridges with imprecise data. *Proceedings of the 8th International Conference on Civil and Structural Engineering Computing*, ed. B.H.V. Topping, Civil-Comp Press: Stirlingshire, UK, Paper 100, 2001.
[33] Negrão, J.H.O. & Simões, L.M.C., Reliability-based optimum design of cable-stayed bridges. *Structural and Multidisciplinary Optimization*, **28**(2–3), pp. 214–220, 2004.
[34] Simões, L.M.C. & Negrão, J.H.O., Reliability-based optimum design of glulam cable-stayed footbridges. *Journal of Bridge Engineering*, **10**(1), pp. 39–44, 2005.
[35] Martins, A.M.B., Simões, L.M.C. & Negrão, J.H.J.O., Optimization of extradosed concrete bridges subjected to seismic action. *Computers and Structures*, **245**, 2021.
[36] Martins, A.M.B., Simões, L.M.C. & Negrão, J.H.J.O., Optimization of under-deck concrete cable-stayed bridges. *Proceedings of the 11th International Conference on Engineering Computational Technology*, eds B.H.V. Topping & P. Iványi, Civil-Comp Press: Stirlingshire, UK, to be published.
[37] Ferreira, F. & Simões, L., Least cost design of curved cable-stayed footbridges with control devices. *Structures*, **19**, pp. 68–83, 2019.
[38] Ferreira, F. & Simões, L., Automated synthesis of controlled cable-stayed footbridges with S-shaped deck. *Advances in Engineering Software*, **149**, 2020.
[39] Ferreira, F. & Simões, L., Optimum seismic design of curved cable-stayed bridges. *Structures*, **43**, pp. 131–148, 2022.
[40] Cid, C., Baldomir, A. & Hernandez, S., Optimum crossing cable system in multi-span cable-stayed bridges. *Engineering Structures*, **160**, pp. 342–355, 2018.
[41] Atmaca, B., Size and post-tensioning cable force optimization of cable-stayed footbridge. *Structures*, **33**, pp. 2036–2049, 2021.
[42] Feng, Y., Lan, C., Briseghella, B., Fenu, L. & Zordan, T., Cable optimization of a cable-stayed bridge based on genetic algorithms and the influence matrix method. *Engineering Optimization*, **54**(1), pp. 20–39, 2022.
[43] Franchini, A., Sebastian, W. & D'Ayala, D., Surrogate-based fragility analysis and probabilistic optimisation of cable-stayed bridges subject to seismic loads. *Engineering Structures*, **256**, 2022.
[44] Cid Montoya, M., Hernández, S. & Nieto, F., Shape optimization of streamlined decks of cable-stayed bridges considering aeroelastic and structural constraints. *Journal of Wind Engineering and Industrial Aerodynamics*, **177**, pp. 429–455, 2018.
[45] Cid Montoya, M., Nieto, F., Hernández, S., Fontán, A., Jurado, J.A. & Kareem, A., Optimization of bridges with short gap streamlined twin-box decks considering structural, flutter and buffeting performance. *Journal of Wind Engineering and Industrial Aerodynamics*, **208**, 104316, 2021.
[46] Cid Montoya, M., Hernández, S. & Kareem, A., Aero-structural optimization-based tailoring of bridge deck geometry. *Engineering Structures*, 114067, 2022.