

OPTIMIZATION OF STEEL AND TIMBER HALL STRUCTURES

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ABSTRACT

The paper deals with the optimization of single-storey hall structures consisting of the same main frames to which steel purlins, façade rails and façade columns are connected. The frames can be steel or timber portal frames. While the steel frames are made of steel I-sections, the timber frames are made of glulam with rectangular cross-sections. The hall structure is optimized using mixed-integer nonlinear programming (MINLP), a combined continuous-discrete optimization technique. MINLP optimization is performed in three steps. It starts with defining the hall superstructure, modelling the optimization model of the structure, and solving the defined optimization problem. The superstructure includes all discrete alternatives of topologies, standard dimensions and material qualities competing for a feasible and optimal result. The optimization model includes continuous and discrete binary variables. The continuous variables represent dimensions, cross-sections, material grades, loads, etc., while the binary variables are used to optimize the topology of the structure and to select standard dimensions/profiles and material grades. The objective function of the material cost of the structure is subject to a system of (in)equality constraints of structural analysis and dimensioning. The dimensioning constraints are defined according to the Eurocode regulations. In order to solve the defined optimization problem, the modified outer-approximation/equality-relaxation (OA/ER) algorithm was used. A numerical example of MINLP optimization of a steel and timber frame hall structure is presented at the end of the article. *Keywords: steel hall, timber hall, steel structures, timber structures, optimization, mixed-integer non-linear programming, MINLP.*

1 INTRODUCTION

The optimization of steel and timber frames/hall structures represents a modern field within structural optimization. To achieve optimal frame design, researchers have developed a number of useful optimization methods suitable for both continuous and discrete optimization. O'Brien and Dixon [1] proposed a linear programming approach for optimal portal frame design. Guerlement et al. [2] presented a practical method in which they minimized the mass of a steel hall using Eurocode 3 [3]. Saka [4] and McKinstry et al. [5] calculated the optimal steel frame design using a genetic algorithm. Using mixed integer nonlinear programming, MINLP, Kravanja and Žula [6] optimized the production cost of the structure of a steel hall. Recently, Kravanja et al. [7] presented the parametric optimization of steel industrial halls. One of the latest research contributions in this field is the work of McKinstry et al. [8], in which the authors achieve the optimal shape of the main steel frame with minimal mass.

According to the number of available references, the area of optimization of hall structures with timber frames is less frequently discussed than the area with steel halls. In this area, Topping and Robinson [9] have performed optimization of timber frames with sequential linear programming and Kravanja and Žula performed optimization of hall structures with timber portal frames with mixed integer nonlinear programming, MINLP [10], [11].

The paper deals with the optimization of a single-storey hall structure with steel or timber main frames (see Fig. 1). The mentioned hall structures are built for industrial, sports and commercial purposes. We optimize the hall structures with mixed-integer nonlinear programming, MINLP. MINLP is a discrete/continuous method of mathematical



programming. MINLP contains both continuous and discrete variables. Continuous variables are defined for calculating continuous parameters, and discrete variables are used for discrete decision making. In MINLP, continuous and discrete optimization take place simultaneously. The results of such optimization are optimal continuous parameters (the optimal mass or production cost of the structure) and discrete parameters (the optimal topology of the structure, the strength classes of the different materials, and the discrete standard/rounded dimensions).

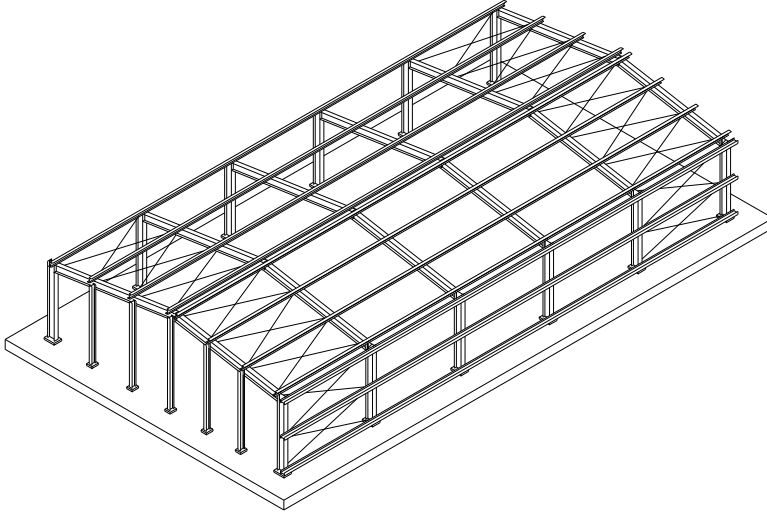


Figure 1: Structure of a steel hall.

2 MINLP PROBLEM FORMULATION

The optimization problem of hall structures is non-linear, non-convex, continuous, and discrete. Therefore, MINLP is chosen for the optimization. It is assumed that a general non-convex, non-linear, discrete, and continuous optimization problem can be formulated as a MINLP problem in the form of:

$$\min z = \mathbf{c}^T \mathbf{y} + f(\mathbf{x})$$

subjected to:

$$\mathbf{h}(\mathbf{x}) = \mathbf{0}$$

$$\mathbf{g}(\mathbf{x}) \leq \mathbf{0}$$

$$\mathbf{B}\mathbf{y} + \mathbf{C}\mathbf{x} \leq \mathbf{b}$$

$$\mathbf{x} \in X = \{\mathbf{x} \in R^n: \mathbf{x}_{LO} \leq \mathbf{x} \leq \mathbf{x}_{UP}\}$$

$$\mathbf{y} \in Y = \{0,1\}^m$$

(MINLP)

The above mathematical MINLP formulation contains the objective function z , the nonlinear functions $f(\mathbf{x})$, $\mathbf{h}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$, the mixed linear equality/inequality constraints $\mathbf{B}\mathbf{y} + \mathbf{C}\mathbf{x} \leq \mathbf{b}$, the vector of continuous variables \mathbf{x} and the vector of discrete binary variables \mathbf{y} .

In structural optimization, the continuous variables x define the dimensions, strains, stresses, costs etc., while the binary variables y represent the potential existence of structural elements within the defined superstructure and the choice of discrete/standard materials and sizes. Non-linear equality and inequality constraints and the bounds of the continuous variables represent the rigorous system of design, loading, resistance, and deflection constraints known from structural analysis.

The general MINLP model formulation has been adapted for the optimization of mechanical superstructures (MINLP-SMS). The resulted formulation is more specific, particularly in variables and constraints. It was used also for the modelling of steel and timber framed structures, see also Žula and Kravanja [12]. This formulation is given in the following form:

$$\begin{aligned}
 &\min z = \mathbf{c}^T \mathbf{y} + f(\mathbf{x}) \\
 &\text{subjected to:} \\
 &\mathbf{h}(\mathbf{x}) = \mathbf{0} \\
 &\mathbf{g}(\mathbf{x}) \leq \mathbf{0} \\
 &\mathbf{E}\mathbf{y} \leq \mathbf{e} \\
 &\mathbf{D}\mathbf{y}^e + \mathbf{R}(\mathbf{x}) \leq \mathbf{r} \\
 &\mathbf{K}\mathbf{y}^e + \mathbf{L}(\mathbf{d}^{cn}) \leq \mathbf{k} \\
 &\mathbf{P}\mathbf{y} + \mathbf{S}(\mathbf{d}^{st}) \leq \mathbf{s} \\
 &\mathbf{A}\mathbf{y} + \mathbf{B}(\mathbf{d}^{mat}) \leq \mathbf{a} \\
 &\mathbf{x} \in X = \{\mathbf{x} \in R^n: x_{LO} \leq x \leq x_{UP}\} \\
 &\mathbf{y} \in Y = \{0,1\}^m
 \end{aligned} \tag{MINLP-SMS}$$

In the model formulation, included are continuous variables $\mathbf{x} = \{\mathbf{d}, \mathbf{p}\}$ and discrete binary variables $\mathbf{y} = \{\mathbf{y}^e, \mathbf{y}^{st}, \mathbf{y}^{mat}\}$. The continuous variables are partitioned into design variables $\mathbf{d} = \{\mathbf{d}^{cn}, \mathbf{d}^{st}, \mathbf{d}^{mat}\}$ and into performance (non-design) variables \mathbf{p} , where sub-vectors \mathbf{d}^{cn} , \mathbf{d}^{st} and \mathbf{d}^{mat} stand for the continuous dimension, standard dimensions and standard material strengths, respectively. Sub-vectors of binary variables \mathbf{y}^e , \mathbf{y}^{st} and \mathbf{y}^{mat} denote the potential existence of structural elements inside the superstructure (the topology determination), the potential selection of standard dimension alternatives and standard material strengths, respectively.

The economical objective function z involves fixed cost charges in the linear term $\mathbf{c}^T \mathbf{y}$ and dimension dependent costs in the term $f(\mathbf{x})$.

The parameter nonlinear and linear constraints $\mathbf{h}(\mathbf{x}) = \mathbf{0}$ and $\mathbf{g}(\mathbf{x}) \leq \mathbf{0}$ represent the rigorous system of the design, loading, resistance, stress, deflection, etc., constraints known from the structural analysis.

The integer linear constraints $\mathbf{E}\mathbf{y} \leq \mathbf{e}$ are proposed to describe the relations between binary variables.

The mixed linear constraints $\mathbf{D}\mathbf{y}^e + \mathbf{R}(\mathbf{x}) \leq \mathbf{r}$ restore interconnection relations between currently selected or existing structural elements (corresponding $y^e = 1$) and cancel relations for currently disappearing or non-existent elements (corresponding $y^e = 0$).

The mixed linear constraints $\mathbf{K}\mathbf{y}^e + \mathbf{L}(\mathbf{d}^{cn}) \leq \mathbf{k}$ are proposed to define the continuous design variables for each existing structural element. The space is defined only when the corresponding structure element exists ($y^e = 1$), otherwise it is empty.

The mixed linear constraints $\mathbf{P}\mathbf{y} + \mathbf{S}(\mathbf{d}^{st}) \leq \mathbf{s}$ define the standard design variables \mathbf{d}^{st} . Each standard dimension d^{st} is determined as a scalar product between its vector of standard/discrete dimension constants \mathbf{q} and its vector of binary variables \mathbf{y}^{st} , see eqn (1). Only one discrete value can be selected for each standard dimension, see eqn (2).

The mixed linear constraints $\mathbf{A}\mathbf{y} + \mathbf{B}(\mathbf{d}^{mat}) \leq \mathbf{a}$ determine the standard material strengths of steel, timber and concrete. These conditions are defined in a similar way as the standard design variables (sections). For more on the MINLP model formulation and how to handle the equations and variables, see Kravanja et al. [13]–[15].

$$d^{st} = \sum_{i \in I} q_i y_i^{st} \quad (1)$$

$$\sum_{i \in I} y_i^{st} = 1 \quad (2)$$

The hall structure under study consists of the same main frames on which the roof purlins, façade rails, and front and rear façade columns are connected. Purlins, rails and façade columns are made of hot-rolled IPE or HEA steel sections. The main frames may be steel or timber portal frames. The steel frames are made of HEA profiles and the timber frames are made of glued laminated timber with rectangular cross-section. The columns of the portal frames are supported by concrete pad foundations. The MINLP superstructure of the hall is defined, which presents a variety of different topological/structural alternatives with:

- sets and binary variables for topology alternatives of frames, purlins, rails and façade columns;
- sets and binary variables for standard-dimension alternatives of steel profiles, timber cross-sections and concrete pad foundation;
- sets and binary variables for standard strength classes of different materials.

3 OPTIMIZATION MODELS

The MINLP optimization models HALLOPT (HALL OPTimization) were developed for the optimization of hall structures, an extra model for the hall with steel frame and an extra model for the hall with timber frames. These models were developed based on the presented MINLP model formulation. The Optimization models include input data (constants), variables, and a cost objective function subject to load, stress, resistance, and deflection (in)equality constraints for dimensioning, and integer logical constraints for topology, standard dimensions, and strength class calculations.

The optimization of the considered structure is performed by the combined action of the self-weight of the frame elements, the vertical uniformly distributed variable load (snow) and the horizontal concentrated variable load located at the top of the columns (wind). The internal forces are calculated according to the first-order elastic theory. Steel frames are defined as non-sway frames ($\alpha_{cr} \geq 10$) and timber frames are treated as sway frames. Longitudinal stability is provided by a bracing system. Eurocode 3 [3] has been used for the dimensioning of structural steel elements, where all conditions for ultimate limit state (ULS) and serviceability limit state (SLS) are fulfilled. For ULS, the elements are checked for the following:

- axial force;
- shear force;
- bending moment;



- resistance to compression buckling;
- lateral-torsional buckling resistances;
- interaction between compression buckling resistance and lateral-torsional buckling resistance, see Kravanja et al. [7].

Eurocode 5 [16] has been used for the dimensioning of a glulam frame. These equations include the cross-sectional resistances of the columns and beams for:

- axial compression force;
- bending moment;
- shear force;
- compression buckling resistance;
- lateral torsional stability;
- interaction between compression buckling resistance and lateral-torsional buckling resistance, see Kravanja et al. [7].

Stresses in the apex zone have been also checked, see Kravanja and Žula [11]. For SLS, the vertical deflections of steel and timber elements and the horizontal displacements of the main frames are checked.

Input data (constants) in the models include span, height and length of the hall structure, alternatives (strengths) of standard materials used (steel, timber, concrete), standard/discrete section alternatives, vertical load (snow), horizontal load (wind), weights of the roof and cladding, prices of steel, timber and concrete, safety factors, etc. The variables included in the models are continuous and discrete binary variables. The continuous variables represent the material costs of fabricating the structure, the number of main frames, purlins, and rails, the intermediate spacing between main frames, purlins, and rails, the section dimensions, areas, section moduli, torsional constants, etc. Note that the above mentioned (in)equality constraints for the ultimate and serviceability limit states (the axial and shear forces, bending moments, deflections, etc.) depend on the input data and the variables calculated.

3.1 Objective function

The material cost of the structure MAT_{COST} is defined as the sum of the material costs for the fabrication of the main frames, roof purlins, façade rails, façade columns, and foundations, see eqn (3):

$$\begin{aligned}
 MAT_{COST} = & \{2 \cdot [(A_C \cdot H_C \cdot \rho_{FRAME}) \cdot NO_{FRAME} + (A_B \cdot L_B \cdot \rho_{FRAME}) \cdot NO_{FRAME}]\} \cdot ECM_{FRAME} \quad (3) \\
 & + \{(A_P \cdot L_{TOT} \cdot \rho_{STEEL}) \cdot NO_{PURL} + (A_R \cdot L_{TOT} \cdot \rho_{STEEL}) \cdot NO_{RAIL} \\
 & + 2 \cdot (A_{FC} \cdot H_{FC} \cdot \rho_{STEEL}) \cdot (NO_{PURL} - 1)\} \cdot ECM_{STEEL} \\
 & + 2 \cdot (H_F \cdot B_F \cdot B_F) \cdot NO_{FRAME} \cdot ECM_{CONCR}
 \end{aligned}$$

where A_C in eqn (3) is the cross-section of the frame column, H_C is the height of the column, ρ_{FRAME} is the volume density of the frame material (steel or glulam), NO_{FRAME} is the number of main frames, A_B is the cross-section of the frame beam, L_B is half the length of the inclined beam (approx. span/2), and ECM_{FRAME} is the unit price of steel or glulam frame material (€/kg). In addition, A_P is the cross-section of the roof purlin, L_{TOT} is the length of the hall, ρ_{STEEL} is the volume mass of the steel, NO_{PURL} represents the number of purlins, A_R is the cross-section of the façade rail, NO_{RAIL} represents the number of façade rails, A_{FC} is the cross-section of the front and rear façade columns, H_{FC} stands for the height of the façade column,

ECM_{STEEL} is the unit price of steel (€/kg), H_F is the height of the square pad foundations, B_F is the width of the foundations and ECM_{CONCR} is the unit price of concrete (€/m³). The mentioned numbers of structural elements and cross-sections are defined as variables that are calculated when the objective function MAT_{COST} converges.

4 NUMERICAL EXAMPLE

In the paper, the numerical example of MINLP optimization of a hall structure with a span of 16 m, a length of 80 m and a height of 4.5 m is presented. The structure of the hall is loaded with its own weight, the weight of the roofing 0.20 kN/m², the weight of the façade cladding 0.15 kN/m², snow 0.80 kN/m² and with the horizontal wind 0.50 kN/m². The unit price of the steel is 1.5 €/kg, the price of the glulam is 1000 €/m³ and that of the concrete is 150 €/m³.

The MINLP optimization models of a single-storey steel and timber hall structures are modelled in the GAMS (General Algebraic Modelling System) environment [17]. The defined MINLP optimization problem is solved using the Modified Outer-Approximation/Equality-Relaxation algorithm of Kravanja and Grossmann [18], see also Kravanja et al. [13]. The algorithm works as an alternative sequence of non-linear programming (NLP) and mixed-integer linear programming (MILP) subproblems. The MINLP computer program MIPSYN [19] is applied for the optimization. GAMS/CONOPT (generalized reduced-gradient method) [20] is used for NLP continuous calculations and GAMS/CPLEX (branch and bound) [21] for MILP discrete optimizations.

4.1 Hall structure with steel frames

In the optimization model HALLOPT, we defined the superstructure of the steel hall, which represents variety of different alternatives of structural elements, cross-sections and strength classes of materials: 70 portal frames, 2×25 purlins, 2×5 façade rails, 18 hot rolled IPE profiles (from IPE 80 to IPE 600, extra for purlins and rails), 24 hot-rolled HEA sections (from HEA 100 to HEA 1000, extra for frame columns, frame beams, and the front and rear wall façade columns), three structural steel grades (S 235, S 275 and S 355) and 121 different discrete alternative widths for the 1.20 m deep square foundations (from 50 to 350 cm, in 2.5 cm increments). The defined superstructure of the hall contains $1.4226 \cdot 10^{13}$ different alternative structures – one of which is the optimal one.

The MINLP optimization was carried out using the computer program MIPSYN, which required a working time of about 20 minutes for the calculation. The optimal result of €100,245 was found in the 19th major MINLP iteration, see the convergence in Table 1.

Note that the OA/ER algorithm consists of an alternative sequence of non-linear programming (NLP) optimization subproblems and mixed-integer linear programming (MILP) main problems. The discrete optimization MILP involves a global linear approximation to the structure and predicts a new set of binary variables, i.e., a new topology and standard dimensions/materials. The NLP subproblem corresponds to the continuous optimization of the structure for the computed topology and standard dimensions/materials given at the corresponding MILP. From iteration to the iteration (MILP and NLP), the NLPs get better results, and the MILPs get worse results. The search is terminated when the NLP matches the MILP result. In the case of the non-convex problem, the search is stopped when the NLP can no longer be improved.

A three-phase MINLP strategy was used for the optimization. The first phase corresponds to the initialization, in which the continuous NLP optimization is performed. The first NLP solution of €84,308 is used as a good starting point for further discrete optimization. The

Table 1: Convergence to the optimal result for the steel hall.

MINLP iteration	MINLP subphase	Result €
Phase 1: Continuous optimization		
1	Initialization 1.NLP	84,308
Phase 2: Discrete topology and material optimization		
2	1.MILP 2.NLP	85,142 85,475
3	2.MILP 3.NLP	85,484 86,546
Phase 3: Discrete topology, material and standard dimension optimization		
4	3.MILP 4.NLP	2,068,470 98,664 loc. infeas.
.	.	.
.	.	.
.	.	.
19	18.MILP 19.NLP	2,070,029 100,245
20	19.MILP 20.NLP	2,070,531 100,973 loc. infeas.
.	.	.
.	.	.
.	.	.
25	24.MILP 25.NLP	2,070,549 100,788

second phase corresponds to the simultaneous topology and material discrete optimization. The dimensions/sections are still continuous (not discrete) in this phase. The optimal solution of the second phase is found in the 2nd MINLP iteration and yields €85,475. Since this problem is highly non-linear and non-convex optimization problem, the search is terminated if the NLP does not yield an improvement (the next 3rd iteration shows a worse solution of €86,546). After the optimal solution of the second phase is calculated, the optimization continues with the overall topology, material and standard dimension optimization of the structure at the third phase. In this phase, the best NLP solution of €100,245 is found in the 19th iteration, since the first real subsequent NLP solution shows a worse result of €100,788 at the 25th iteration (note that all other subsequent solutions before the 25th iteration are locally infeasible).

The optimal result represents the lowest possible material cost of the hall structure, namely €100,245, the optimal number of 16 portal frames, 12 purlins and eight rails. Obtained are the optimal profiles HEA 280 for the frame columns, HEA 320 for the frame beams, IPE 120 for the purlins, IPE 140 for the rails, HEA 120 for the façade columns and the square concrete pad foundations $1.20 \times 1.675 \times 1.675 \text{ m}^3$. Steel class S 355 is calculated (see Fig. 2).

4.2 Hall structure with timber frames

The second calculation deals with the optimization of the same hall structure as above, but the main frames are made of a glulam with a rectangular cross-section. The paper deals with

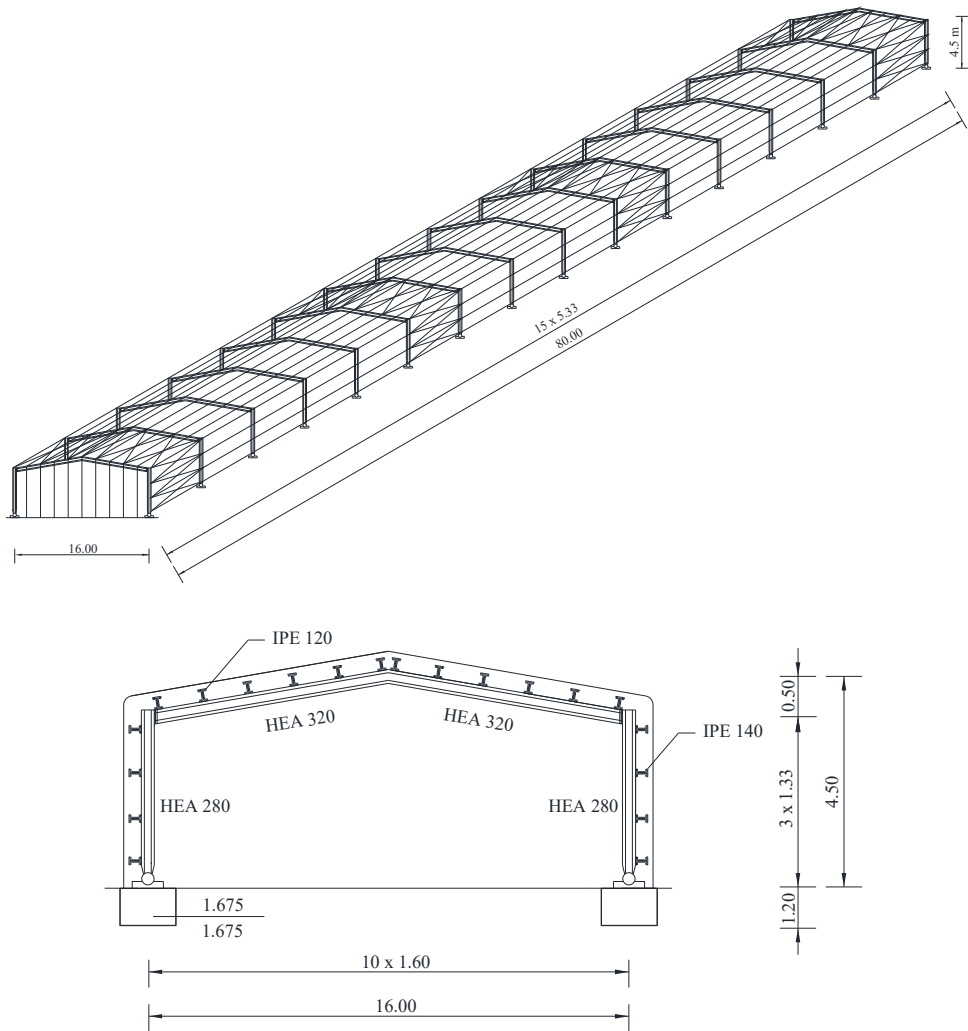


Figure 2: Optimal structure of the hall with main steel frames.

a special case, where the columns and beams of the timber frames have the same cross-section. The superstructure of the hall contains 70 portal frames, 2×25 purlins, 2×5 rails, 17 discrete alternatives for the width of the timber rectangular cross-section of the frame (from 10 to 50 cm, in 2.5 cm steps), 51 alternatives for the height of the timber frame cross-section (from 25 to 150 cm, in 2.5 cm steps), 18 hot-rolled IPE profiles (from IPE 80 to IPE 600 for purlins and rails separately), 24 hot-rolled HEA sections (from HEA 100 to HEA 1000 for façade columns), three structural steel classes (S 235, S 275 and S 355) and 121 different discrete alternative widths for the 1.20 m deep square foundations (from 50 to 350 cm, 2.5 cm increments). The material of the glulam is GL28h. The superstructure of the hall contains $6.4240 \cdot 10^{13}$ different structural alternatives – one of them is optimal.

The minimal calculated material cost of the hall structure with main frame made of timber resulted in €138,207. The computer program MIPSYN needed a working time of about 15 minutes for the calculation. The optimal result was found in the 9th major MINLP iteration, see the convergence in Table 2. The optimal number of 12 portal frames, 12 purlins and eight rails was calculated. The optimal cross-sectional dimensions of the timber frames are 200/1,200 mm². The optimal profiles IPE 160 for the purlins, IPE 200 for the rails and HEA 120 for the façade columns were determined (see Fig. 3). The dimensions of the square concrete foundation are 1.20 × 1.925 × 1.925 m³. The steel grade S 355 was calculated.

Table 2: Convergence to the optimal result for the timber hall.

MINLP iteration	MINLP subphase	Result €
Phase 1: Continuous optimization		
1	Initialization 1.NLP	123,279
Phase 2: Discrete topology and material optimization		
2	1.MILP 2.NLP	96,712 129,021
3	2.MILP 3.NLP	101,529 125,370
4	3.MILP 4.NLP	104,320 124,990
5	4.MILP 5.NLP	105,772 127,602
Phase 3: Discrete topology, material and standard dimension optimization		
6	5.MILP 6.NLP	2,954,022 137,290 loc. infeas.
7	6.MILP 7.NLP	2,954,455 137,745 loc. infeas.
8	7.MILP 8.NLP	2,954,615 137,884 loc. infeas.
9	8.MILP 9.NLP	2,954,889 138,207
10	9.MILP 10.NLP	2,955,049 138,377

5 CONCLUSIONS

The paper deals with the optimization of the structure of a single-storey hall consisting of the same main frames to which steel purlins, façade rails and façade columns are connected. The frames may be made of steel profiles or glulam. The optimization of the hall structure is performed using mixed-integer non-linear programming, MINLP. The objective function of the material cost of the structure is subject to a system of (in)equality constraints of statics and dimensioning. The modified outer-approximation/equality-relaxation algorithm (OA/ER) is applied to solve the optimization problem. The computer program MIPSYN is used. In addition to the determined minimal material cost of the structure, the optimal topology of the hall structure, the strength classes of the materials used, the standard steel profiles, and the discrete/rounded cross-sections of the glulam frames and of the concrete foundations are calculated.



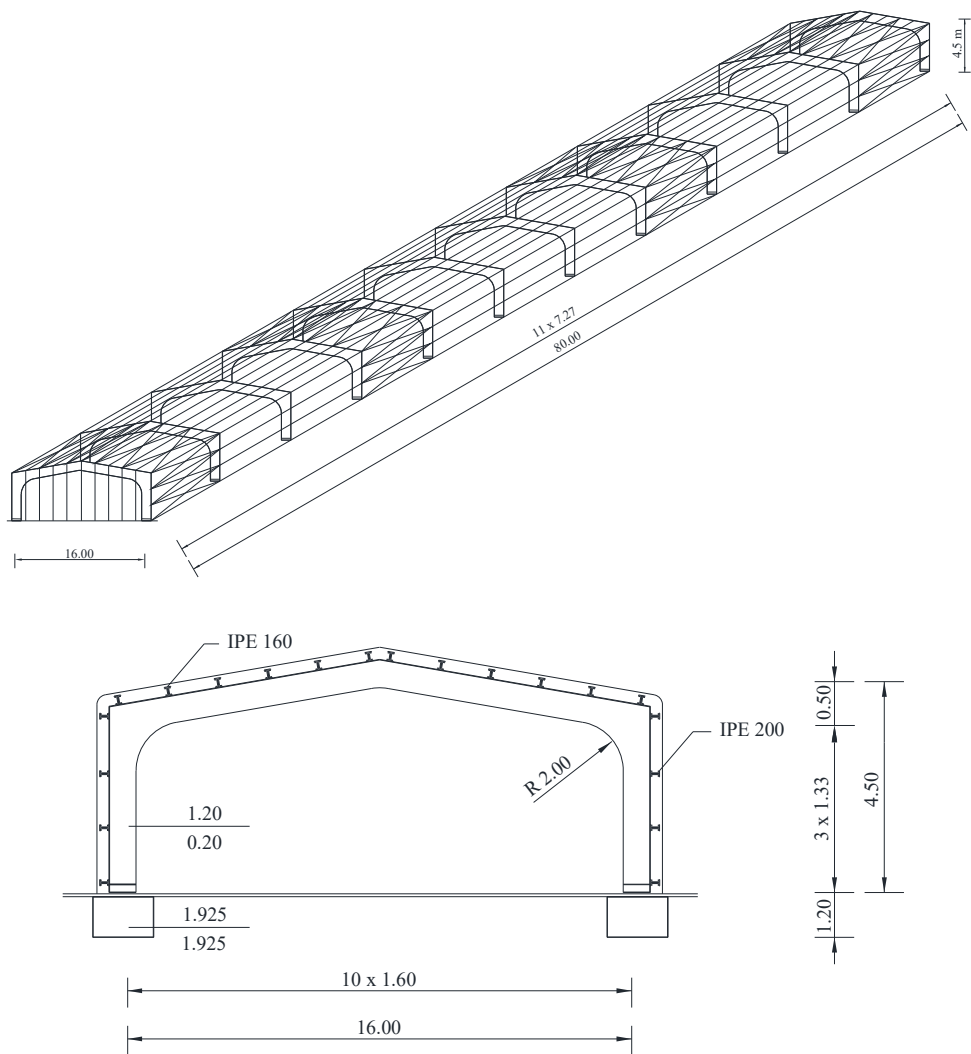


Figure 3: Optimal structure of the hall with main timber frames.

A numerical example of MINLP optimization of a hall structure is presented at the end of the article. For the given span, load, unit prices of steel, timber and concrete, we calculated the minimal material cost of the hall, separately for the steel-framed hall type and separately for the timber-framed hall version. Such cost optimization is very useful for quick comparison and selection of the optimal construction variant. Note that the steel frame hall version turned out to be more advantageous than the timber frame version, especially because of the high price of the glulam used.

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