DEALING WITH DISCRETE BINARY VARIABLES IN MINLP STRUCTURAL OPTIMIZATION

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ABSTRACT

This paper presents the rational use of discrete binary variables in mixed-integer non-linear programming (MINLP) structural optimization. MINLP is a powerful optimization method that integrates both discrete and continuous variables while considering non-linear constraints. MINLP is used in many areas, such as chemical engineering, energy systems, supply chain and logistics and financial portfolio management. MINLP also plays a crucial role in structural optimization. In this area, it can be used to optimize various structures such as steel, concrete, timber and composite systems such as frames, bridges and trusses. However, structural optimization problems using MINLP often involve an immense number of binary variables, sometimes in the tens of thousands, making the problems computationally intractable. To get a handle on this complexity, the number of binary variables must be reduced to a manageable size. This can be achieved by techniques such as incorporating special logical constraints, implementing multilevel MINLP strategies or performing preselection of binary variables. This paper specifically examines how many binary variables can be eliminated by pre-screening, a method that has been shown to be particularly effective. Five structural optimization case studies are examined: three large problems with up to tens of thousands of binary variables and two medium-sized problems with several hundred variables. The analysed structures include a steel truss, two hydraulic steel gates, a steel hall and a timber hall. The results show that pre-screening leads to a greater reduction in binary variables for large problems than for medium problems. On average, only about 5% of the original binary variables were retained in the MINLP optimization process after pre-screening. This significant reduction contributed to a significant improvement in the convergence speed of the OA/ER algorithm.

Keywords: structural optimization, mixed-integer non-linear programming, MINLP, discrete optimization, sizing optimization, topology optimization, multilevel strategy, pre-screening.

1 INTRODUCTION

Mixed-integer non-linear programming (MINLP) is a powerful optimization technique for solving combined discrete, non-linear and non-convex optimization problems. The latter can include both integer and continuous decision variables as well as non-linear objective functions and/or constraints. MINLP extends and combines the capabilities of integer programming (IP) or mixed-integer linear programming (MILP) with non-linear programming (NLP) by enabling more realistic modelling of complex systems where some decisions can be defined as binary or categorical. MINLP is therefore particularly valuable for real-world applications where decisions are not purely discrete or continuous, but are a combination of both, such as in structural optimization, industrial process optimization, supply chain management, and resource allocation.

MINLP is widely used in areas such as structural optimization, energy system design, chemical process optimization, logistics and finance, where decision making must consider both discrete decisions and non-linear relationships between variables. For example, in a manufacturing environment, a plant must decide which machines to turn on (discrete) and how much material to allocate to each process (continuous), while at the same time considering nonlinear physical or economic constraints.

In structural optimization, it is usually desirable to calculate for a given structure which structural elements make up the structure and where (discrete topology), which standard/

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discrete profiles and strength classes/materials are optimal; and at the same time, it is also necessary to calculate some continuous dimensions (usually the lengths of some elements, e.g. diagonals). These design parameters are defined as variables that are calculated in the optimization when the objective function converges. The latter is usually defined as a mass or cost objective function. Binary 0–1 variables are defined for the discrete decisions. For example, a binary variable is assigned to each structural element, which is then selected to build the structure if the value of the calculated binary variable is 1, otherwise (0) it is removed.

For large MINLP problems, a major difficulty is the large number of discrete, binary 0–1 variables that are defined to solve discrete decisions. This number can be up to several tens of thousands, which usually prevents the problems from being solved successfully. To overcome this difficulty, the number of binary variables should be reduced. The latter can be done by defining special equations to reduce the number of binary variables, by using a suitable multilevel MINLP strategy and by preselecting the binary variables. The paper answers the question to what extent this reduction should be performed and how high the percentage of active binary variables is, that is particularly reduced by the pre-screening procedure.

2 APPLICATIONS OF THE MINLP

The MINLP method has become an indispensable tool for solving complex optimization problems in various industries, e.g. chemical engineering, energy system optimization, supply chain and logistics management, financial portfolio optimization and structural optimization.

In process optimization in chemical engineering, MINLP plays a crucial role in process design and optimization (see for example Grossmann and Kravanja [1] and Ye et al. [2]). These processes often involve discrete decisions – e.g. the selection of suitable equipment or the definition of operating modes – as well as continuous parameters such as flow rates, temperature and pressure that need to be fine-tuned. The aim is often to reduce energy consumption, increase product yield or reduce operating costs. Here, the models include non-linearities such as reaction kinetics and thermodynamic relationships, leading to more efficient and economically viable designs.

Energy production and distribution systems, ranging from electricity generation to gas distribution and renewable energy integration, represent comprehensive optimization problems (see for example Nguyen et al. [3] and Moretti et al. [4]). MINLP is often used to support both the planning and operational phases of these systems. Strategic decisions, e.g. where and whether to install energy-generating facilities such as power plants or wind turbines, are inherently discrete. Operational aspects, such as energy distribution and fuel consumption, on the other hand, involve continuous variables. These systems are further complicated by non-linear elements such as fuel cost curves and environmental compliance.

Supply chain optimization is another important application of MINLP that offers significant advantages in managing complex logistics networks (see for example Holler Branco et al. [5] and Billal et al. [6]). The goal is typically to reduce overall costs while ensuring customer demand and maintaining service standards across suppliers, warehouses and distribution hubs. Integer variables are used to model discrete decisions, such as the selection of warehouse locations, the choice of transportation methods and the number of vehicles. At the same time, continuous variables are used to capture aspects such as shipping volumes, production rates and inventory levels. Non-linearities often result from dynamic elements such as variable transportation costs, shifting demand patterns and lead times.

In the financial sector, MINLPs are often used for portfolio optimization to help investors maximize their returns under risk management constraints (see for example Díaz et al. [7] and Bayat et al. [8]). Integer variables are used to model decisions such as which assets to include in the portfolio, while continuous variables determine how much capital to allocate to each selected asset. The optimization process is often complicated by non-linear elements such as returns, individual risk preferences and transaction costs.

3 APPLICATIONS IN STRUCTURAL OPTIMIZATION

The MINLP method has become an indispensable tool for solving complex optimization problems for steel, concrete and timber structures such as frames, bridges, trusses, composite structures, energy efficient buildings and others.

MINLP optimization of frames includes discrete and non-linear continuous decisions such as the choice of material and cross-sectional profiles and the determination of load assignments. These optimizations are applied to both multi-storey frame structures and single-storey frames (halls). While the multi-storey frame structures are usually optimized for the fixed topology with a fixed number of storeys and bays, the optimization of hall structures is also performed to find an optimal number of portal frames, purlins and rails (topology) (see Klanšek et. al. [9] and Kravanja et. al. [10]).

MINLP optimization of bridges requires the simultaneous calculation of discrete design variables (e.g. beam types, column sizes) and continuous parameters (e.g. load conditions, displacement limits). MINLP has been applied to bridge design problems, where it optimizes both structural configuration and material selection considering traffic and wind loads as well as safety requirements (see Kusano et al. [11] and Jin et al. [12]).

The optimization of trusses typically requires a combination of discrete and non-linear continuous decisions – for example, the optimization of the number of members (topology), the selection of materials or cross-sectional profiles, and the determination of load distribution (see for example Ohsaki and Katoh [13] and Šilih et al. [14]). MINLP has been widely used in this field to find optimal configurations that reduce weight or minimize cost while meeting structural safety and performance requirements. Its ability to deal with both variable types and non-linear constraints makes it particularly well suited to handle the complexity of truss optimization.

MINLP optimization of composite structures combines both continuous variables (such as intermediate spacing between I-beams) and integer variables (such as the selection of discrete materials and standard sizes) while dealing with nonlinear relationships (such as resistance conditions and deflection constraints). It is useful for finding the best design that meets performance requirements while minimizing weight, cost, or other objectives (see Kravanja et al. [15] and Kırdar et al. [16]).

MINLP optimization of energy efficient buildings balances structural integrity, thermal performance and energy consumption. The MINLP is used to optimize the structural and energy systems of buildings to ensure minimum energy consumption while maintaining structural strength and stability (see for example Lešnik Nedelko et al. [17] and Kravanja et al. [18]).

MINLP optimization is also performed for the optimization of structures for seismic design, for offshore structures, hydrotechnical structures (steel gates and high-pressure penstocks), cranes and others.

4 SOLVING THE MINLP PROBLEM

MINLP optimization problems are combined discrete, continuous, non-linear and non-convex problems that can be mathematically formulated as follows:

$$\min f(\boldsymbol{x},\boldsymbol{y})$$

Subject to:

$$\boldsymbol{h}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{0} \qquad \text{(MINLP)}$$

$$\boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}) \le \boldsymbol{0}$$

$$\boldsymbol{x} \in X = \{\boldsymbol{x} \mid \boldsymbol{x} \in R^{n};\ \boldsymbol{x}^{lo} \le \boldsymbol{x} \le \boldsymbol{x}^{up}\}$$

$$\boldsymbol{y} \in Y = \{0, 1\}^{m}$$

where:

- $\boldsymbol{x}$ is a vector of continuous variables,
- $\boldsymbol{y}$ is a vector of discrete/binary variables,
- $f(\boldsymbol{x},\boldsymbol{y})$ is (usually) a non-linear objective function,
- $\boldsymbol{h}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{0}$ is a set of equality constraints, and
- $\boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}) \le \boldsymbol{0}$ is a set of inequality constraints.

Note that at least one function (or objective or constraint) must be non-linear. The presence of both discrete and continuous variables significantly increases the difficulty of the problem, since non-linear programming (NLP) solvers cannot explicitly handle discrete variables, and integer programming (IP) solvers are not designed for non-linear constraints. These problems can be effectively solved with the following MINLP algorithms:

- Generalized Benders Decomposition by Benders [19] and Geoffrion [20],
- Outer-Approximation/Equality-Relaxation (OA/ER) algorithm by Kocis and Grossmann [21],
- Sequential Linear Discrete Programming method by Olsen and Vanderplaats [22] and Bremicker et al. [23],
- Extended Cutting Plane algorithm by Westerlund and Petterson [24], and
- the LP/NLP Branch and Bound algorithm by Quesada and Grossmann [25].

4.1 The modified OA/ER algorithm

To overcome the challenges posed by non-convexities in MINLP problems, the improved version, the modified outer-approximation/equality-relaxation (OA/ER) algorithm developed by Kravanja and Grossmann [26], is used together with the MIPSYN computational platform [27]. This algorithm builds on the basic OA/ER method by introducing advanced convexification strategies to mitigate the negative effects of non-convexities during the iterative optimization process. This process alternates between two types of optimization methods: nonlinear programming (NLP) sub-problems and mixed-integer linear programming (MILP) main problems (see Fig. 1).

In this optimization process, each NLP sub-problem focuses on optimizing the continuous variables associated with a mechanical structure whose topology, material selection, and standard dimensional parameters were specified in the previous MILP step. These NLP steps contribute to the refinement of the global approximation of the entire mechanical design space. After solving each NLP, the MILP main problem takes over. It performs a discrete optimization of the updated linear model of the superstructure to propose a new configuration – topology, materials and standard dimensions – for the next NLP iteration. In this two-layer

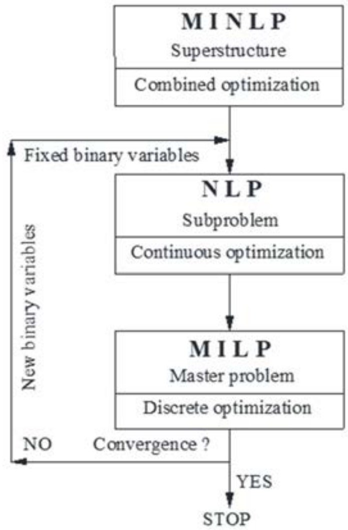

Figure 1: Steps of the OA/ER algorithm.

process, the main MILP problem provides a lower bound, while the NLP sub-problem provides an upper bound on the overall MINLP objective. Convergence is achieved when the two bounds coincide, signifying the optimal solution for convex problems. In non-convex cases, optimization is complete when successive NLP iterations no longer yield any improvements.

The NLP steps are solved using the GAMS/CONOPT solver, which applies the generalized reduced gradient algorithm [28]. Meanwhile, the MILP problems are treated with GAMS/CPLEX, which uses a branch and bound method [29]. GAMS (general algebraic modelling system) is proposed for modelling [30].

5 HANDLING OF DISCRETE BINARY VARIABLES

In discrete optimization, a major problem is the large number of discrete binary 0–1 variables defined in the problems, since the number of all combinations (e.g. different designs due to different topologies, materials, and standard dimensions) is 2^m, where m is the number of binary variables in the problem. This problem is even more pronounced with MINLP, which contains both linear and nonlinear functions. In order to obtain a better result or an optimal solution for large problems, it is necessary to reduce the binary variables. This is done by introducing special logical equations to reduce the number of binary variables, by using a suitable MINLP strategy and by preselecting the binary variables.

5.1 Special logical equation for reducing the binary variables

A special logical equation in which binary variables are multiplied and summed with different integer scalars can reduce the number of binary variables in problems. With eqn (1), for example, it is possible to define eight possible integers of *NUM* with a choice of only three binary variables:

$$NUM = y_1 + 2y_2 + 4y_3 \tag{1}$$

If all three calculated binary 0–1 variables are equal to 0, the number *NUM* is also equal to zero according to eqn (1). *NUM* is 1 if y_1 is 1 and the other two binary variables are 0. *NUM* is 2 if y_2 is 1 and the other two are 0. *NUM* is 3 if y_1 and y_2 are 1 and y_3 is 0. *NUM* is 4 if y_3 is 1 and the other two variables are 0. *NUM* is 5 if y_1 and y_3 are 1 and y_2 is 0. *NUM* is 6 if y_2 and y_3 are 1 and y_1 is 0. And finally, *NUM* is 7 if all binary variables are calculated as 1. In this way, eqn (1) requires fewer binary variables for the same condition than eqn (2):

$$NUM = y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 \tag{2}$$

With a very high number of *NUM*, see eqn (3), this notation allows the use of significantly fewer binary variables, resulting in a much faster calculation.

$$NUM = \sum_i^N 2^{(i-1)} \cdot y_i \quad i \in I \tag{3}$$

5.2 Linked multilevel hierarchical strategy (LMHS)

To streamline the solution process and support faster convergence within the OA/ER framework, the multilevel MINLP strategies, such as the linked multilevel hierarchical strategy (LMHS) [31], [32], are used.

LMHS decomposes the original MINLP problem into a structured hierarchy of levels, both in terms of variable space and optimization stages. Instead of dealing with the entire set of discrete variables Y_{TOT} at once, the discrete decision space is expanded step by step. At each level, a new subset of binary variables is introduced and pre-evaluated, while variables from other subsets are approximated by relaxed 0–1 representations. With each level, the accumulated outer approximations refine the model and help generate stronger lower bounds for the next stage of optimization.

In structural optimization, this hierarchy is generally composed of four decision levels:

- Level 1 focuses on discrete choices concerning the topology of the structure; here only binary variables y from the set for topology determination Y_{top} are used, while all other design parameters (materials, discrete dimensions) are continuous: $y \in Y_{top}$.
- Level 2 handles discrete decisions related to material qualities; here discrete optimization of topology and material is performed simultaneously, while all other parameters (discrete dimensions) remain continuous; binary variables are enriched here with the set of binary variables Y_{mat} for discrete materials: $y \in (Y_{top} \cup Y_{mat})$.
- Level 3 adds discrete decisions for standard dimensions; the simultaneous discrete optimization of topology, material and standard dimensions is performed at this level; a set of binary variables for standard dimensions Y_{st} is added: $y \in (Y_{top} \cup Y_{mat} \cup Y_{st})$.
- Level 4 refines the design using rounded continuous dimension values; the entire discrete topology, material, standard dimension and rounded dimension optimization is then applied; since a set of binary variables for rounded dimensions Y_{rd} is added, the entire set of binary variables Y_{TOT} is used here: $y \in (Y_{TOT} = Y_{top} \cup Y_{mat} \cup Y_{st} \cup Y_{rd})$.

The optimization process starts with the level 1 decisions (topology optimization), assuming relaxed values for the materials and dimensions. Once the optimal selection is made, the algorithm moves to the next level (level 2) to optimize both the discrete topology and the material selection. Once the optimal solution is found, the optimization process moves to discrete optimization at level 3 (topology, materials and standard dimensions). At the final level (level 4), the model incorporates all discrete variables Y_{TOT}, including the rounded dimensional alternatives derived from the earlier optimization phases, and the entire discrete optimization is performed.

Like the OA/ER algorithm, the LMHS ensures that the final solutions for convex problems are globally optimal (for non-convex problems they are usually local optima and practically acceptable).

5.3 Pre-screening of binary variables

Since the total number of binary variables, especially those linked to rounded dimensions, can run into tens of thousands, LMHS uses an automatic variable reduction procedure. This technique filters out less relevant binary variables and focuses only on those that closely match the previously obtained continuous solutions. For example, for each design parameter (topology, material or dimension), 3 (or 2) discrete variables below and 3 (or 2) above the calculated continuous value are considered. In this way, only 6 (or 4) binary variables are used to calculate a design parameter. This pre-screening of binary variables can be performed at any level and for any design parameter.

6 ANALYSIS OF PRE-SCREENED BINARY VARIABLES IN EXAMPLES

How many binary variables are reduced by pre-screening varies from problem to problem. It is assumed that the percentages of reduced-active binary variables are low for large optimization problems and higher for medium or small problems. To determine the number and percentage of reduced-active binary variables in structural optimization problems, some previously performed and published MINLP examples are analysed [32]–[34].

For example, Kravanja [32] presents the simultaneous MINLP optimization of mass, shape and standard dimensions of a simply supported 45-bar steel truss over a span of 30 m (see Fig. 2). Since the topology of the structure was not changed in the optimization, a single level of LMHS strategy was applied (with all binary variables organised in a single set). In this problem, 2,093 binary variables were reduced to 160 by the pre-screening procedure. An optimal mass of 2,250 kg was determined.

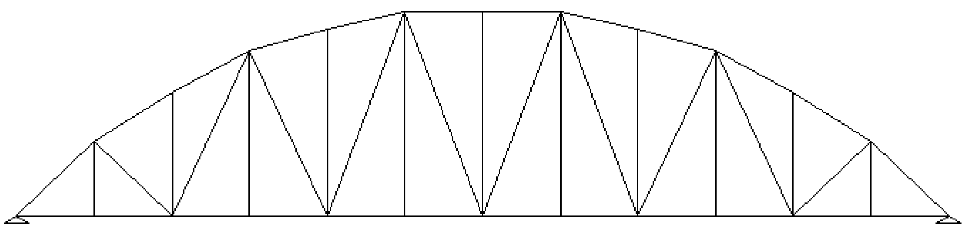

Figure 2: Optimal shape of the truss [32].

In Kravanja [32], the simultaneous MINLP optimization of costs, topology, standard dimensions and rounded dimensions of a roller hydraulic steel gate is presented. The gate was manufactured for the Hydroelectric Project Blanda in Iceland (see Fig. 3). The optimization problem was very extensive as it involved 19,622 binary and 59,097 continuous variables. A three-level LMHS strategy was applied (level 1: topology, level 2: standard dimensions, level 3: rounded dimensions) for the steel S275 (St 44-3) used. 19,622 of the binary variables were pre-screened to 224 variables. The optimal self-manufacturing costs of €8,804 were obtained.

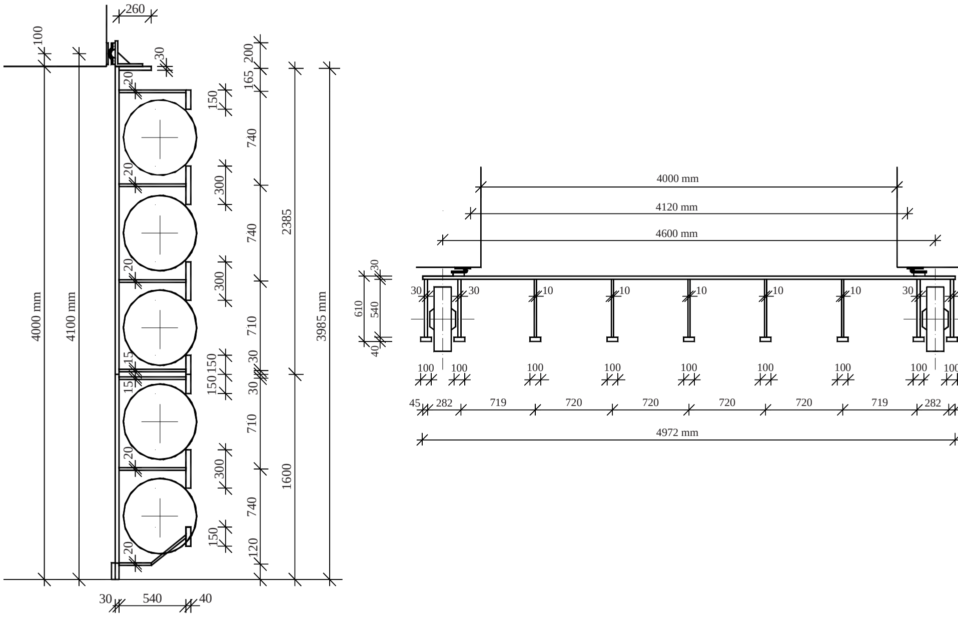

Figure 3: Vertical and horizontal cross-sections of the gate [32].

The MINLP optimization of another roller hydraulic steel gate is reported: the optimization of production costs, topology, material, standard and rounded dimensions of the Intake gate. Eight such gates were manufactured for Aswan II, Egypt [33]. A four-level LMHS strategy was used (level 1: topology, level 2: materials, level 3: standard dimensions, level 4: rounded dimensions). 35,052 of the binary variables were pre-screened to 666 variables which enabled the calculation of the optimal result. The optimal self-manufacturing costs of $48,811 were determined in this way.

Kravanja and Žula [34] present the MINLP optimization of production costs, topology, material, standard dimensions and rounded dimensions for a single-storey steel hall structure with a span of 16 m, a length of 80 m and a height of 4.5 m. A two-level LMHS strategy was applied (level 1: topology plus materials, level 2: standard dimensions together with rounded dimensions). 360 binary variables were pre-screened/reduced to 27 variables. The optimal result for the minimal production costs of the structure was €100,245 (see Fig. 4).

In Kravanja and Žula [34], the simultaneous MINLP optimization of material costs, topology, materials, standard dimensions and rounded dimensions of a timber hall structure with the same global geometry as above is also presented. A two-level LMHS strategy was applied (the same as above). Here, 426 binary variables were pre-screened to 33 variables. The minimal material cost of the hall structure reached €138,207 (see Fig. 5).

While the first three examples (the steel truss and the gates) represent large and very extensive MINLP optimization problems with several thousand discrete binary variables, the last two examples represent medium-sized problems. The percentages of binary variables reduced by the pre-screening procedure are listed for each example in Table 1. As hypothesized, the percentage of reduced-active binary variables is low for the large problems: 3.5%, while it is higher for the medium problems: 7.6%. The average percentage for all

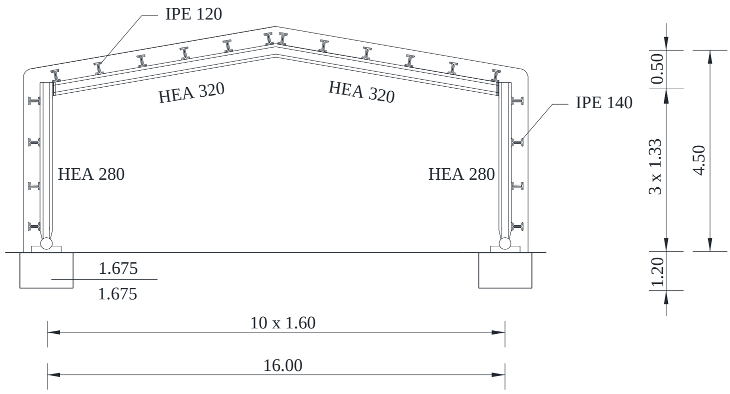

Figure 4: Optimal structure of the steel hall frame [34].

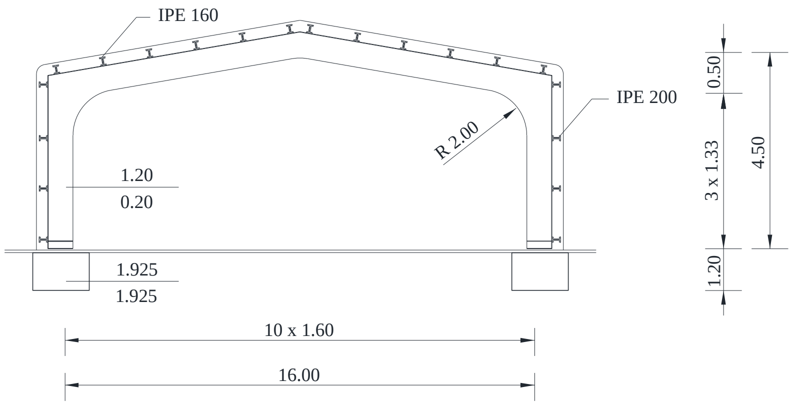

Figure 5: Optimal structure of the timber hall frame [34].

Table 1: Percentage of reduced-active binary variables by the pre-screening procedure.

Example: Size of the problem	Type of structure and reference	No. of LMHS levels	Total no. of binary variables	No. of prescreened binary variables	Percentage of reduced-active binary variables
1: large	Truss [32]	1 level	2,093	160	7.6%
2: large	Gate Blanda [32]	3 levels	19,622	224	1.1%
3: large	Gate Aswan [33]	4 levels	35,052	666	1.9%
Average percentage of reduced binary variables for large problems:					**3.5%**
4: medium	Steel hall [34]	2 levels	360	27	7.5%
5: medium	Timber hall [34]	2 levels	426	33	7.7%
Average percentage of reduced binary variables for medium problems:					**7.6%**
Average percentage of reduced binary variables for all problems:					**5.2%**

structures/examples considered is 5.2%. This means that not the entire sets of binary variables, but only about 5% of them were involved in the MINLP optimization processes after pre-screening, which significantly accelerated the convergences of the OA/ER algorithm.

7 CONCLUSIONS

This paper deals with MINLP, which is used in structural optimization. MINLP is a combined discrete and continuous optimization technique which also takes into account nonlinear constraints. For this reason, both discrete and continuous variables are used. MINLP is used to solve optimization problems in chemical engineering, the optimization of energy systems, supply chain and logistics management, the optimization of financial portfolios and also in structural optimization. In structural optimization, MINLP can be used for the optimization of steel, concrete, timber and composite structures such as trusses, floor systems, frames, bridges and others.

MINLP problems in structural optimization can contain up to several tens of thousands of binary variables, which makes the problems unsolvable. For this reason, the number of binary variables should be reduced to a reasonable level by introducing special logical equations, or by using a suitable multilevel MINLP strategy or by pre-screening the binary variables. The article answers the question of how many binary variables are reduced by pre-screening. Five MINLP optimization problems are analysed, three large problems with up to several tens of thousands of binary variables and two medium problems with several hundred variables. A steel truss, two hydraulic steel gates, a steel hall structure and a timber hall structure are analysed. It was found that after pre-screening, large problems have a lower percentage of active binary variables than medium problems. It was also found that only about 5% of the binary variables were involved in the MINLP optimization processes after pre-screening, which significantly accelerated the convergences of the OA/ER algorithm.

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