

COST OPTIMIZATION OF A CANTILEVER ROOF STRUCTURE FOR A SMALL FOOTBALL STADIUM IN LIMBUŠ, SLOVENIA

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ABSTRACT

The paper deals with the cost optimization of a cantilever roof structure, designed for a small football stadium in Limbuš, located close to the city of Maribor, Slovenia. The structure was proposed to be designed from steel I sections for columns, beams and struts, and from reinforced concrete bases. The cost optimization of the structure was calculated by the Mixed-Integer Non-linear Programming (MINLP). The MINLP optimization model of the structure was developed. A cost objective function was applied and subjected to the design, resistance and deflection constraints. The dimensioning constraints were defined in accordance with Eurocodes 2 and 3. The Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm was used.

Keywords: cost optimization, roof structure, mixed-integer non-linear programming, MINLP.

1 INTRODUCTION

Many different methods for structural optimization have been developed and used in the near past. Kalanta et al. [1] have performed discrete optimization of steel bar structures by the branch and bound method. Cicconi et al. [2] have used virtual prototyping tools and the genetic algorithm to reduce the weight and cost of steel structures. Mela and Heinisuo [3] have introduced the particle swarm optimization method for the weight minimization and the minimum cost optimization of welded high strength steel beams. Recently, Hasançeb [4] has considered the sizing optimization of steel frames by the evolution strategy method.

This paper deals with the material, standard and rounded dimension optimization of a cantilever roof structure for the small football stadium in Limbuš, located five kilometers far from the city of Maribor, Slovenia. The optimization of the cantilever is calculated by the Mixed-Integer Non-linear Programming (MINLP). The MINLP performs the discrete standard dimension optimization of columns, beams and struts (standard sizes optimization) as well as the rounded dimension optimization of reinforced concrete bases (the bases' dimensions are rounded on whole 10 centimeters) simultaneously with the continuous optimization of the structure self-manufacturing costs, internal forces and deflections.

The task of the optimization is to minimize the self-manufacturing costs of the cantilever roof structure. A cost objective function is applied and subjected to the design, resistance and deflection constraints. The dimensioning constraints are defined in accordance with Eurocode 3 [5] for steel, Eurocode 2 [6] for concrete and Eurocode 7 [7] for geotechnics.

The Modified Outer-Approximation/Equality-Relaxation algorithm is applied, see Kravanja and Grossmann [8], and Kravanja et al. [9], [10]. A three-phase MINLP optimization is used. The calculation begins with the overall continuous optimization. The first result is used as the starting point for the further second step, where the discrete optimization of steel grades is executed (standard and rounded dimensions are still continuous). After the optimal steel grade is obtained, the overall discrete material, standard and rounded dimension optimization of steel sections and concrete base is calculated.



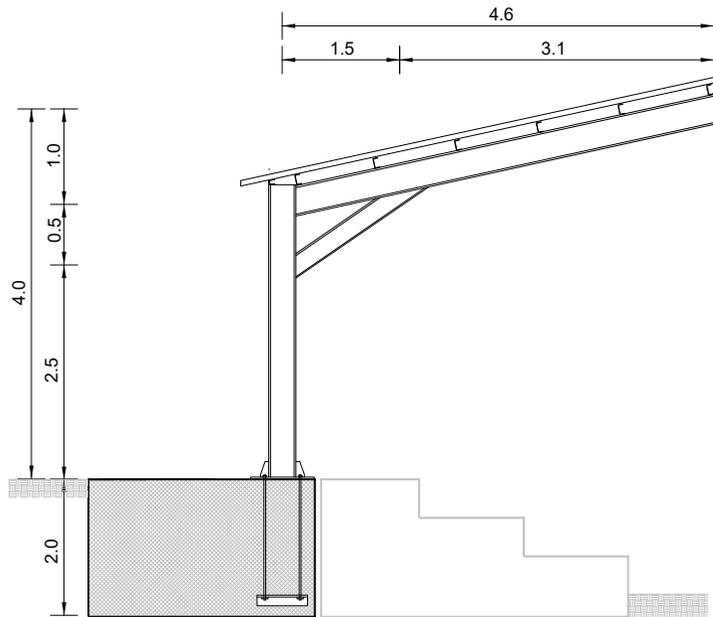


Figure 1: Cantilever roof structure.

2 CANTILEVER ROOF STRUCTURE

The paper presents the material, standard and rounded dimension optimization of a steel cantilever roof structure for the small football stadium in Limbuš. The treated steel structure is 16.0 metres long, 4.0 metres high and 4.6 metres wide. The structure is consisted from four equal cantilever frames with the intermediate distances of 5.33 meters, clamped onto four reinforced concrete bases (Fig. 1). The cantilever frame structure is proposed to be designed from standard hot rolled steel I sections for columns, beams and struts. Purlins which are mutually connected onto cantilever frames are not included in the optimization. Each reinforced concrete base is 2.0 meters high and 1.5 meters wide, placed behind the existed concrete seat structure. The in-plane length of the base L is treated in the optimization as a variable.

The cantilever frame structure is subjected to self-weight g and uniformly distributed variable load q (snow $s=1.5$ kN/m² and wind $w=0.14$ kN/m²). The design loads are defined: $1.35 \cdot g + 1.50 \cdot s + 1.50 \cdot 0.6 \cdot w$ for ultimate limit states, and $1.00 \cdot g + 1.00 \cdot s + 1.00 \cdot 0.6 \cdot w$ for serviceability limit states. Constraints for internal forces and deflections are defined by the force method. Eurocodes 2, 3 and 7 are used. When the ultimate limit state of structural elements is considered, the steel members are checked for:

- Axial resistance, see eqn (1). N_{Ed} stands for the design axial force, A represents the cross-section area of the member, f_y is the yield strength of steel and γ_{M0} is the resistance partial safety factor

$$N_{Ed} \leq \frac{A \cdot f_y}{\gamma_{M0}}. \quad (1)$$

- Shear resistance, see eqn (2), where V_{Ed} denotes the design shear force and A_v is the effective shear area of cross-sections

$$V_{Ed} \leq A_v \cdot \frac{f_y}{\sqrt{3}} \cdot \frac{1}{\gamma_{M0}}. \quad (2)$$

- Bending moment resistance, see eqn (3). M_{Ed} represents the design bending moment and W is the section modulus of the steel member

$$M_{Ed} \leq \frac{W \cdot f_y}{\gamma_{M0}}. \quad (3)$$

- Compression/buckling resistance, determined by eqn (4), χ is the reduction factor due to the flexural buckling and γ_{M1} is the resistance partial safety coefficient

$$N_{Ed} \leq \chi \frac{A \cdot f_y}{\gamma_{M1}}. \quad (4)$$

- The interaction between the compression and bending moment resistance, defined by eqn (5)

$$\frac{N_{Ed}}{A \cdot f_y / \gamma_{M0}} + \frac{M_{Ed}}{W \cdot f_y / \gamma_{M0}} \leq 1.0. \quad (5)$$

- The interaction between the buckling and the lateral-torsional buckling resistance, checked by eqn (6), where χ_{LT} is the reduction factor due to the lateral-torsional buckling

$$\frac{N_{Ed}}{\chi \cdot A \cdot f_y / \gamma_{M1}} + \frac{M_{Ed}}{\chi_{LT} \cdot W \cdot f_y / \gamma_{M1}} \leq 1.0. \quad (6)$$

- The bearing resistance failure in the ground under the reinforced concrete bases, checked by eqn (7). N presents the vertical force and M is the bending moment. A_b , L and b_b stand for the surface area, length and width of the base, respectively. While the surface area is calculated by eqn (8), the section modulus of the base W_b is defined by eqn (9)

$$\sigma = \frac{N}{A_b} + \frac{M}{W_b} \leq 200kPa, \quad (7)$$

$$A_b = L \cdot b_b, \quad (8)$$

$$W_b = \frac{b_b \cdot L^2}{6}. \quad (9)$$

- Overturning resistance of the reinforced concrete base, see eqn (10), where G is the self-weight of the reinforced concrete base, F_{tot} is the total action force (the self-weight plus snow plus wind), e is the distance between the total force and the point of overturning (the right bottom base edge) and γ is the safety factor ($\gamma = 2.5$)

$$G \cdot \frac{L}{2} \geq F_{tot} \cdot e \cdot \gamma. \quad (10)$$



When the serviceability limit state is verified, the cantilever total vertical deflection δ is checked by eqn (11), where L_{cr} is the span of cantilever roof structure

$$\delta \leq \frac{2 \cdot L_{cr}}{200}. \quad (11)$$

3 MINLP MODEL FORMULATION

Since the problem of the roof structure is the non-linear discrete-continuous optimization problem, the MINLP is applied for the solution. The general MINLP optimization problem can be formulated as follows:

$$\begin{aligned} \min \quad & z = \mathbf{c}^T \mathbf{y} + f(\mathbf{x}) \\ \text{s.t.} \quad & \\ & \mathbf{g}(\mathbf{x}) \leq \mathbf{0} \\ & \mathbf{B}\mathbf{y} + \mathbf{C}\mathbf{x} \leq \mathbf{b} \\ \mathbf{x} \in X = & \{\mathbf{x} \in R^n: \mathbf{xLO} \leq \mathbf{x} \leq \mathbf{xUP}\} \\ \mathbf{y} \in Y = & \{0, 1\}^m, \end{aligned} \quad (\text{MINLP-G})$$

where \mathbf{x} is a vector of continuous variables and \mathbf{y} is a vector of discrete/binary 0–1 variables. While $f(\mathbf{x})$ presents a dimension dependent non-linear function in the objective, $\mathbf{c}^T \mathbf{y}$ is linear fixed item, $\mathbf{g}(\mathbf{x})$ stand for non-linear functions in the (in)equality constraints. $\mathbf{B}\mathbf{y} + \mathbf{C}\mathbf{x} \leq \mathbf{b}$ are mixed linear (in)equality constraints.

The general MINLP-G model formulation is extended for the optimization of mechanical structures (MINLP-MS). The MINLP-MS model formulation is defined as follows:

$$\begin{aligned} \min \quad & z = \mathbf{c}^T \mathbf{y} + f(\mathbf{x}) \\ \text{s.t.} \quad & \\ & \mathbf{g}(\mathbf{x}) \leq \mathbf{0} \\ & \mathbf{A}(\mathbf{x}) \leq \mathbf{a} \\ & \mathbf{E}\mathbf{y} \leq \mathbf{e} \\ & \mathbf{P}\mathbf{y} + \mathbf{M}(\mathbf{d}^{\text{mat}}) \leq \mathbf{m} \\ & \mathbf{P}\mathbf{y} + \mathbf{S}(\mathbf{d}^{\text{st}}) \leq \mathbf{s} \\ & \mathbf{P}\mathbf{y} + \mathbf{R}(\mathbf{d}^{\text{rd}}) \leq \mathbf{r} \\ \mathbf{x} \in X = & \{\mathbf{x} \in R^n: \mathbf{xLO} \leq \mathbf{x} \leq \mathbf{xUP}\} \\ \mathbf{y} \in Y = & \{0, 1\}^m, \end{aligned} \quad (\text{MINLP-MS})$$

where $\mathbf{g}(\mathbf{x}) \leq \mathbf{0}$ and $\mathbf{A}(\mathbf{x}) \leq \mathbf{a}$ stand for the design, resistance and deflection constraints, $\mathbf{E}\mathbf{y} \leq \mathbf{e}$ define the relations between binary variables, $\mathbf{P}\mathbf{y} + \mathbf{M}(\mathbf{d}^{\text{mat}}) \leq \mathbf{m}$ determine discrete materials (grades), $\mathbf{P}\mathbf{y} + \mathbf{S}(\mathbf{d}^{\text{st}}) \leq \mathbf{s}$ calculate standard dimensions and $\mathbf{P}\mathbf{y} + \mathbf{R}(\mathbf{d}^{\text{rd}}) \leq \mathbf{r}$ define rounded dimensions (length of the concrete base is rounded up to whole 10 centimetres). See the formulation in detail in [11], [12].

4 MINLP OPTIMIZATION

The optimization of the structure is carried out by the MINLP computer program MIPSYN, the extension of PROSYN by Kravanja and Grossmann [8]. The optimization model is modelled according to the above model formulation with GAMS (General Algebraic



Modelling System) by Brooke et al. [13]. The Modified OA/ER algorithm and the three-phased optimization are applied. While GAMS/CONOPT2 (Generalized Reduced-Gradient method) by Drudd [14] is used to solve NLP problems, GAMS/CPLEX [15] (Branch and Bound) is used to solve MILP problems.

The cost objective function comprises the self-manufacturing costs of the material, anti-corrosion resistant painting, panelling, connections and assembling, see eqn (12). Table 1 presents the defined material and labour prices/costs.

$$\min: COST = C_c \cdot V_c + C_s \cdot \rho_s \cdot V_s + C_r \cdot \rho_s \cdot V_r + C_{ac} \cdot A_{ac} + C_{sc} + 0.15 \cdot C_s \cdot \rho_s \cdot V_s + C_p \cdot V_p + C_{pl} \cdot A_{pl}, \quad (12)$$

where *COST* denotes the self-manufacturing costs of the cantilever roof structure; C_c , C_s , C_r , C_{ac} , C_{sc} , C_p and C_{pl} are the prices of the concrete and steel materials, of the anti-corrosion resistant painting, of the steel connections as well as the prices of the construction pit and of the panelling, see Table 1. V_c , V_s , V_r and V_p represent the volumes of concrete base, steel I sections, reinforcing steel and excavation, respectively; ρ_s is the unit mass of steel; A_{ac} is the exposed area of steel members; and A_{pl} is the area of panelling. Assembling costs of the steel structure is defined to be 15% of the calculated material costs.

The cantilever roof superstructure includes different material, standard dimension and rounded dimension discrete alternatives. The superstructure comprises 3 different structural steel grades (S235, S275, S355), 24 different standard hot rolled European wide flange sections (from HEA 100 to HEA 1000) for column, 18 different standard hot rolled European I sections (from IPE 80 to IPE 600) separately for beam and strut, and 61 various discrete alternatives for rounding up on whole 10 centimeters (from 100 to 700 cm) for the length of the concrete base.

The optimal discrete design of the cantilever roof structure is gained in the 24th MINLP iteration. The minimal self-manufacturing costs of 4545 € per cantilever frame are obtained, and 18180 € for all four structures (the selling price is approx. 2.5 times higher than these production costs). The calculated optimal design comprises the structural steel grade S355, steel sections HEA 400 for columns, sections IPE 300 for beams and IPE 200 for struts. Calculated is also the concrete base length of 540 cm. The calculated design is shown in Fig. 2. The results obtained with the optimization yielded 24% of savings in steel and 30% in concrete when compared to the classical structural analysis calculations.

5 CONCLUSIONS

The paper presents the cost optimization of a cantilever roof structure, designed for the small football stadium in Limbuš, located close to the city of Maribor, Slovenia. The MINLP optimization model of the roof structure and the cost objective function were developed. The optimization was performed by the MINLP computer program MIPSYN. The Modified OA/ER algorithm and the three-phased optimization strategy were applied.

Table 1: Material and labour costs.

C_c Material and erection costs for concrete C 25/30	125.0	EUR/m ³
C_s Material costs for structural steel S 235	1.25	EUR/kg
C_r Material costs for reinforcing steel B 500	1.30	EUR/kg
C_{ac} Anti-corrosion resistant painting costs	20.0	EUR/m ²
C_{sc} Costs per steel connection	300.0	EUR
C_p Construction pit (incl. excavation, removal, planning)	10.0	EUR/m ³
C_{pl} Panelling costs (incl. material and erection)	5.0	EUR/m ²



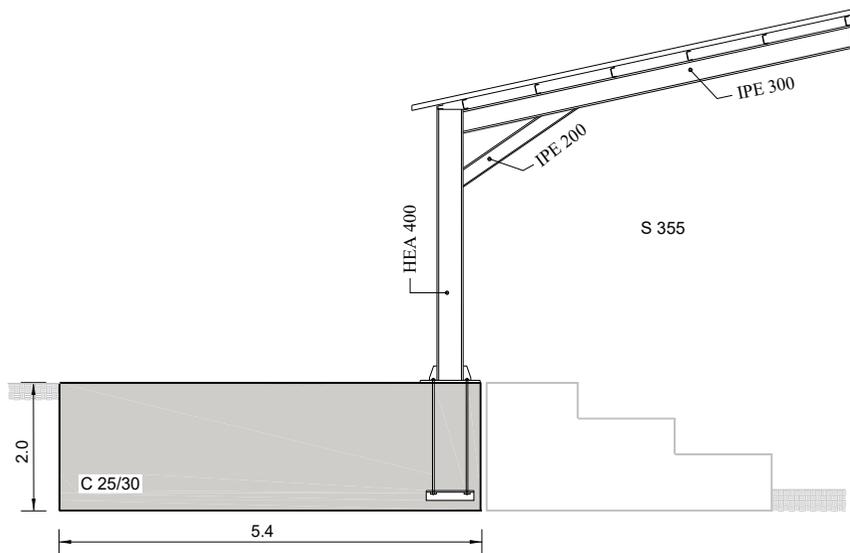


Figure 2: Optimum design of the cantilever roof structure.

The obtained optimal/minimal self-manufacturing costs of the structure represent 24% of savings in steel and 30% in concrete when compared to the classical structural analysis calculations. The example clearly shows the advantage of the proposed MINLP optimization approach.

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