Prediction modeling of power and torque in end-milling

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Abstract

This paper presents the development of mathematical models for torque and power in milling 618 stainless steel using coated carbide cutting tools. The response surface method was used to predict the effect of power and torque in the end-milling. From the model, the relationship between the manufacturing process factors, including the cutting speed, feed rate, axial depth and radial depth with the responses such as torque and power, can be developed. Beside the relationship, the effect of the factors can be investigated from the equation developed. It can be seen that the torque increases with a decrease of cutting speed with an increase of the feed rate, axial depth and radial depth. The acquired results also show that the power increase with an increase in cutting speed, feed rate, axial depth and radial depth. It can be found that the second order is more accurate based on the variance analysis and the predicted value is closely matched with the experimental result. Third- and fourth-order models are generated for both responses to investigate the 3- and 4-way interaction between the factors. The third- and fourth-order models show that 3- and 4-way interaction was found to be less significant for the variables.
Keywords: torque, power, end-milling, response surface method.

1 Introduction

In this work, experimental results were used for modeling using response surface roughness methodology (RSM) [1]. The RSM is practical, economical and relatively easy to use and it has been used by many researchers for modeling

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machining processes [2–4]. Mead and Pike [5] and Hicks [6] reviewed the earliest work on response surface methodology. RSM is a combination of experimental and regression analysis and statistical inferences. The concept of a response surface involves a dependent variable y called the response variable and several independent variables x_1, x_2 ... x_k [7]. The main aim of the paper is to investigate the effect of variables towards the responses and investigate the 3- and 4-way interaction between the factors.

2 Torque and power model

The proposed relationship between the responses (torque and power) and machining independent variables can be represented by the following:

$$\tau = C\,(V^m F^n A_x^y A_r^z)\varepsilon' \tag{1}$$

$$P = C\,(V^m F^n A_x^y A_r^z)\varepsilon \tag{2}$$

where τ is the torque in Nm, P is the power in watts, V, F, A_x, and A_r are the cutting speed (m/s), feed rate (mm/rev), axial depth (mm) and radial depth (mm). C, m, n, y and z are the constants. Equations (1) and (2) can be written in the following logarithmic form:

$$\ln \tau = \ln C + m \ln V + n \ln F + y \ln A_x + z \ln A_r + \ln \varepsilon' \tag{3}$$

$$\ln P = \ln C + m \ln V + n \ln F + y \ln A_x + z \ln A_r + \ln \varepsilon' \tag{4}$$

Equations (3) and (4) can be written as a linear form:

$$\tau = \beta_0 x_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon \tag{5}$$

$$P = \beta_0 x_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon \tag{6}$$

where τ is the torque in Nm, P is the power in watts, $x_0 = 1$(dummy variables), $x_1 = \ln V$, $x_2 = \ln F$, $x_3 = \ln A_x$, $x_4 = \ln A_r$ and $\varepsilon = \ln \varepsilon$, where ε is assumed to be a normally-distributed uncorrelated random error with zero mean and constant variance, $\beta_0 = \ln C$ and β_1, β_2, β_3, and β_4 are the model parameters. The second model can be expressed as:

$$y'' = \beta_0 x_o + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_{11} x^2{}_1 + \beta_{22} x^2{}_2 + \beta_{33} x^2{}_3 + \beta_{44} x^2{}_4 + \beta_{11} x_1 x_2 + \beta_{12} x_1 x_3 + \beta_{13} x_1 x_4 + \beta_{14} x_2 x_3 + \beta_{15} x_3 x_4 \tag{7}$$

The values of β_1, β_2, β_3, and β_4 are to be estimated by the method of least squares. The basic formula is:

$$\beta = (x^T x)^{-1} x^T y \tag{8}$$

where x^T is the transpose of the matrix x and $(x^Tx)^{-1}$ is the inverse of the matrix (x^Tx) and y is the value from experiment. The details of the solution by this matrix approach are explained in [1]. The parameters have been estimated by the method of least-squares using a Matlab computer package.

2.1 Experimental design

To develop the first-order, a design consisting of 27 experiments was conducted. Box-Behnken Design is normally used when performing non-sequential experiments, which are, performing the experiment only once. These designs allow efficient estimation of the first- and second-order coefficients. Because Box-Behnken Design has fewer design points, it is less expensive to run than central composite designs with the same number of factors. Box-Behnken Design does not have axial points, thus one can be sure that all design points fall within the safe operating parameters. Box-Behnken Design also ensures that all factors are never set at their high levels simultaneously [8–10]. Preliminary tests were carried out to find the suitable cutting speed, federate, axial depth and radial depth as shown in table 1.

2.1.1 Experimental details

The 618 stainless steel workpieces were provided in fully annealed condition in sizes of 65x170 mm. The tools used in this study are carbide inserts PVD coated with one layer of TiN. The inserts are manufactured by Kennametal with ISO designation of KC 735M.They are specially developed for milling applications where stainless steel is the major machined material. The end-milling tests were conducted on an Okuma CNC machining center MX-45VA. Every one passes (one pass is equal to 85mm), the cutting test was stopped. The same experiment has been repeated three times to get a more accurate result.

3 Results and discussion

3.1 First-order model for the torque and power model

The machining power is the product of cutting speed, υ and the cutting force, F_c. Thus the equation for the power is:

$$P = F_c \, \upsilon \tag{9}$$

Table 1: Levels of independent variables.

Factors \ Coding of Levels	-1	0	1
Speed, V_c (m/s)	100	140	180
Feed, f (mm/rev)	0.1	0.2	0.3
Axial depth of cut, a_a, (mm)	1	1.5	2
Radial depth of cut, a_r, (mm)	2	3.5	5

where P is the power in watts, υ is the cutting speed in m/min and F_c is the cutting force from the experiment in N. From the equation (9), the power can be calculated. The first order model from Matlab for power and torque are:

$$P' = 6.1993 + 0.1633x_1 + 0.3025x_2 + 0.26x_3 + 0.2592x_4 \quad (10)$$

$$T' = 2.6215 - 0.1308x_1 + 0.2292x_2 + 0.1408x_3 + 0.2142x_4 \quad (11)$$

The predicted result from the first order model for power and torque are shown in figure 1(a) and 1(b). Tables 2 and 3 show the 95% confidence interval for the experiments and analysis of variance. For the linear model, the p-values for lack of fit are 0.196 and 0.123. Therefore, the model is adequate. The transforming equations for each of the independent variables are:

$$x_1 = \frac{\ln(V) - \ln(v)centre}{\ln(v)high - \ln(v)centre} \quad x_2 = \frac{\ln(F) - \ln(f)centre}{\ln(f)high - \ln(f)centre}$$

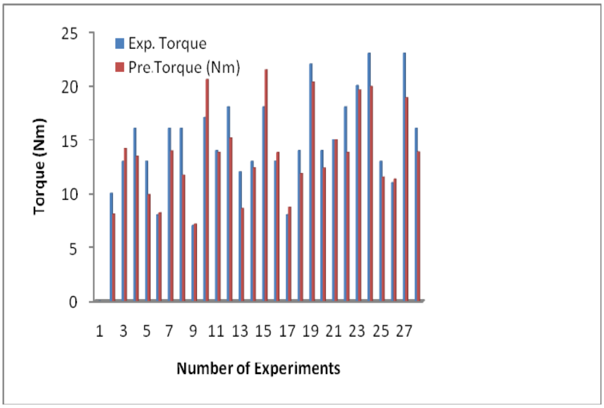
$$x_3 = \frac{\ln(A_x) - \ln(a_x)centre}{\ln(a_x)high - \ln(a_x)centre} \quad x_4 = \frac{\ln(A_r) - \ln(a_r)centre}{\ln(a_r)high - \ln(a_r)centre} \quad (12)$$

Table 2: ANOVA for power.

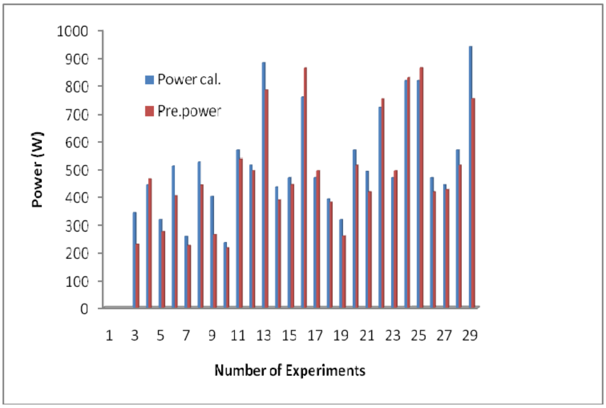
Source	**Degree of freedom**	**F ratio**	**P-value**
Regression	4	186.37	0
Linear	4	186.37	0
Residual Error	22		
Lack-of-Fit	20	5.1033	0.196
Total	26		

Table 3: ANOVA for torque.

Source	**Degree of freedom**	**F ratio**	**P-value**
Regression	4	39.69	0
Linear	4	39.69	0
Residual Error	22		
Lack-of-Fit	20	7.56	0.123
Total	26		



(a)



(b)

Figure 1: Comparison between predicted value and experimental for: (a) power, (b) torque.

Equation (9) describing the torque and power model can be transformed using Equation (12) into the following form:

$$T' = 315.23(V^{-0.5204}F^{0.796719}A_x^{\ 0.489432}A_r^{\ 0.60055}) \quad (13)$$

$$P' = 3.7065(V^{0.6498}F^{1.0515}A_x^{\ 0.9037}A_r^{\ 0.7267}) \quad (14)$$

This result shows that feed rate has the most significant effect on the torque, follow by axial depth, radial depth and cutting speed. The equation shows that the torque increase with reducing the cutting speeds. Equation (13) is utilized to develop torque contour at the selected cutting speed, and feed rate. Figure 2 shows the torque contour with selected cutting speed and feed rate. These contours help to predict the torque at any zone of experimental zone. From the contour, the torque reaches the highest value when the value of cutting speed at its lower value, feed rate, axial depth and radial depth are at their maximum

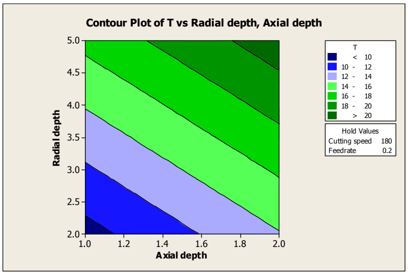


Figure 2: Torque contours in the axial depth-radial depth plane for cutting speed 180m/s and feed rate 0.2mm/rev.

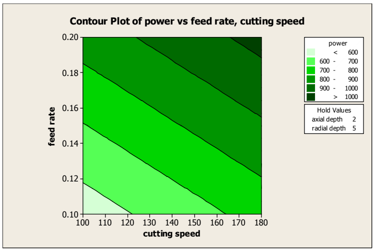


Figure 3: Power surface plot in the cutting speed-feed rate plane for axial depth 2 mm and radial depth 5mm.

value. From this contour plot, the safety zone of torque can be selected for any experiment. The equation shows that the power increases with increasing feed rate, axial depth and radial depth. Equation (14) is utilized to develop power surface plot at the selected axial depth, radial depth. Figure 3 shows the power plot with selected axial and radial depth.

3.2 Second-order and third-order model for torque and power

The second-order model was postulated in obtaining the relationship between the responses and the machine independent variables. The model equations are:

$$P'' = 2.05074 - 0.031x_1 + 47.37x_2 + 2.97x_3 + 1.60x_4 + 0.00029x^2{}_1 - 50.17x^2{}_2 - 0.78x^2{}_3 - 0.14x^2{}_4 - 0.29x_1x_2 - 0.018x_1x_3 - 0.0094x_1x_4 + 24.3x_2x_3 + 12.8x_2x_4 + 0.80x_3x_4 \quad (15)$$

$$T'' = -2080 - 17.22x_1 - 3099.72x_2 - 945.20x_3 - 113.22x_4 + 0.036x^2{}_1 - 3315.17x^2{}_2 + 146.30x^2{}_3 + 2.55x^2{}_4 + 30x_1x_2 + 2.25x_1x_3 + 0.49x_1x_4 + 1633.40x_2x_3 + 116.63x_2x_4 + 63.52x_3x_4 \quad (16)$$

The third-order model obtained to investigate the 3-way interaction between the variables. The third-order model as shown below:

$$y''' = c + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x^2{}_1 \beta 6 x^2{}_2 + \beta_7 x^2{}_3 + \beta_8 x^2{}_4 + \beta_9 x^3{}_1 + \beta_{10} x^3{}_2 + \beta_{11} x^3{}_3$$
$$\beta_{12} x^3{}_4 + \beta_{13} x_1 x_2 + \beta_{14} x_1 x_3 + \beta_{15} x_1 x_4 + \beta_{16} x_2 x_3 + \beta_{17} x_2 x_4 + \beta_{18} x_3 x_4 + \beta_{19} x_1 x_2 x_3 + \beta_{20} x_1 x_2 x_4 +$$
$$\beta_{21} x_1 x_3 x_4 + \beta_{22} x_2 x_3 x_4 \quad (17)$$

From this model, the most important points are the main effect, 2-way interaction and 3-way interaction. So the third order model can be reduced as below:

$$y''' = c + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_{13} x_1 x_2 + \beta_{14} x_1 x_3 + \beta_{15} x_1 x_4 + \beta_{16} x_2 x_3 + \beta_{17} x_2 x_4 + \beta_{18} x_3 x_4$$
$$+ \beta_{19} x_1 x_2 x_3 + \beta_{20} x_1 x_2 x_4 + \beta_{21} x_1 x_3 x_4 + \beta_{22} x_2 x_3 x_4 \quad (18)$$

These model parameters can be solved using least squares method. β are the model parameters, x_1= cutting speed, x_2=feedrate, x_3=axial depth and x_4=radial depth. The third order model for torque and power are:

$$T''' = -176.95 + 1.3922 x_1 + 1103.97 x_2 - 7.6632 x_3 + 56.7540 x_4 - 7.8022 x_1 x_2 - 0.05 x_1 x_3$$
$$- 0.4237 x_1 x_4 + 50 x_2 x_3 - 353.753 x_2 x_4 + 3.4863 x_3 x_4 + 2.4792 x_1 x_2 x_3 \quad (19)$$

$$P''' = -9728 + 70 x_1 + 1024 x_2 - 2048 x_3 + 4096 x_4 - 19 x_1 x_3 - 31 x_1 x_4 + 23.552 x_2 x_3 - 10496 x_2 x_4$$
$$-1408 x_3 x_4 - 168 x_1 x_2 x_3 + 82 x_1 x_2 x_4 + 14 x_1 x_3 x_4 + 512 x_2 x_3 x_4 \quad (20)$$

The variance analysis for the torque and power carried out to determine the model is adequate and significant of 3-way interaction for both models are shown in tables 4 and 5. From the variance analysis both models are not significant to the 3-way interaction since the p value>0.05. The third-order model is adequate for torque and power since the p-value for lack of fit for torque is 0.818 and for power is 0.135. F-static for torque and power are 0.52 and 6.77.

Table 4: Variance analysis for third-order torque model.

Source	Degree of freedom	F ratio	P-value
Main effect	4	11.80	0
2-Way Interactions	6	1.85	0.156
3-Way Interactions	1	3.96	0.065
Residual error	15		
Lack of Fit	12	0.52	0.818
Total	26		

Table 5: Variance analysis for third-order power model.

Source	Degree of freedom	F ratio	P-value
Main effect	4	35.60	0
2-Way Interactions	6	1.55	0.244
3-Way Interactions	4	1.10	0.402
Residual error	12		
Lack of Fit	10	6.77	0.135
Total	26		

3.3 Fourth-order model for torque and power

The fourth-order model obtained to investigate the 4-way interaction between the variables. The fourth-order model as shown below:

$$y'''' = c + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x^2_1 \beta_6 x^2_2 + \beta_7 x^2_3 + \beta_8 x^2_4 + \beta_9 x^3_1 + \beta_{10} x^3_2 + \beta_{11} x^3_3 + \beta_{12} x^3_4 + \beta_{13} x^4_1 + \beta_{14} x^4_2 + \beta_{15} x^4_3 + \beta_{16} x^4_4 + \beta_{17} x_1 x_2 + \beta_{18} x_1 x_3 + \beta_{19} x_1 x_4 + \beta_{20} x_2 x_3 + \beta_{21} x_2 x_4 + \beta_{22} x_3 x_4 + \beta_{23} x_1 x_2 x_3 + \beta_{24} x_1 x_2 x_4 + \beta_{25} x_1 x_3 x_4 + \beta_{26} x_2 x_3 x_4 + \beta_{27} x_1 x_2 x_3 x_4 \quad (21)$$

From this model the most important points are the main effect, 2-way interaction, 3-way interaction and 4-way interaction. So the fourth order model can be reduced as below:

$$y'''' = c + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_6 x_1 x_2 + \beta_7 x_1 x_3 + \beta_8 x_1 x_4 + \beta_9 x_2 x_3 + \beta_{10} x_2 x_4 + \beta_{11} x_3 x_4 + \beta_{12} x_1 x_2 x_3 + \beta_{13} x_1 x_2 x_4 + \beta_{14} x_1 x_3 x_4 + \beta_{15} x_2 x_3 x_4 + \beta_{16} x_1 x_2 x_3 x_4 \quad (22)$$

This model parameters can be solved using least squares method. β are the model parameters, x_1= cutting speed, x_2=feedrate, x_3=axial depth and x_4=radial depth. The fourth order model for torque and power are:

$$T'''' = -216 - 0.0625\, x_1 + 512\, x_2 + 30 x_3 + 63.5 x_4 + 5.5 x_1 x_2 + 0.7031 x_1 x_3 - 0.0625 x_1 x_4 + 444 x_2 x_3 - 155 x_2 x_4 - 3.75\, x_3 x_4 - 10 x_1 x_2 x_3 - 1 x_1 x_2 x_4 - 0.2266 x_1 x_3 x_4 - 139 x_2 x_3 x_4 + 2.75\, x_1 x_2 x_3 x_4 \quad (23)$$

$$P'''' = -6272 + 18176 x_2 + 768 x_3 + 1856 x_4 + 144 x_1 x_2 + 18 x_1 x_3 - 4 x_1 x_4 + 12672 x_2 x_3 - 4608 x_2 x_4 - 72\, x_3 x_4 - 272 x_1 x_2 x_3 - 20 x_1 x_2 x_4 - 5 x_1 x_3 x_4 - 4736 x_2 x_3 x_4 + 90 x_1 x_2 x_3 x_4 \quad (24)$$

The variance analysis for the torque and power carried out to determine the model adequate and significant of 4-way interaction for both model are shown in table 6 and 7. From the variance analysis both model not significant to the 4-way interaction since the p value>0.05. The fourth-order model is adequate for torque and power since the p-value for lack of fit for torque is 0.599 and for power is 0.123. F-static for torque and power are 0.99 and 7.53.

Table 6: Variance analysis for fourth-order torque model.

Source	Degree of freedom	F ratio	P-value
Main effect	4	6.38	0.
2-Way Interactions	6	0.55	0.761
3-Way Interactions	4	0.16	0.956
4-Way Interactions	1	0	1
Residual error	11		
Lack of Fit	9	0.99	0.599
Total	26		

Table 7: Variance analysis for fourth-order power model.

Source	Degree of freedom	F ratio	P-value
Main effect	4	20.22	0.
2-Way Interactions	6	1.41	0.294
3-Way Interactions	4	0.97	0.463
4-Way Interactions	1	0	1
Residual error	11		
Lack of Fit	9	7.53	0.123
Total	26		

4 Conclusion

Reliable torque models have been developed and utilized to enhance the efficiency of the milling 618 stainless steel. The torque equation shows that feed rate, cutting speed, axial depth and radial depth play the major role in producing the torque. The higher the feed rate, axial depth and radial depth, the torque generated is very high compared with low value of feed rate, axial depth and radial depth. Contours of the torque outputs were constructed in planes containing two of the independent variables. These contours were further developed to select the proper combination of cutting speed, feed, axial depth and radial depth to produce the optimum torque. The higher the feed rate, cutting speed, axial depth and radial depth, the power generated is very high compared with low value of feed rate, cutting speed, axial depth and radial depth. Dual response contours of torque and power are very useful in assessing the maximum attainable torque. The third order model and fourth order model very important to investigate the 3-way interaction and 2-way interaction. The third order model and fourth order model shows that the 3-way interaction and 4-way interaction are not significant.

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