Homogenization of regular cross-ply polymer-matrix laminates

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Abstract

In the classic laminate theory, a polymer-matrix laminate is modelled as a thin anisotropic plate of a constant thickness, subject to the Kirchhoff–Love kinematical hypothesis. However, this hypothesis is mostly unsatisfied because of relatively low values of the shear moduli. The laminate elastic response is better simulated under the Reissner–Mindlin kinematical hypothesis. Reformulation of the classic laminate theory to the first – order shear – deformation theory is very sophisticated but useless from the point of view of the Finite Element Method. This paper formulates the exact stiffness theory of a regular cross-ply polymer-matrix laminate, denoted with the symbol CP xFRP, of the plies' configuration $[0/90]_{n\mathrm{S}}$, $n \geq 4$. The plies, made of a specified woven fabric, are repeatable with respect to their thickness and microstructure. This type of laminates is widely used in engineering practice. On the mesomechanics level, a regular CP xFRP laminate is modelled as a homogeneous orthotropic plate. The theory employs respective boundary-value problems related to a representative volume element and the most advanced variant of the exact stiffness theory of a unidirectional xFRP composite. The final results are presented in the form of analytic formulae for nine effective elastic constants of the laminate, describing an orthotropic model of the homogenized CP xFRP laminate. A numerical example of homogenisation is also presented.

Keywords: cross-ply polymer-matrix laminates, homogenisation, exact stiffness theory, linear elasticity, orthotropy.

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1 Introduction

In the classic laminate theory, a laminate is modelled as a thin anisotropic plate of a constant thickness, subject to the Kirchhoff–Love kinematical hypothesis [1–3]. The classic laminate theory assumes an arbitrary plies' configuration in reference to thickness, lamination angle and microstructure. Moreover, there are assumed planar stress states in the plies and external load applied to the midplane. The laminate remains in the normal isothermal conditions. Each ply is homogenized and modelled as a monotropic solid body. Arbitrary configuration of the plies induces coupling between the disk state and the plate state. The classic laminate theory results in a system of differential equations of static equilibrium of an anisotropic plate.

In practice, a polymer-matrix laminate is a thin plate. However, the Kirchhoff – Love kinematical hypothesis is mostly unsatisfied because of relatively low values of the shear moduli. The laminate elastic response is better simulated under the Reissner–Mindlin kinematical hypothesis also termed as Hencky–Mindlin hypothesis [4]. According to this hypothesis a straight line perpendicular to the midplane is inextensible, remains straight, and rotates but does not remain perpendicular to the tangent of the deformed midplane. Reformulation of the classic laminate theory to the first – order shear – deformation theory is very sophisticated [5] but useless from the finite element method's (FEM) point of view.

Nowadays, the FEM is the only effective tool for simulation of the elastic behaviour of laminate structures with shear deformation taken into consideration. Preprocessors of CAE systems, such as MSC, ADINA, ANSYS, COSMOS, ABAQUS, require values of all effective elasticity constants (EECs) of an anisotropic material modelling a laminate. These values can be identified experimentally, but theoretical analytical prediction of the EECs by respective stiffness theory is much more cheaper, faster and more important in mechanics of laminates.

The paper formulates a new exact stiffness theory of a regular cross-ply laminate, denoted with the symbol CP xFRP, of the plies' configuration $[0/90]_{n\mathrm{S}}$, $n \geq 4$. The plies, made of a specified woven fabric, are repeatable with respect to their thickness and microstructure. This type of laminates is widely used in engineering practice. On the mesomechanics level, a regular CP xFRP laminate is modelled as a homogeneous orthotropic plate. All EECs of the laminate have been determined analytically.

2 Assumptions and standard constitutive equations of linear elasticity of the homogenized laminate

A regular CP xFRP laminate is created by a stack of plies of the $[0/90]_{n\mathrm{S}}$, $n \geq 4$ configuration. Each ply is a UD xFRP composite, i.e. a thermoset reinforced with long fibres aligned unidirectionally. The following assumptions are valid to each ply [6]:

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- a UD xFRP composite is a two – phase material,
- there are considered quasi-static isothermal processes, at room temperature,
- both constituents, i.e. a matrix and a fibre, are homogeneous,
- stresses are restricted to the levels protecting linear behaviour of the constituents,
- a matrix is a thermoset (chemically hardening plastic) made of a crosslinked polymer, modelled as an isotropic material,
- a fibre is modelled as a monotropic material (isotropic, in particular),
- fibres have identical solid circular cross-section; they are rectilinear and embedded uniformly in the matrix, in a hexagonal scheme,
- the matrix – fibre interface is a cylindrical surface,
- preparation of the fibres protects perfect bonding of the fibres to the matrix,
- residual stresses resulting from the manufacturing process are neglected.

Under these assumptions, a UD xFRP composite is modelled macroscopically as a homogeneous monotropic (transversely isotropic) material, with the monotropy direction coinciding the fibres' alignment direction (the laminate moulding direction). Each ply is described in the $x_1x_2x_3$ - Cartesian coordinate system, where x_1 - axis coincides the laminating direction, and x_2x_3 is termed as a transverse isotropy plane. The constituents of each ply are fully characterized by the following elasticity constants: E, ν (a Young's modulus, a Poisson's ratio of an isotropic matrix), $\overline{E}_1, \overline{E}_2, \overline{\nu}_{32}, \overline{\nu}_{21}, \overline{G}_{12}$ (longitudinal and transverse Young's moduli, Poisson's ratios in respective planes, a shear modulus in the monotropy plane of a monotropic fibre). A ply is also described by a real fibre volume fraction f.

A monotropic material modelling the homogenized ply is described by five independent EECs, i.e. $E_1, E_2, \nu_{32}, \nu_{21}, G_{12}$ (effective longitudinal and transverse Young's moduli, effective Poisson's ratios in respective planes, an effective shear modulus in the monotropy plane). The EECs are determined analytically by the exact stiffness theory of a UD xFRP composite. The most advanced variant of this theory is presented in [6].

A regular CP xFRP laminate is modelled by a homogeneous orthotropic material described in the xyz - Cartesian coordinate system. Axes x, y coincide the lamination directions of respective plies, whereas axis z is perpendicular to the xy midplane. Standard constitutive equations of linear elasticity of the homogenized regular CP xFRP laminate have the following form [1–4]

$$\boldsymbol{\varepsilon} = \mathbf{S}\boldsymbol{\sigma}, \tag{1}$$

where $\boldsymbol{\sigma} = \mathrm{col}(\sigma_x, \sigma_y, \sigma_z, \sigma_{yz}, \sigma_{xz}, \sigma_{xy})$, $\boldsymbol{\varepsilon} = \mathrm{col}(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{yz}, \gamma_{xz}, \gamma_{xy})$ are stress and strain vectors in the xyz - system, with the following components: $\sigma_x, \sigma_y, \sigma_z$ - normal stresses, $\sigma_{yz}, \sigma_{xz}, \sigma_{xy}$ - shear stresses, $\varepsilon_x, \varepsilon_y, \varepsilon_z$ - directional strains, $\gamma_{yz}, \gamma_{xz}, \gamma_{xy}$ - shear strains. The elasticity compliance matrix

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \tag{2}$$

contains nine elasticity compliances

$$S_{11} = \frac{1}{E_x},\ S_{22} = \frac{1}{E_y},\ S_{33} = \frac{1}{E_z},\ S_{12} = -\frac{\nu_{yx}}{E_x},\ S_{13} = -\frac{\nu_{zx}}{E_x},\ S_{23} = -\frac{\nu_{zy}}{E_y}, \\ S_{44} = \frac{1}{G_{yz}},\ S_{55} = \frac{1}{G_{xz}},\ S_{66} = \frac{1}{G_{xy}}, \tag{3}$$

expressed in terms of the EECs of the homogenized laminate, i.e. E_x, E_y, E_z – Young's moduli in the x, y, z directions, $\nu_{zy}, \nu_{zx}, \nu_{yx}$ – Poisson's ratios in respective planes, G_{yz}, G_{xz}, G_{xy} – shear moduli in respective planes.

For a regular CP xFRP laminate only six EECs take different values, i.e. $E_x, E_z, \nu_{zx}, \nu_{yx}, G_{xz}, G_{xy}$. The remaining constants equal $E_y = E_x$, $\nu_{zy} = \nu_{zx}$, $G_{yz} = G_{xz}$ and result in $S_{11} = S_{22}$, $S_{13} = S_{23}$, $S_{44} = S_{55}$.

For the plies with 0° laminating direction one obtains $x_1 = x,\ x_2 = y,\ x_3 = z$, while the plies with 90° laminating direction are defined by the relations $x_1 = y,\ x_2 = x,\ x_3 = z$.

3 The exact stiffness theory of a regular CP xFRP laminate

3.1 A representative volume element

There is considered a representative volume element (RVE) cut from the whole laminate at point $A(x, y, 0)$, as shown in Fig. 1 for exemplary value $n = 4$. Before homogenization the RVE is a stack of plies of the $[0/90]_{n\mathrm{S}}$, $n \geq 4$ configuration of cubicoidal global geometry. After homogenization, the RVE is a homogeneous orthotropic cubicoid of dimensions $1 \times 1 \times h$ where h is a thickness of the laminate. The orthotropy directions coincide the x, y, z directions, respectively.

The RVE is considered in selected boundary-value problems (BVPs) under the following requirements:

- elastic behaviour of the unhomogenized and homogenized RVE must be compatible with the elastic behaviour of the whole laminate,
- the unhomogenized and homogenized RVE satisfy the compatibility conditions put on the stress and displacement states.

In further considerations, a' denotes a quantity related to the plies of 0° arrangement, a'' - a quantity related to the plies of 90° arrangement, and a – a quantity related to the homogenized laminate.

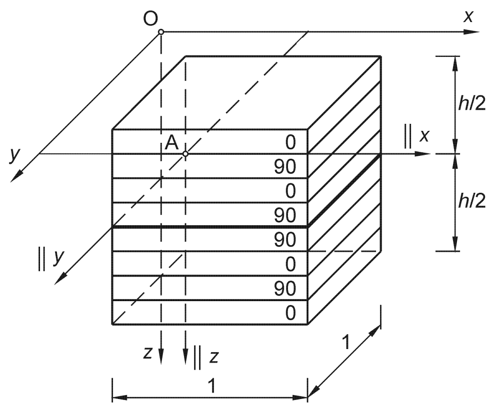

Figure 1: A representative volume element for $n = 4$.

3.2 BVP 1: Uniform tension in the *x* direction

Following the behaviour of the whole laminate, the RVE walls remain planar. BVP 1 is related to the constrained RVE, namely the walls parallel to the xz plane are unmovable in the y direction. The shear deformations vanish. The stress and strain components of the homogenized RVE equal

$$\sigma_x = \sigma\ , \quad \sigma_y \neq 0\ , \quad \sigma_z = 0\ , \quad \varepsilon_x = \varepsilon\ , \quad \varepsilon_y = 0\ , \quad \varepsilon_z \neq 0\ . \tag{4}$$

After inserting eqn (4) into eqn $(1)_{1,2,3}$, one obtains constitutive equations

$$\varepsilon = \frac{1}{E_x}\sigma - \frac{\nu_{yx}}{E_x}\sigma_y\ , \quad 0 = -\frac{\nu_{yx}}{E_x}\sigma + \frac{1}{E_x}\sigma_y\ , \quad \varepsilon_z = -\frac{\nu_{zx}}{E_x}\sigma - \frac{\nu_{zx}}{E_x}\sigma_y\ , \tag{5}$$

which can be transformed to the form

$$E_x\varepsilon = (1-\nu_{yx}^2)\sigma\ , \quad \sigma_y = \nu_{yx}\sigma\ , \quad \varepsilon_z = -\frac{\nu_{zx}}{E_x}(1+\nu_{yx})\sigma\ . \tag{6}$$

Before homogenization, the stress and strain components in each ply of 0° orientation equal

$$\begin{aligned} &\sigma_1 = \sigma_x' \neq 0\ , \quad \sigma_2 = \sigma_y' \neq 0\ , \quad \sigma_3 = \sigma_z' = 0\ , \\ &\varepsilon_1 = \varepsilon_x' = \varepsilon\ , \quad \varepsilon_2 = \varepsilon_y' = 0\ , \quad \varepsilon_3 = \varepsilon_z' \neq 0\ . \end{aligned} \tag{7}$$

Inserting eqn (7) into eqn $(1)_{1,2,3}$ results in constitutive equations

$$\varepsilon = \frac{1}{E_1}\sigma_x' - \frac{\nu_{21}}{E_1}\sigma_y'\ , \quad 0 = -\frac{\nu_{21}}{E_1}\sigma_x' + \frac{1}{E_2}\sigma_y'\ , \quad \varepsilon_z' = -\frac{\nu_{21}}{E_1}\sigma_x' - \frac{\nu_{32}}{E_2}\sigma_y'\ , \tag{8}$$

which can be transformed to the form

$$E_1\varepsilon=\left(1-\nu_{21}^2\frac{E_2}{E_1}\right)\sigma_x', \quad \sigma_y'=\nu_{21}\frac{E_2}{E_1}\sigma_x', \quad \varepsilon_z'=-\frac{\nu_{21}}{E_1}(1+\nu_{32})\sigma_x'. \tag{9}$$

Before homogenization, the stress and strain components in each ply of 90° orientation equal

$$\begin{aligned}&\sigma_1=\sigma_y''\neq 0, \quad \sigma_2=\sigma_x''\neq 0, \quad \sigma_3=\sigma_z''=0, \\ &\varepsilon_1=\varepsilon_y''=0, \quad \varepsilon_2=\varepsilon_x''=\varepsilon, \quad \varepsilon_3=\varepsilon_z''\neq 0.\end{aligned} \tag{10}$$

Inserting eqn (10) into eqn $(1)_{1,2,3}$ results in constitutive equations

$$0=\frac{1}{E_1}\sigma_y''-\frac{\nu_{21}}{E_1}\sigma_x'', \quad \varepsilon=-\frac{\nu_{21}}{E_1}\sigma_y''+\frac{1}{E_2}\sigma_x'', \quad \varepsilon_z''=-\frac{\nu_{21}}{E_1}\sigma_y''-\frac{\nu_{32}}{E_2}\sigma_x'', \tag{11}$$

which can be transformed to the form

$$E_2\varepsilon=\left(1-\nu_{21}^2\frac{E_2}{E_1}\right)\sigma_x'', \quad \sigma_y''=\nu_{21}\sigma_x'', \quad \varepsilon_z'=-\left(\frac{\nu_{21}^2}{E_1}+\frac{\nu_{32}}{E_2}\right)\sigma_x''. \tag{12}$$

In BVP 1, the RVE satisfies the following compatibility conditions:
a) compatibility of the stress resultant in the x direction:

$$\sigma_x'\frac{h}{2}+\sigma_x''\frac{h}{2}=\sigma_x h, \tag{13}$$

b) compatibility of the stress resultant in the y direction:

$$\sigma_y'\frac{h}{2}+\sigma_y''\frac{h}{2}=\sigma_y h, \tag{14}$$

c) compatibility of the elongation of the RVE in the z direction:

$$\varepsilon_z'\frac{h}{2}+\varepsilon_z''\frac{h}{2}=\varepsilon_z h, \tag{15}$$

Equations (13–15) can be rewritten in the form

$$\sigma_x'+\sigma_x''=2\sigma, \quad \sigma_y'+\sigma_y''=2\sigma_y, \quad \varepsilon_z'+\varepsilon_z''=2\varepsilon_z. \tag{16}$$

Taking into account eqns (6), (9), (12), one transforms eqn (16) to the final form

$$\frac{E_1+E_2}{1-\nu_{21}\nu_{12}}=\frac{2E_x}{1-\nu_{yx}^2}, \quad \frac{E_1\nu_{12}+E_2\nu_{21}}{1-\nu_{21}\nu_{12}}=\frac{2E_x\nu_{yx}}{1-\nu_{yx}^2}, \quad \frac{\nu_{21}(1+\nu_{12}+\nu_{32})+\nu_{32}}{1-\nu_{21}\nu_{12}}=\frac{2\nu_{zx}}{1-\nu_{yx}} \tag{17}$$

3.3 BVP 2: Uniform tension in the *z* direction

Following the behaviour of the whole laminate, the RVE walls remain planar. BVP 2 is related to the constrained RVE, i.e. vertical walls are unmovable in the

x and y direction respectively. The shear deformations vanish. The stress and strain components of the homogenized RVE equal

$$\sigma_x \neq 0\,,\ \ \sigma_y = \sigma_x\,,\ \ \sigma_z = \sigma\,,\ \ \varepsilon_x = 0\,,\ \ \varepsilon_y = 0\,,\ \ \varepsilon_z = \varepsilon\,. \qquad (18)$$

The RVE satisfies the compatibility condition put on the elongation of the RVE in the z direction, i.e.

$$\varepsilon_z'\frac{h}{2} + \varepsilon_z'\frac{h}{2} = \varepsilon_z h \qquad (19)$$

which can be reduced to the form

$$\varepsilon_z' = \varepsilon\ \ . \qquad (20)$$

Performing the homogenization procedure analogous to that for BVP 1 (see Section 3.2), one obtains the final relation

$$\frac{1 - 2\nu_{21}\nu_{12}(1+\nu_{32}) - \nu_{32}^2}{E_2(1-\nu_{21}\nu_{12})} = \frac{1}{E_z} - \frac{2\nu_{zx}^2}{E_x(1-\nu_{yx})}\ \ . \qquad (21)$$

3.4 Prediction of constants $E_x, E_z, \nu_{zx}, \nu_{yx}$

Equations (17), (21) constitute a set of four nonlinear algebraic equations with unknowns $E_x, E_z, \nu_{zx}, \nu_{yx}$. One can rewrite them in the form

$$2E_x = (1-\nu_{yx}^2)a\,,\ \ 2E_x\nu_{yx} = (1-\nu_{yx}^2)e\,,\ \ 2\nu_{zx} = (1-\nu_{yx})c\,,\ \ \frac{1}{E_z} - \frac{2\nu_{zx}^2}{E_x(1-\nu_{yx})} = d\ , \qquad (22)$$

where

$$a = \frac{E_1 + E_2}{1-\nu_{21}\nu_{12}}\,,\quad e = \frac{E_1\nu_{12} + E_2\nu_{21}}{1-\nu_{21}\nu_{12}}\,,\quad b = \frac{e}{a} = \frac{E_1\nu_{12} + E_2\nu_{21}}{E_1 + E_2}\,,$$
$$c = \frac{\nu_{21}(1+\nu_{12}+\nu_{32}) + \nu_{32}}{1-\nu_{21}\nu_{12}}\,,\quad d = \frac{1 - 2\nu_{21}\nu_{12}(1+\nu_{32}) - \nu_{32}^2}{E_2(1-\nu_{21}\nu_{12})}\,. \qquad (23)$$

The analytical solution of eqn (22) has the form

$$E_x = \tfrac{1}{2}a(1-b^2)\,,\ \ E_z = \frac{a(1+b)}{a(1+b)d + c^2}\,,\ \ \nu_{zx} = \tfrac{1}{2}c(1-b)\,,\ \ \nu_{yx} = b\,. \qquad (24)$$

3.5 BVP 3: Pure shear in the *xz* plane

Following the behaviour of the whole laminate, shear deformation of the RVE before and after homogenization takes the form shown in Fig. 2. Shear angles related to plies of 0° and 90° orientation are different and independent. The horizontal walls of the RVE remain planar. The strength task is related to the unconstrained RVE. The bulk deformations vanish.

The stress and strain components of the homogenized RVE equal

$$\sigma_{xz} = \tau \ , \quad \gamma_{xz} = \gamma \ . \tag{25}$$

Inserting eqn (25) into eqn $(1)_5$ results in

$$\gamma = \frac{1}{G_{xz}} \tau \ . \tag{26}$$

Before homogenization, the stress and strain components in each ply of 0° orientation equal

$$\sigma_{12} = \tau \ , \quad \gamma_{12} = \gamma' \ . \tag{27}$$

After inserting eqn (27) into eqn $(1)_6$, one obtains the physical relation

$$\gamma' = \frac{1}{G_{12}} \tau \ . \tag{28}$$

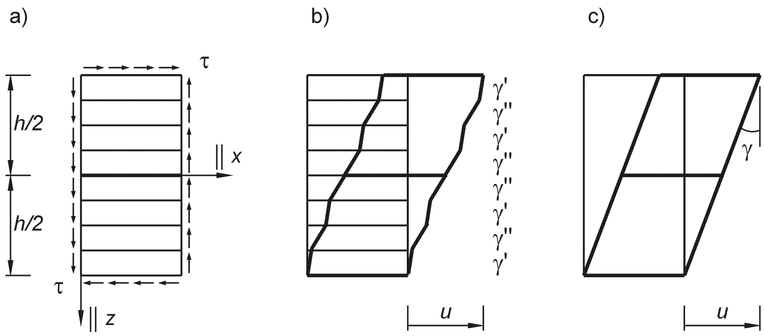

Figure 2: Pure shear of the RVE in the *xz* plane: a) distribution of shear stresses; b) deformation of the unhomogenized RVE; c) deformation of the homogenized RVE.

Similarly, the stress and strain components in each ply of 90° orientation equal

$$\sigma_{23} = \tau \ , \quad \gamma_{23} = \gamma'' \ . \tag{29}$$

Inserting eqn (29) into eqn $(1)_4$ gives the physical relation

$$\gamma'' = \frac{1}{G_{23}} \tau \ . \tag{30}$$

The compatibility condition is put on the horizontal shift of the top wall of the RVE, i.e.

$$\gamma' \frac{h}{2} + \gamma'' \frac{h}{2} = \gamma h \ . \tag{31}$$

Equation (31) can be rewritten as

$$\gamma' + \gamma'' = 2\gamma \quad . \tag{32}$$

Inserting eqns (26), (28), (30) into eqn (32) results in the equation

$$\frac{1}{G_{12}} + \frac{1}{G_{23}} = \frac{2}{G_{xz}} \tag{33}$$

giving a shear modulus in the *xz* plane, i.e.

$$G_{xz} = \frac{2G_{12}G_{23}}{G_{12} + G_{23}} \tag{34}$$

3.6 BVP 4: Pure shear in the *xy* plane

In the horizontal plane, both groups of the plies are described by the same shear modulus G_{12}. Shear deformations of the unconstrained RVE before and after homogenization are identical – the cubicoid metamorphoses into parallelepiped. It results in the simple relation

$$G_{xy} = G_{12} \quad . \tag{35}$$

4 An example of homogenization of the specified laminate

Based on the exact stiffness theory of UD xFRP composites [6] and the exact stiffness theory of regular CP xFRP laminates presented in this study, the authors have written a computer programme in PASCAL for predicting the ECCs of these materials.

As an example, there is considered a regular CP U/E53 laminate of $[0/90]_{nS}$, $n \geq 4$ configuration. The matrix (E53 thermoset) is made of Epidian 53 epoxide resin and reinforced with UTS 5631 carbon fibres produced by Tenax Fibers. The elasticity constants of the constituents equal [7]

$$E = 3.1 \text{ GPa} \ , \quad \nu = 0.42 \ ,$$

$$\overline{E}_1 = 234 \text{ GPa} \ , \ \overline{E}_2 = 6.6 \text{ GPa}, \ \ \overline{\nu}_{32} = 0.36 \ , \ \ \overline{\nu}_{21} = 0.11 \ , \ \overline{G}_{12} = 10.6 \text{ GPa}.$$

The real fibre volume fraction equals $f = 0.50$.

The EECs of a single ply, calculated according to the stiffness theory presented in [6], equal

$$E_1 = 118.6 \text{ GPa}, \ E_2 = 5.4 \text{ GPa}, \ \nu_{32} = 0.54, \ \ \nu_{21} = 0.27, \ \ G_{12} = 2.6 \text{ GPa} .$$

The EECs of the CP U/E53 laminate, calculated according to the stiffness theory presented in this study, equal

$$E_x = E_y = 62.2 \text{ GPa} \ , \ E_z = 7.3 \text{ GPa},$$

$$\nu_{zx} = \nu_{zy} = 0.47, \ \ \nu_{yx} = \nu_{xy} = 0.02,$$

$$G_{xz} = G_{yz} = 2.1 \text{ GPa}, \ \ G_{xy} = 2.6 \text{GPa}.$$

5 Final conclusions

The study concerns regular CP xFRP laminates, i.e. a stack of plies of $[0/90]_{nS}$, $n \geq 4$ configuration. Each ply is a UD xFRP composite, i.e. an isotropic thermoset reinforced with long monotropic fibres packed unidirectionally in a hexagonal scheme. The plies are identical with respect to their thickness and microstructure. The considerations are limited to stress levels protecting geometrically and physically linear elastic behaviour of the material.

A new exact stiffness theory of a regular CP xFRP laminate has been formulated. Effective elasticity constants of the homogenized orthotropic laminate have been derived analytically from respective boundary-value problems related to the representative volume element of the material. A numerical example of homogenization of the specified laminate is attached as well.

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