Wind tunnel sectional tests for the identification of flutter derivatives and vortex shedding in long span bridges

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Abstract

Aeroelastic phenomena are of major concern for design of long span bridges. Flutter is a critical condition that can be computationally evaluated with a previous identification of 18 functions which relate forces with movements. These functions are called flutter derivatives, which can be extracted by means of a free vibration sectional test of the deck considering three degree of freedom. Two different identification methods are used: Modified Ibrahim Time Domain, and Iterative Least Squares. On the other hand, vortex shedding excitation is an aerodynamic effect which frequently occurs to cable supported bridges. It is due to synchronization of the frequency of the alternating vortices shed from the deck and one of the natural frequencies of the structure. Both phenomena have been studied using a sectional model with a shape similar to the Great Belt Bridge deck.
Keywords: flutter derivatives, vortex shedding, bridges, sectional models, wind tunnel.

1 Flutter derivatives identification

Flutter is a critical condition during the design of long-span bridges. Sectional models of the deck are initially tested in an aerodynamic wind tunnel to obtain the flutter derivatives. These coefficients are then used in the computational analysis of the aeroelastic behaviour of the completed bridge (Jurado *et al*. [1]). Figure 1 shows the three forces acting on a deck.

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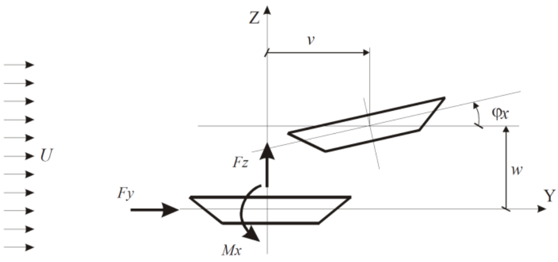

Figure 1: Forces and displacements of a sectional model.

According to Scanlan formulation, these actions are linealized as functions of the displacements and velocities of the system for vertical *w*, lateral *v* and torsional rotation φ_x degrees of freedom (DOF). The expressions can be written as

$$\mathbf{f}_a = \mathbf{C}_a\dot{\mathbf{u}} + \mathbf{K}_a\mathbf{u} = \tag{1}$$

$$\begin{Bmatrix} F_y \\ F_z \\ M_x \end{Bmatrix} = \frac{1}{2}\rho U^2 KB \begin{pmatrix} P_1^* & -P_5^* & -BP_2^* \\ -H_5^* & H_1^* & BH_2^* \\ -BA_5^* & BA_1^* & B^2A_2^* \end{pmatrix} \begin{Bmatrix} \dot{v} \\ \dot{w} \\ \dot{\varphi}_x \end{Bmatrix} + \frac{1}{2}\rho U^2 K^2 \begin{pmatrix} P_4^* & -P_6^* & -BP_3^* \\ -H_6^* & H_4^* & BH_3^* \\ -BA_6^* & BA_4^* & B^2A_3^* \end{pmatrix} \begin{Bmatrix} v \\ w \\ \varphi_x \end{Bmatrix}$$

where B is the deck width, ρ is the air density, U is the mean wind speed, $K=B\omega/U$ is the reduced frequency with ω the frequency of the response, A_i^*, H_i^*, P_i^*, i=1...6 are the flutter derivatives which are functions of *K*. As Figure 2 shows, the support system is set of vertical and horizontal springs which permit the three movements *v*, *w*, φ_x. The frequency similarity is not necessary to evaluate flutter derivatives because they are functions of the reduced frequency. By changing the wind speed in the tunnel and the stiffness constants of the springs, a wide range of reduced velocities can be simulated. The dynamic balance equation for the sectional deck model is

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}_a = \mathbf{C}_a\dot{\mathbf{u}} + \mathbf{K}_a\mathbf{u} \tag{2}$$

M, **C** and **K** are respectively the mass, damping and stiffness structural matrices, which depend on mass *m*, polar inertia *I* of the model and the springs constants. $\mathbf{f}_a$ is the aeroelastic forces vector, which can be written as functions of the aeroelastic matrices according with expression (1). The geometric similarity from prototype considering the deck shape, barriers and aerodynamic appendages is essential.

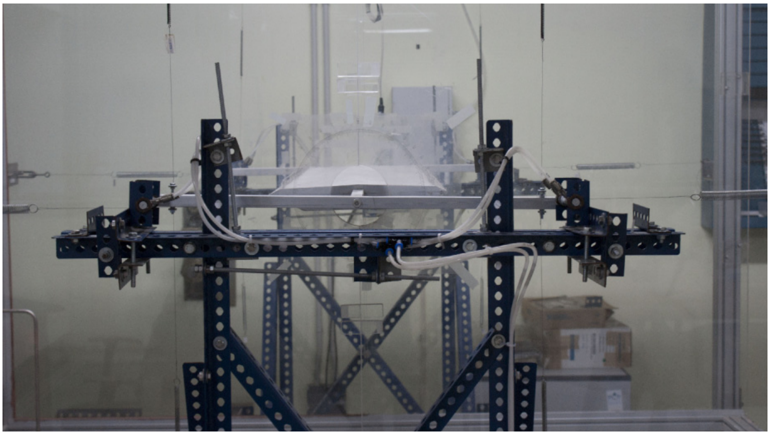

Figure 2: Sectional test in the University of Coruña wind tunnel.

Equation (2) can be written as

$$\ddot{\mathbf{u}} + \mathbf{C}_m\dot{\mathbf{u}} + \mathbf{K}_m\mathbf{u} = \mathbf{0} \tag{3}$$

where $\mathbf{C}_m = \mathbf{M}^{-1}(\mathbf{C}-\mathbf{C}_a)$ and $\mathbf{K}_m = \mathbf{M}^{-1}(\mathbf{K}-\mathbf{K}_a)$. To obtain the flutter derivatives, all terms of $\mathbf{C}_m$ and $\mathbf{K}_m$ must be calculated. Denoting K_{ij}^U and C_{ij}^U the terms for stiffness and damping with wind in the tunnel and K_{ij}^0 and C_{ij}^0 the terms for mechanical stiffness and damping without wind in the tunnel, any flutter derivative can be evaluated by subtraction. For example,

$$A_2^* = -\frac{2I}{\rho B^4\omega}\left(C_{22}{}^U - C_{22}{}^0\right);\ H_4^* = -\frac{2m}{\rho B^2\omega^2}\left(K_{22}{}^U - K_{22}{}^0\right) \tag{4}$$

There are several methods for system identification of flutter derivatives. In the following, Modified Ibrahim Time Domain (MITD) and Iterative Least Squares Method (ILS) will be used.

1.1 Modified Ibrahim Time Domain (MITD) method

This method is based on the Ibrahim Time Domain method (ITD) proposed by Ibrahim and Mikulcik [4]. ITD is suitable for time history responses of free vibrations which decay in an exponential way, assuming as a solution of (2) $\mathbf{u} = \mathbf{p}e^{\lambda t}$. Transforming the problem into an eigenvalue problem results

$$\left[\begin{pmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{K}_m & -\mathbf{C}_m \end{pmatrix} - \lambda\begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}\right]\begin{Bmatrix} \mathbf{p} \\ \lambda\mathbf{p} \end{Bmatrix} = \mathbf{0} \ \text{ or } \ (\mathbf{A} - \lambda\mathbf{I})\mathbf{p}_\lambda = \mathbf{0} \tag{5}$$

It can be proved that there is another eigenproblem which has eigenvalues directly related with those of (5), which is

$$[\mathbf{A}^{*} - \exp(\lambda n_1 \Delta t)\,\mathbf{I}]\mathbf{q} = \mathbf{0} \tag{6}$$

where the matrix $\mathbf{A}^*$ is built with the time history of the model displacements. The eigen-vectors of both problems are therefore related. The frequency ω and the damping ς for each DOF are calculated from the eigenvalues λ. The ITD method does not run well for high noise level signals such as the measured displacements of the sectional model in the wind tunnel. Sarkar *et al.* [2] proposed a modification, known as the Modified Ibrahim Time Domain (MITD). First of all, the signals are fitted using ITD for each degree of freedom. The matrix $\mathbf{A}^*$ is built mixing the original signal and the fitted functions and new λ are calculated. This process is iteratively repeated until convergence. Finally, the matrices $\mathbf{C}_m$ and $\mathbf{K}_m$ are built with the frequencies ω and the dampings ς calculated from converged λ.

1.2 Iterative Least Squares (ILS) method

In 2003, Chowdhury and Sarkar [3] proposed a method based on the representation of equation (3) as the state-space model

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} \quad \text{where} \quad \mathbf{X} = \begin{Bmatrix} \mathbf{u} \\ \dot{\mathbf{u}} \end{Bmatrix} \tag{7}$$

The $\mathbf{A}$ matrix can be identified if accelerations, velocities and displacements data are recorded for all DOF n for at least $N = 2n$ different instants of time. The method starts eliminating high frequency noise components of the signals measured in the wind tunnel with a numerical filter. After that, the velocity and acceleration time-histories are obtained by finite differences. The next step applies a process of windowing which consists of discarding both the first and the last quarters of the time histories, in which numerical errors during filtering are more important. The matrices $\mathbf{X}$ and $\dot{\mathbf{X}}$ are assembled with the displacements velocities and accelerations for the considered interval times. The matrix $\mathbf{A}$ for the first iteration is generated by least squares according to

$$\mathbf{A}^0 = \left(\dot{\mathbf{X}}\mathbf{X}^{\mathrm{T}}\right)\left(\dot{\mathbf{X}}\mathbf{X}^{\mathrm{T}}\right)^{-1} \tag{8}$$

Using initial conditions $\mathbf{X}_0$, it is possible to simulate $\mathbf{X}^1 = \mathrm{e}^{\mathbf{A}^0 t}\mathbf{X}_0$, updating afterwards the matrix $\mathbf{A}$ using least squares

$$\mathbf{A}^1 = \left(\dot{\mathbf{X}}\mathbf{X}^{1\,\mathrm{T}}\right)\left(\dot{\mathbf{X}}\mathbf{X}^{1\,\mathrm{T}}\right)^{-1} \tag{9}$$

The process continues until all the terms of matrices $\mathbf{C}_m$ and $\mathbf{K}_m$ contained in $\mathbf{A}$ converge. The great advantage of the ILS method is that does not need to calculate complex eigenvalues as the MITD method, because it directly estimates the terms of the matrices.

1.3 Example of the Great Belt sectional model

A sectional model with a shape similar to the Great Belt Bridge deck (Figure 2) has been tested in the wind tunnel of the School of Civil Engineering at the University of La Coruña. The mass of the model is 2.58 kg, the polar inertia is 0.048 kgm^2 and the experiments were carried out using two sets of springs (Table 1) in three DOF, so the whole set of 18 flutter derivatives is evaluated. Set 1 has been designed to simulate the vortex shedding effect so it has high vertical stiffness. Therefore the flutter derivatives values corresponding to low reduce velocities. Set 2 has a relatively low vertical stiffness to get the flutter derivatives values for high reduced velocities. Also a two degree of freedom tests were carried out using Set 1. For this last case only A^*_1, A^*_2, A^*_3, A^*_4 and H^*_1, H^*_2, H^*_3, H^*_4 can be obtained. An example of time histories obtained for Set 1 are shown in Figure 3.

Table 1: Properties of the springs sets.

	Set 1	Set 2
horizontal stiffness	902.73 N/m	463.47 N/m
vertical stiffness	3572.22 N/m	179.4 N/m
rotational stiffness	364.75 Nm/m	12.76 Nm/m
horizontal frequency	9.58 Hz	1.53 Hz
vertical frequency	5.55 Hz	0.95 Hz
rotational frequency	14.1 Hz	2.89 Hz

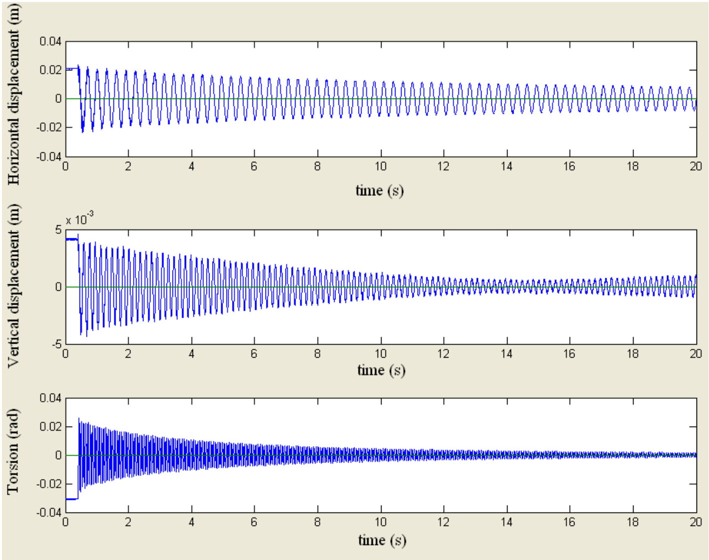

Figure 3: Time histories for 2m/s of speed in the wind tunnel.

Figure 4 shows the relation between the original signal and the obtained displacements after convergence of the methods. ILS method produces lower percentages than MITD except at high reduce velocities, where they have similar values. The results for the flutter derivatives are shown in Figures 5 to 7. There is a good agreement for the points calculated with both methods. Also results considering only vertical and rotation have been calculated to check the three degree of freedom results.

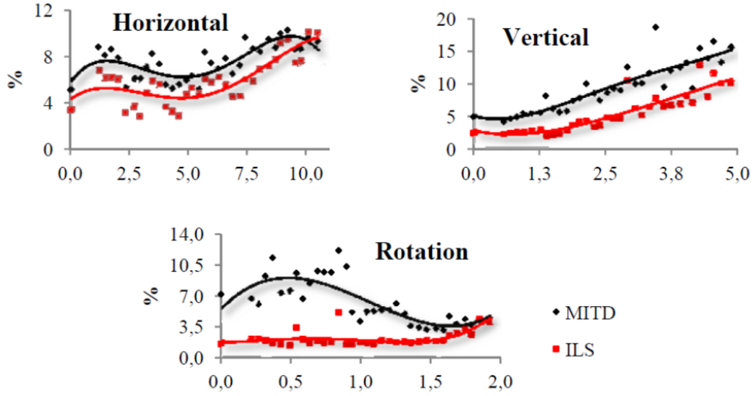

Figure 4: Rate between original data and fitted signals by MITD and ILS.

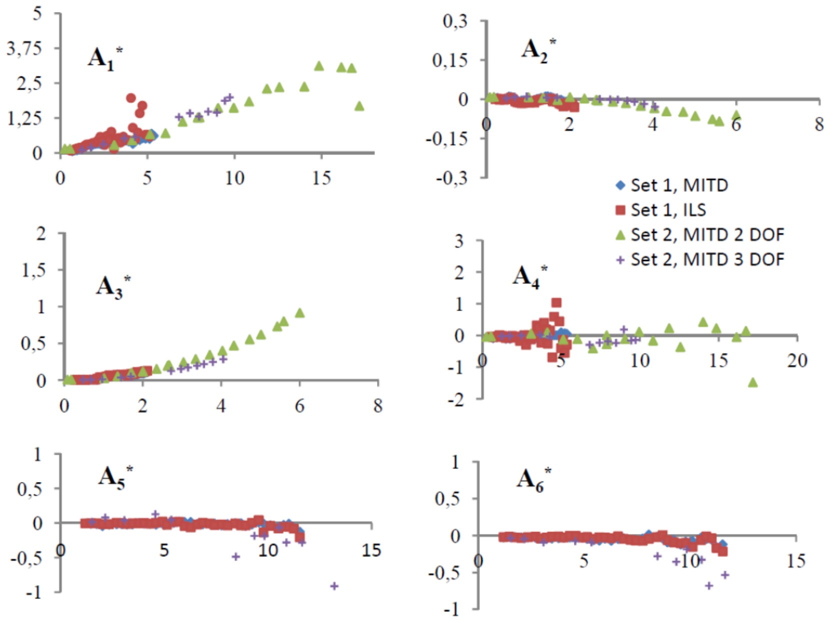

Figure 5: Flutter derivatives relates with the moment.

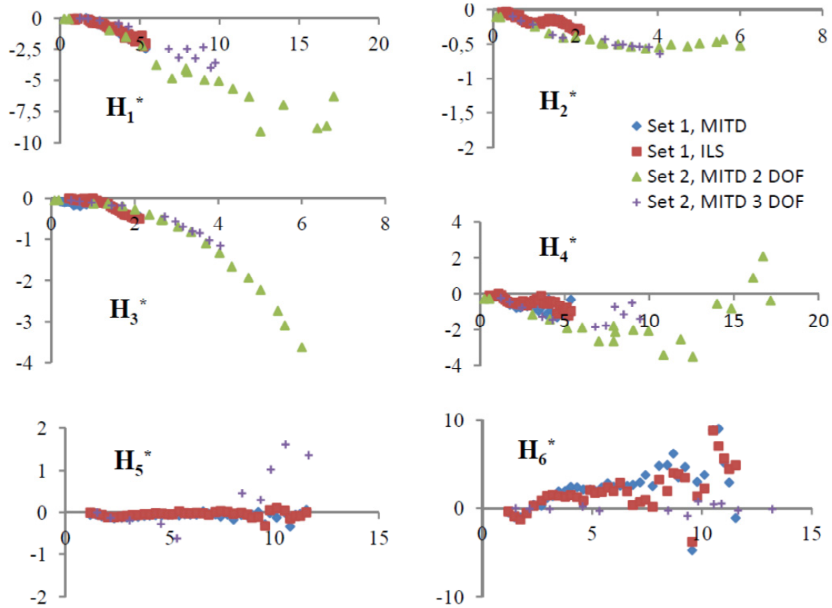

Figure 6: Flutter derivatives relates with the lift.

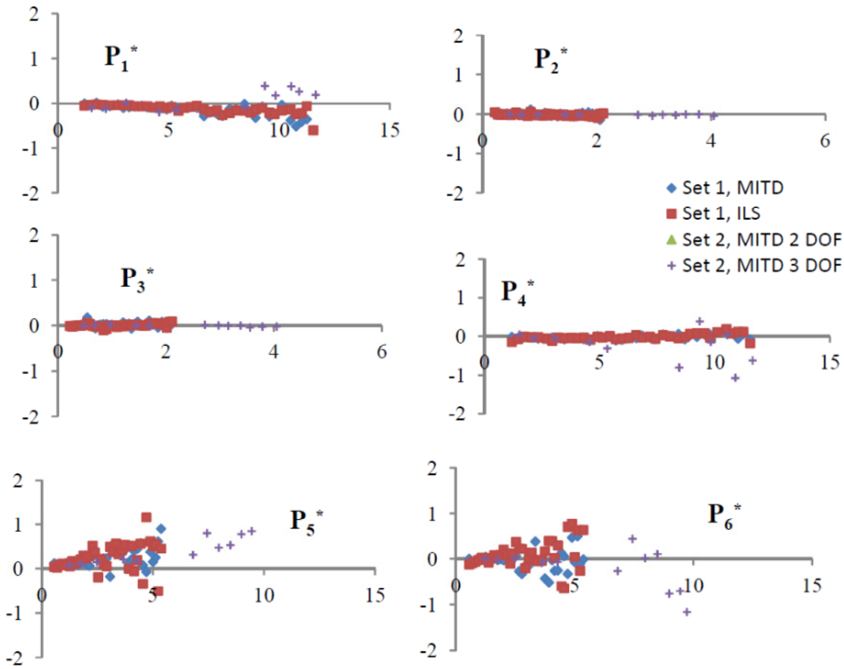

Figure 7: Flutter derivatives relates with the drag.

2 Vortex shedding analysis

Vortex shedding is perhaps one of the most studied phenomena of fluid mechanics, especially its interaction with circular cylinders. Nevertheless, still many bridges have had important response issues because of vortex shedding vibrations (Jurado *et al*. [5]). A bluff body immersed in a wind flow sheds alternating vortices of frequency *n*, which mainly depends on the Reynolds number and body geometry. Under certain conditions, the vorticity in the shear layers of the body may become periodic, forming alternating vortices downstream, which induce transverse forces on the body. It is of a particular interest to establish a relationship between flow velocity and the frequency of the fluctuating forces, which will permit the definition of a critical velocity at which vortex shedding frequency is close to a natural frequency of the structure. The relationship between flow velocity and the frequency of the fluctuating forces on the body is defined by Strouhal Number $St = nD/U$, where n is the frequency of the forces. Vortex shedding is affected by Reynolds number *Re*. Its effects on the separation point of this boundary layer, are the basis to explain the body behaviour immersed in a flow. This separation point can either be a fixed point, whenever there is a sharp edge on the body, or variable, due to a high curvature surface or smooth edges of the body. There are several possibilities to determine the wind speed for vortex shedding excitation and to evaluate if the oscillation amplitude is dangerous for the structure. Numerical methods based on CFD analysis have nowadays great expectation and give rapid results, but wind tunnel testing remains to be the most reliable tool for studying the phenomenon.

Suspension bridges such as Great Belt Bridge in Denmark (Larsen *et al*. [6]) have suffered important oscillations due to vortex shedding. In this particular case, there were registered events of vortex shedding for a mean velocity of $5 < U < 13$ m/s. All the events occurred at a reduced velocity of $1 < U/nB < 1.5$, exciting the third (0.13 Hz), the fifth (0.209 Hz) and the sixth mode (0.242 Hz).

The sectional model of Great Belt Bridge deck supported by Set 2 springs permits the investigation of the vortex shedding excitation. Figure 8 shows the acceleration time history. It can be observed that for the reduced velocity of

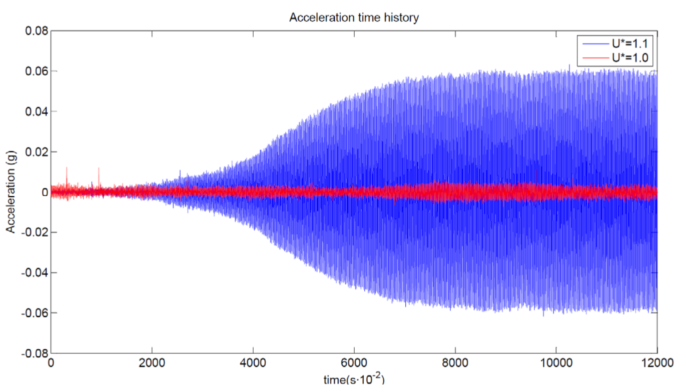

Figure 8: Time histories of the accelerations for Set 1.

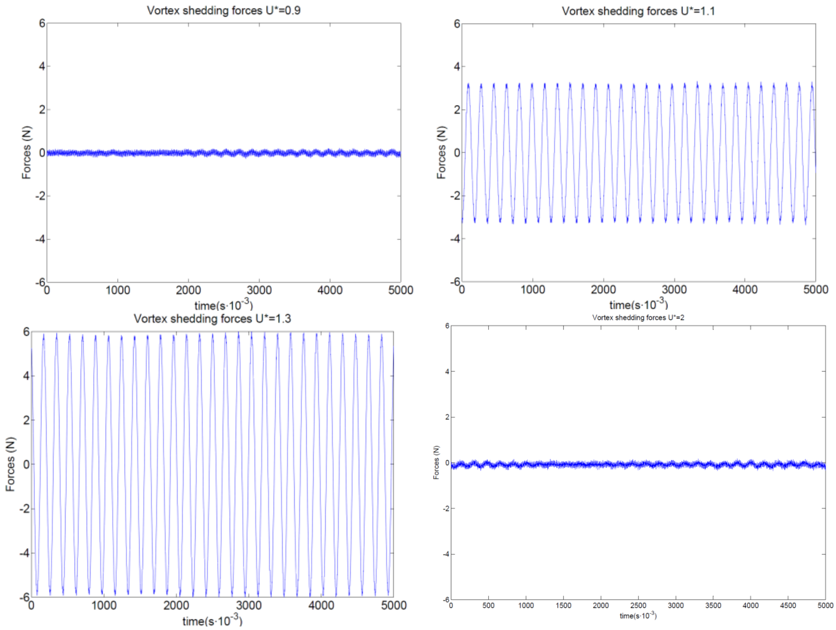

Figure 9: Time histories of the forces measured for Set 1.

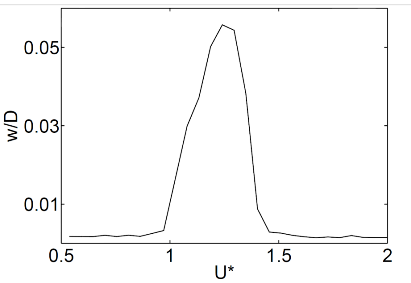

Figure 10: Non-dimensional vertical displacements with respect to reduce velocity.

*U**=1.0, the sectional model remains within low accelerations. However, for the reduced velocity of *U**=1.1, the amplitude of acceleration increased with time until reaching a steady-state. It takes as many as 70 seconds to reach the stability in vibration, which indicates the importance of stable environmental conditions for the development of vortex shedding. Time histories of the vertical forces are shown in Figure 9. The amplitudes of forces at a reduced velocity slightly lower than critical vortex shedding velocity (*U**=0.9) are much lower than those measured at reduced velocities that generate vortex shedding (*U**=1.1 and *U**=1.3). At higher reduce velocities (*U**=2), the amplitudes of forces decrease. As shown in Figure 10, the range of vortex shedding reduced velocity obtained for this model is between 1.1 and 1.4, which is in good agreement with Larsen's results.

3 Conclusions

A sectional model in the shape of Great Belt Bridge deck has been tested in free vibration using MITD and ILS identification methods to obtain the flutter derivatives.

MITD and ILS algorithms give very similar results but displacements fitted by ILS have a lower rate with the original signals.

The complete set of 18 flutter derivatives is calculated considering three DOF, vertical, lateral and torsional rotation. Tests measuring only vertical and rotation components are also carried out to recheck the results.

The test configuration with Set 1 springs permits the reproduction of vortex shedding phenomenon that affected the section of Great Belt Bridge deck.
The maximum oscillation amplitude for vortex shedding becomes steady after a certain time.

The amplitudes of vertical forces increase considerably during vortex shedding excitation and it causes vibration problems on the bridge that had been eliminated.

Acknowledgement

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