Hydrodynamic aspects of ship collisions with tension leg platforms

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Abstract

Collisions of tension leg platforms with ships are numerically simulated and their consequences predicted. A non-linear mathematical model is formulated; it accounts for two-dimensional kinematics of the colliding bodies, the local force-deformation characteristics within the impact zone of both structures as well as the restoring forces induced by the pre-tensioned legs. Particular emphasis is given to the rigorous modelling of the hydrodynamic interaction between the floating vessels and the fluid in their vicinity. This is achieved through unit impulse response functions that provide the damping forces as convolution integrals involving the histories of fluid motion and inertia terms with constant coefficients giving rise to added masses. The derived integro-differential equations are numerically solved in the time domain. Various collision scenarios are analysed and results are obtained for collision and hydrodynamic forces as well as the extent of local damage. The hydrodynamic effect is linked to the duration of impact. Thus, the problem is addressed in a more rational and mathematically rigorous manner than that of the classical impact theory.
Keywords: tension leg platform, collision, hydrodynamic interaction, time domain integration.

1 Introduction

Collisions between offshore structures and other floating bodies are considered a significant risk due to the high density of offshore installations, as well as services and marine traffic in their vicinity. The protection of structures against accidental damage requires, among others, the study of their behaviour under the action of collision forces in a marine environment.

WIT Transactions on The Built Environment, Vol 129,
www.witpress.com, ISSN 1743-3509 (on-line)
doi:10.2495/FSI130031

Drilling and exploration in deep waters has led to the design of tension leg platforms (TLPs). A TLP operates on the principle of excess buoyancy whereby a positively buoyant hull, forcibly held down by tethers, supports the superstructure. One end of each of the taut but flexible tethers is attached to the base of a column, while the other to either a gravity foundation or a group of tension anchor piles driven into the seabed. Vertical movement is thus negligible compared with horizontal motion, which is to a lesser extent restricted by the tensioned tethers. Such platforms are huge and have significant draught; therefore even large ships are more likely to collide with the cylindrical columns supporting the superstructure rather than the pontoons bracing the columns together at the base of the hull.

Investigations on ship-to-ship collisions have provided a knowledge base for also dealing with collisions between ships and platforms. In the latter case, differences arise from the platform geometry, its influence on hydrodynamic interaction and the local response to impact forces; the presence of restoring forces also affect the subsequent global dynamics of the platform.

The collision problem is here addressed through the formulation and integration of the equations of motion. This approach allows the assessment of the response of the colliding bodies during impact; this can lead to a more accurate prediction of the consequences of collision than that obtained by the classical impact theory.

Davies [1] and Davis and Mavrides [2] modelled the collision interaction between a concrete gravity platform and a ship as a single-degree-of-freedom system accounting for added mass, viscous damping and the stiffness characteristics of the platform. The caisson was assumed rigid but a feedback effect led to the adoption of a two-degree-of-freedom system accounting for the distortions of the caisson. Time-dependent fluid forces have been considered in the analysis of tension leg platforms [3]. This hydrodynamic interaction model is appropriate for forces incident on the moving bodies over the relatively short duration of an offshore collision. In such a rigorous analysis of ship-platform impacts [4], the hydrodynamic forces comprised both inertia and damping terms.

This paper focuses on modelling the interactions of the floating bodies with the surrounding fluid during their short period of contact and assessing the effects of these interactions on the motion and deformation of the colliding bodies. Due to the complexity of the problem, the time-dependent solution for the motion and deformation is obtained by a numerical time-stepping procedure.

A typical collision scenario between a TLP and a ship is envisaged whereby a corner leg of a TLP, initially at rest, is struck by a vessel. Various modelling assumptions regarding the orientation and geometry of colliding bodies, material properties and hydrodynamic interaction lead to the formulation of two coupled sets of integro-differential equations. Their numerical solution is extended to the post-impact period to determine the maximum displacement of the TLP under hydrodynamic and restoring forces.

Results were obtained for various collision scenarios involving a built TLP and three types of ship namely, a typical supply boat, a barge and a tanker. The effect of time-dependent hydrodynamic forces on the predictions of deformation and motion was assessed by comparisons with corresponding results obtained using the classical impact theory.

2 Formulation

The analysis of a collision event involving a ship and a TLP requires the input of the geometries and the material properties of the colliding bodies, as well as the modelling of the hydrodynamic forces, load-indentation characteristics in the zone of impact and the TLP restoring forces.

The ship and TLP motion in the horizontal plane is considered uncoupled from that in the other planes, thus both colliding bodies have three degrees of freedom, namely, surge, sway and yaw. The motion is described relative to moving frames of reference with origin at the respective geometric centres of ship and TLP. Relative to these axes, the hydrodynamic coefficients are constants at a given time or frequency. Possible eccentricity of the centre mass relative to the geometric centre is taken into account in the analysis. For a ship, in particular, the x axis is in the surge direction, the z axis in the sway direction while yaw is measured about the y axis; also, due to symmetry, its centre of mass lies on the x axis.

It is assumed that (i) the ship has drifted at a constant speed towards the platform, thus propeller and rudder forces can be ignored; (ii) the platform is struck while at rest; (iii) the collision occurs under calm sea conditions, hence no wind, wave or current forces need to be considered; (iv) hydrodynamic interaction exists between each floating body and the fluid in its immediate vicinity, any such interaction generated by one body on another is neglected; (v) the geometric characteristics of the colliding bodies are not affected by the local deformations within the impact zone; (vi) the impact duration is much higher than the fundamental period of either the TLP or the ship structure.

The equations governing the two-dimensional motion of either a platform or a ship are

$$F_x\mathbf{i} + F_z\mathbf{k} = m\mathbf{a} = m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = m\frac{\mathrm{d}^2(\mathbf{r} + \bar{x}\mathbf{i} + \bar{z}\mathbf{k})}{\mathrm{d}t^2} \tag{1}$$

$$M_y\mathbf{j} = I_G\frac{\mathrm{d}^2\psi}{\mathrm{d}t^2}\mathbf{j} + (\bar{x}\mathbf{i} + \bar{z}\mathbf{k}) \times (F_x\mathbf{i} + F_z\mathbf{k}) \tag{2}$$

where $\mathbf{i}$ and $\mathbf{k}$ are unit vectors along, respectively, the x and z directions of the moving frame of reference, F_x and F_z the corresponding components of the total external force, m is the mass, $\mathbf{r} = x\mathbf{i} + z\mathbf{k}$ the displacement vector of the geometric centre O, $\bar{x}$ and $\bar{z}$ the co-ordinates of the centre of mass, M_y the total moment about O, I_G the moment of inertia about the centre of mass and ψ the rotation of the body.

Taking into account the relations

$$\frac{\mathrm{d}\mathbf{i}}{\mathrm{d}t} = -q\mathbf{k}\ ,\ \frac{\mathrm{d}\mathbf{k}}{\mathrm{d}t} = q\mathbf{i}\ ,$$

the velocity vector is obtained as

$$\mathbf{v} = (u + \overline{z}q)\mathbf{i} + (w - \overline{x}q)\mathbf{k}$$

where $q = \dot{\psi}$ is the angular velocity, $u = \dot{x} + zq$, $w = \dot{z} - xq$ are the velocity components of O relative to the moving frame of reference and a dot over a symbol represents differentiation of the corresponding variable with respect to time. The acceleration vector is similarly obtained as

$$\mathbf{a} = [(\dot{u} + \overline{z}\dot{q}) + (w - \overline{x}q)q]\mathbf{i} + [(\dot{w} - \overline{x}\dot{q}) - (u + \overline{z}q)q]\mathbf{k} \tag{3}$$

Taking into account eqns (1) and (3), the moment eqn (2) is reduced to

$$M_y = I_y\dot{q} + m[\overline{z}(\dot{u} + wq) - \overline{x}(\dot{w} - uq)] \tag{4}$$

where $I_y = I_G + m(\overline{x}^2 + \overline{z}^2)$ is the moment of inertia about O.

The total external force is made up of the hydrodynamic force (H_x, H_z), the collision force (P_x, P_z) and the restoring force (R_x, R_z) acting on the platform only; the respective moments about the moving y axis are represented by M_p, M_h and M_r.

2.1 Hydrodynamic forces

Fluid forces arise from the impulsive motion of the colliding bodies in calm seas. Since these hydrodynamic forces are not the consequence of some periodic excitation, they cannot be associated with a particular frequency but should be expressed as functions of time. The main assumptions in determining the hydrodynamic forces are that (i) the fluid is incompressible, inviscid and irrotational and (ii) the motions of the body are small.

The general form of the hydrodynamic forces in the case of three-dimensional impulsive motion described relative to a fixed frame of reference is [5]

$$H_k = -\sum_{j=1}^{6} m_{kj} a_j - \int_{-\infty}^{t} K_{jk}(t-\tau)v_j(\tau)\mathrm{d}\tau \; ; k = 1,\ldots,6 \tag{5}$$

where subscripts j and k correspond to translations along and rotations about the x, y and z directions while symbols H, m, a and v also represent moment, moment of inertia, angular acceleration and angular velocity, respectively. The first term on the right hand side of eqn (5) represents the added inertia forces, whose coefficients are constant relative to the adopted frame of reference that rotates with the same angular velocity as the body. The second term represents the damping forces, which account for the free surface effects and the history of water motion. The substitution $t - \tau = \tau'$ leads to the transformation

$$\int_{-\infty}^{t} K_{jk}(t-\tau)v_j(\tau)\mathrm{d}\tau = \int_{0}^{\infty} K_{jk}(\tau)v_j(t-\tau)\mathrm{d}\tau \tag{6}$$

Both m_{jk} and $K_{jk}(t)$ can be calculated if the corresponding frequency dependent added mass coefficients a_{jk} and damping coefficients b_{jk} are known [5]. The latter are given by expressions in terms of fluid velocity potentials arising from rigid body oscillatory motion in various directions. Either the boundary element or the finite element method can be applied for the determination of these potentials.

Combining the inertia terms obtained for a two-dimensional rotating frame of reference [6] with the respective damping forces according to eqn (5), the hydrodynamic forces take the form

$$H_x = -[m_{xx}\dot{u} + m_{x\psi}\dot{q} + (m_{zz}w + m_{z\psi}q)q]$$

$$-\int_0^\infty K_{xx}(\tau)u(t-\tau)\mathrm{d}\tau - \int_0^\infty K_{x\psi}(\tau)q(t-\tau)\mathrm{d}\tau \tag{7}$$

$$H_z = -[m_{zz}\dot{w} + m_{z\psi}\dot{q} - (m_{xx}u + m_{x\psi}q)q]$$

$$-\int_0^\infty K_{zz}(\tau)w(t-\tau)\mathrm{d}\tau - \int_0^\infty K_{z\psi}(\tau)q(t-\tau)\mathrm{d}\tau \tag{8}$$

$$M_h = -[m_{\psi\psi}\dot{q} + m_{x\psi}(\dot{u} + wq) + m_{z\psi}(\dot{w} - uq) - (m_{zz} - m_{xx})uw]$$

$$-\int_0^\infty K_{x\psi}(\tau)u(t-\tau)\mathrm{d}\tau - \int_0^\infty K_{z\psi}(\tau)w(t-\tau)\mathrm{d}\tau - \int_0^\infty K_{\psi\psi}(\tau)q(t-\tau)\mathrm{d}\tau \tag{9}$$

The added mass coefficient m_{xz} and retardation function K_{xz} were set equal to zero since the x axis is an axis of symmetry and, as a consequence, motions in the x and z directions are uncoupled.

The convolution integrals can be simplified by accounting for the initial conditions at impact, namely the initial velocities $u(0)$, $w(0)$ and $q(0)$. Considering, for instance, motion in the x direction, the corresponding convolution integral can be written as

$$\int_0^\infty K_{xx}(\tau)u(t-\tau)\mathrm{d}\tau = \int_0^t K_{xx}(\tau)[u(t-\tau) - u(0)]\mathrm{d}\tau$$

$$+\int_t^\infty K_{xx}(\tau)[u(t-\tau) - u(0)]\mathrm{d}\tau + u(0)\int_0^\infty K_{xx}(\tau)\mathrm{d}\tau \tag{10}$$

But the second integral on the right hand side of eqn (10) vanish since $u(t - \tau) = u(0)$ for $\tau \geq t$, assuming that the body moves with constant velocity before collision, while the vanishing of the third is an intrinsic property of the retardation functions.

Taking into account eqns (3) and (7)-(9), the equations of motion take the form

$$m'_{xx}\dot{u} + m'_{x\psi}\dot{q} + (m'_{zz}w + m'_{z\psi}q)q + D_{xxu} + D_{x\psi q} = P_x + R_x \tag{11}$$

$$m'_{zz}\dot{w} + m'_{z\psi}\dot{q} - (m'_{xx}u + m'_{x\psi}q)q + D_{zzw} + D_{z\psi q} = P_z + R_z \tag{12}$$

$$I'_y\dot{q} + m'_{x\psi}(\dot{u} + wq) + m'_{z\psi}(\dot{w} - uq) + (m_{zz} - m_{xx})uw$$

$$+ D_{x\psi u} + D_{z\psi w} + D_{\psi\psi q} = M_p + M_r \tag{13}$$

where

$$m'_{xx} = m + m_{xx},\ m'_{zz} = m + m_{zz},\ I'_y = I_y + m_{\psi\psi}$$

$$m'_{x\psi} = \bar{z}m + m_{x\psi},\ m'_{z\psi} = -\bar{x}m + m_{z\psi}$$

$$D_{jk\ell} = \int_0^t K_{jk}(\tau)[u_\ell(t-\tau) - u_\ell(0)]\mathrm{d}\tau\ ;\ j,k = x,z,\psi \text{ and } u_\ell = u,w,q$$

2.2 Impact forces

The local material behaviour within the impact zone was represented by a pair of springs [4] whose elasto-plastic properties coincided with the respective structural characteristics of the colliding bodies. Strain rate effects on these relations were assumed negligible. The springs are placed normal and tangential to the surface at the point of impact. The respective orientations of the generated forces and displacements are shown schematically in fig. 1. Referring to the notation in this figure, the collision force is written

$$\mathbf{P} = P_n\mathbf{n} + P_t\mathbf{t} \tag{14}$$

where **n** and **t** are unit vectors in the direction of the non-linear springs and P_n, P_t are known functions of the respective deformations s_n, s_t. The moment of the collision force about the geometric centre of the body is

$$M_p\mathbf{j} = [(x_c\mathbf{i} + z_c\mathbf{k}) + (s_n\mathbf{n} + s_t\mathbf{t})] \times (P_n\mathbf{n} + P_t\mathbf{t}) \tag{15}$$

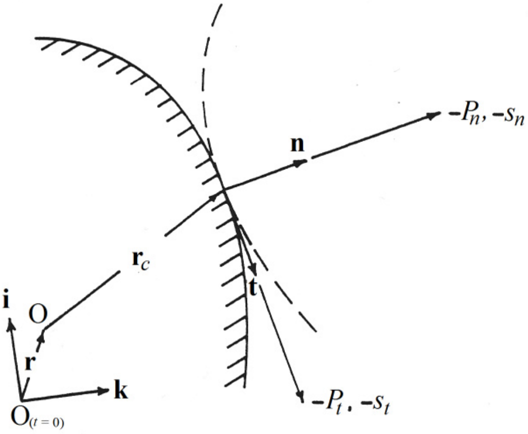

Figure 1: Impact point location and spring force orientations.

where (x_c, z_c) are the co-ordinates of the impact point relative to the geometric centre. Relative to the moving *x-z* frame of reference the impact force components and moment are obtained from

$$\begin{Bmatrix} P_x \\ P_z \\ M_p \end{Bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \\ -x_c \sin\theta + z_c \cos\theta + s_t & x_c \cos\theta - z_c \sin\theta - s_n \end{bmatrix} \begin{Bmatrix} P_n \\ P_t \end{Bmatrix} \tag{16}$$

where θ is the angle between **i** and **n** unit vectors.

Relations $P_n(s_n)$ were derived on the assumption that the local deformation of each colliding body at the point of impact can be considered independently, that is, a flexible TLP leg is dented by a rigid ship side and vice versa. A large fictitious stiffness was assigned to the tangential springs to prevent slip between the two surfaces in contact at the impact zone.

2.3 TLP restoring forces

Restoring forces are generated by the displacement of tethers under tension whose intensity depends on the buoyancy of the platform. For small deviations of the tethers from the vertical position, the relation between platform displacement and restoring force is linear. Larger displacements however increase significantly the buoyancy of the platform, which is partly compensated by the tether extension [7]. Taking these two effects into account, it is possible to derive a non linear force-displacement relation for the j^{th} tether group in the form

$$R_j = c_1 d_j + c_3 d_j^3 +$$

where the constants c_1, c_3, … depend on the initial tension, initial length and stiffness of the tether as well as the relevant dimensional characteristics of the platform. With the adopted notation, the total horizontal displacement is given by

$$d_j = \sqrt{d_{jx}^2 + d_{jz}^2}$$

where

$$d_{jx} = x + x_j(1 - \cos\psi) + z_j \sin\psi$$

$$d_{jz} = z - x_j \sin\psi + z_j(1 - \cos\psi)$$

and (x_j, z_j) are the co-ordinates of the j^{th} tether group relative to the geometric centre. Assuming that the platform is held in place by N tether groups, the total restoring force would be given by

$$R_x\mathbf{i} + R_z\mathbf{k} = -\sum_{j=1}^{N} \frac{R_j}{d_j}(d_{jx}\mathbf{i} + d_{jz}\mathbf{k}) \tag{17}$$

$$M_r\mathbf{j} = -\sum_{j=1}^{N} \frac{R_j}{d_j}(x_j\mathbf{i} + z_j\mathbf{k}) \times (d_{jx}\mathbf{i} + d_{jz}\mathbf{k}) = -\sum_{j=1}^{N} \frac{R_j}{d_j}(z_j d_{jx} - x_j d_{jz})\mathbf{j} \tag{18}$$

3 Numerical solution

The equations of motion are applied to both colliding bodies using subscripts 1 and 2 to distinguish between variables and properties of the ship and the TLP, respectively. Plan views of ship and TLP as well as a possible collision scenario are schematically drawn in fig. 2.

The two sets of governing equations are coupled through the requirements of deformation compatibility and force equilibrium at the point of impact. Using vector notation, the compatibility condition is expressed as

$$(x_1 + x_{1c})\,\mathbf{i}_1 + (z_1 + z_{1c})\mathbf{k}_1 - x_{1c}\mathbf{i}_{10} - z_{1c}\mathbf{k}_{10} + s_{n1}\mathbf{n}_1 + s_{t1}\mathbf{t}_1$$
$$= (x_2 + x_{2c})\mathbf{i}_2 + (z_2 + z_{2c})\mathbf{k}_2 - x_{2c}\mathbf{i}_{20} + z_{2c}\mathbf{k}_{20} + s_{n2}\mathbf{n}_2 + s_{t2}\mathbf{t}_2 \qquad (19)$$

where $\mathbf{i}_0$, $\mathbf{k}_0$ denote the initial orientations of $\mathbf{i}$, $\mathbf{k}$. Equilibrium is satisfied if

$$P_{n1}\mathbf{n}_1 + P_{t1}\mathbf{t}_1 + P_{n2}\mathbf{n}_2 + P_{t2}\mathbf{t}_2 = 0 \qquad (20)$$

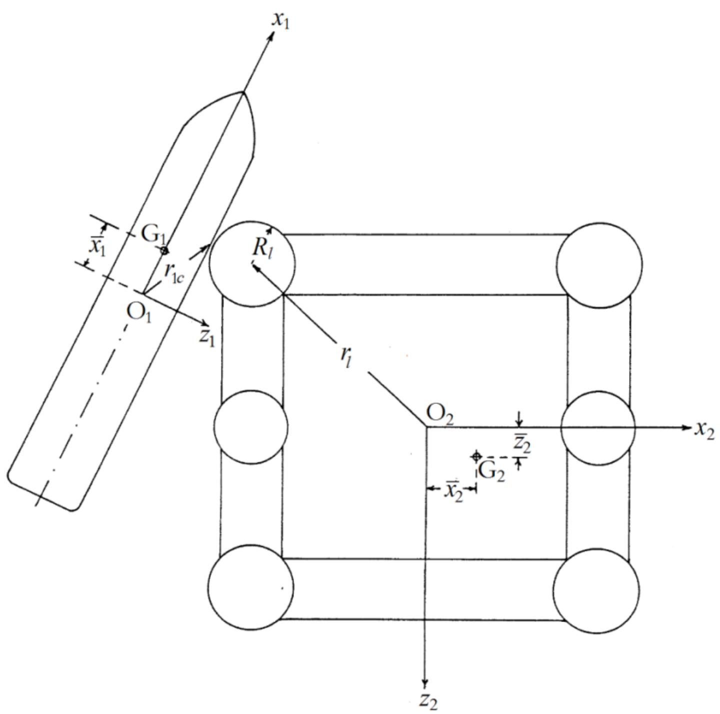

Figure 2: Possible collision scenario for a ship and a platform.

Eqns (19) and (20) can be written in matrix form by referring the vectors to the fixed frame of reference. This leads to the systems of equations

$$\mathbf{T}_{\psi_{10}}[\mathbf{T}_{\psi_1}(\mathbf{r}_1 + \mathbf{r}_{1c}) + \mathbf{T}_{\phi_1}\mathbf{s}_1] = \mathbf{T}_{\psi_{20}}[\mathbf{T}_{\psi_2}(\mathbf{r}_2 + \mathbf{r}_{2c}) + \mathbf{T}_{\phi_2}\mathbf{s}_2] \tag{21}$$

$$\mathbf{T}_{\psi_{10}}\mathbf{T}_{\phi_1}\mathbf{P}_1 + \mathbf{T}_{\psi_{20}}\mathbf{T}_{\phi_2}\mathbf{P}_2 = 0 \tag{22}$$

where ψ_0 describes the initial orientation of the x-z frame, $\phi = \psi + \theta$,

$$\mathbf{r} = \begin{Bmatrix} x \\ z \end{Bmatrix},\ \mathbf{r}_c = \begin{Bmatrix} x_c \\ z_c \end{Bmatrix},\ \mathbf{s} = \begin{Bmatrix} s_n \\ s_t \end{Bmatrix},\ \mathbf{P} = \begin{Bmatrix} P_n \\ P_t \end{Bmatrix} \text{ and } \mathbf{T}_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}.$$

The formulated system of non-linear equations is solved through an incremental approach whereby the variables are replaced by their small variation over a small time interval Δt. Starting from the impact forces, their increments can be expressed as

$$\Delta\mathbf{P} = \mathbf{K}(\mathbf{s})\ \Delta\mathbf{s} \tag{23}$$

where

$$\mathbf{s} = \begin{Bmatrix} s_n \\ s_t \end{Bmatrix},\ \mathbf{K} = \begin{bmatrix} k_n & 0 \\ 0 & k_s \end{bmatrix}$$

and $k_n(s_n)$ and $k_t(s_t)$ are the gradients of the force-deformation curves at s_n and s_t, respectively. It is assumed that the **n** and **t** maintain their orientation relative to the rotating axes during collision. The incremental form of eqns (21) and (22) are

$$\mathbf{T}_{\psi_{10}}\{[\mathbf{T}_{(\psi_1+\pi/2)}(\mathbf{r}_1 + \mathbf{r}_{1c}) + \mathbf{T}_{(\phi_1+\pi/2)}\mathbf{s}_1]\Delta\psi_1 + \mathbf{T}_{\psi_1}\Delta\mathbf{r}_1 + \mathbf{T}_{\phi_1}\Delta\mathbf{s}_1\}$$
$$= \mathbf{T}_{\psi_{20}}\{[\mathbf{T}_{(\psi_2+\pi/2)}(\mathbf{r}_2 + \mathbf{r}_{2c}) + \mathbf{T}_{(\phi_2+\pi/2)}\mathbf{s}_2]\Delta\psi_2 + \mathbf{T}_{\psi_2}\Delta\mathbf{r}_2 + \mathbf{T}_{\phi_2}\Delta\mathbf{s}_2\} \tag{24}$$

$$\mathbf{T}_{\psi_{10}}\ [\mathbf{T}_{(\phi_1+\pi/2)}\mathbf{P}_1\Delta\psi_1 + \mathbf{T}_{\phi_1}\mathbf{K}_1(\mathbf{s})\ \Delta\mathbf{s}_1]$$
$$+ \mathbf{T}_{\psi_{20}}[\mathbf{T}_{(\phi_2+\pi/2)}\mathbf{P}_2\Delta\psi_2 + \mathbf{T}_{\phi_2}\mathbf{K}_2(\mathbf{s})\ \Delta\mathbf{s}_2] = 0 \tag{25}$$

having substituted $\Delta\mathbf{P}_i$ (i = 1,2) from eqn (23) into eqn (25). Eqns (24) and (25) can be re-arranged to provide a single matrix relation in the form

$$\Delta\mathbf{S} = \mathbf{A}\Delta\mathbf{X} \tag{26}$$

between the incremental spring deformations $\Delta\mathbf{S} = \Delta(s_{n1}, s_{t1}, s_{n2}, s_{t2})$ and the incremental displacements and rotations $\Delta\mathbf{X} = \Delta(x_1, z_1, \psi_1, x_2, z_2, \psi_2)$ of the two colliding bodies.

Linear equations governing small increments of all kinematic variables are obtained from eqns (11)-(13) by applying a small time increment Δt; accounting for the kinematic relations between $\Delta\mathbf{u}_i = \{\Delta u, \Delta w, \Delta q\}_i$ and $\Delta\mathbf{x}_i = \{\Delta x, \Delta z, \Delta\psi\}_i$, these incremental equations can be written in matrix form as

$$\mathbf{M}_i\Delta\ddot{\mathbf{x}}_i + \mathbf{C}_i\Delta\dot{\mathbf{x}}_i + \mathbf{B}_i\Delta\mathbf{x} + \dot{\mathbf{D}}_i\Delta t = \Delta\mathbf{P}_{oi} + \Delta\mathbf{R}_i\ ;\ i = 1, 2 \tag{27}$$

where $\mathbf{P}_{oi} = \{P_x, P_z, M_p\}_i$ and the one-dimensional array $\dot{\mathbf{D}}_i$ comprises integrals of the form

$$\dot{D}_{jk\ell} = \int_0^t K_{jk}(\tau)\dot{u}_\ell(t-\tau)]\mathrm{d}\tau \; ; j,k = x,z,\psi \text{ and } u_\ell = u,w,q$$

Incremental equations relating $\Delta\mathbf{P}_{oi}$ with $\Delta\mathbf{s}_i$ and $\Delta\mathbf{P}_i$ can be derived from eqn (16). Taking also into account eqn (23) leads to a matrix relation in the form

$$\Delta\mathbf{P}_{oi} = \mathbf{K}_{oi}\,\Delta\mathbf{s}_i \quad (28)$$

where matrices $\mathbf{K}_{oi}$ depend not only on the local stiffnesses but also on the geometry of contact. Similarly, eqns (17) and (18) are used to generate incremental relations in the form

$$\Delta\mathbf{R} = \Delta\mathbf{R}_2 = -\mathbf{K}_r\,\Delta\mathbf{x}_2 \quad (29)$$

where matrix $\mathbf{K}_r$ depends on c_1, c_3, the geometric arrangement of tethers and the current value of ψ_2. Thus eqns (27) can be written only in terms of kinematic variable increments and eventually coupled using eqn (26).

The increments of the displacements and their time derivatives are eliminated from the final set of equations using the Newmark relations [8]. Thus, these equations can be solved for the increments of the second time derivatives of **X**. This leads to the determination of all kinematic variables at time $t + \Delta t$. The original non-linear equations can then be used to re-calculate the second time derivatives and thus check the accuracy of the procedure. An iterative process can be adopted to establish convergence of the solution. When separation of the two bodies is detected, the post-impact response of only the TLP is carried out until the time when its displacement reaches a maximum.

4 Results

4.1 Ship details

A typical supply ship, a barge and a tanker are considered in the present study, all falling into the category of attendant vessels [9]. Their shapes and sizes are described through the data listed in Table 1. In this table, e is the eccentricity of centre of gravity along the x-axis and m the displacement. The frequency dependent added mass and damping coefficients of these ships have been calculated by an approximate method [10].

Table 1: Ship data used in collision simulations.

Ship category	Gross dimensions (m)				e (m)	m (metric tonnes)	I_G ($\times 10^9$) (kg m^2)
	Length	Breadth	Depth	Draught			
Supply boat	60 [4]	12.5 [4]	6.4 [4]	5.5 [4]	0	2,500	0.5625
Barge	88 [11]	18.3 [11]	6 [11]	5.2 [11]	0	11,000	5.3
Tanker	251 [11]	38.2 [11]	22.2 [11]	15.3 [11]	5.4	150,000	591

The local force-indentation characteristics of ship structures significantly depend on impact location. Such information on the various types of ships listed in table 1 have been experimentally and analytically assessed by many investigators [12, 13]. Published data on ship sides indented by rigid cylindrical columns were retrieved from various sources and entered as input in the performed collision simulations.

4.2 Platform details

A TLP of almost square plan was adopted in the simulated collisions. Its dimensions measured from the centres of the corner legs were 60 m by 57 m. Its cylindrical columns were internally ring-stiffened and 57.7 m long [14]; the diameter of the corner columns was 18 m, while that of the columns situated in the middle of each shorter side 14.5 m. The draught of the platform was 20 m while the depth of water 350 m. Its frequency dependent coefficients were provided by the offshore industry [15].

Platform corner legs in the form of ring-stiffened cylinders were assumed to be deformed by flat rigid indenters representing ship sides. The shell thickness and the dimensions as well as the spacing of stiffeners were selected according to offshore design standards. Non-linear force-deformation relations were numerically obtained using a general-purpose finite element code [16]. The modelling was validated through comparison of its predictions with existing experimental data for unstiffened cylinders. The analysis was performed for two indenter widths so that its output could be adjusted to variable ship side size in subsequent collision simulations.

4.3 Collision scenarios

The adopted fixed frame of reference coincided with that of the platform on impact, that is, at $t = 0$. The assumed impact velocity range was from 0.5 to 3 m/s for ships drifting towards the platform, which was assumed initially at rest. Four collision cases were analysed: central sideways collisions involving all three ship types and an eccentric stern collision with the supply boat only.

For each collision scenario, the obtained results were presented in the form of a table that provided the impact duration, the maximum impact force, the corresponding ship and TLP distortions as well as the absorbed energies predicted by both the present analysis and the classical impact theory. The predicted rate of increase of local deformation with collision speed was not uniform but depended on the local stiffening characteristics. In the case of eccentric collision, low stern damage was due to collision energy partially converted to rotational kinetic energy.

A number of load-indentation curves with different stiffening characteristics were retrieved from the literature for a typical supply boat. The central impact analysis was performed for each of those curves. As expected, the stiffer the ship side, the smaller its damage and the higher the deformation of the platform. Parametric studies were carried out for the case of sideways collision of the TLP with either a supply boat or a tanker. The parameters investigated were the angle of incidence, location of impact zone along the side of the ship, initial velocity of the platform and transverse rigidity at the point of impact.

Both maximum local indentation and TLP displacement were predicted to vary non-linearly collision speed in the case of tanker impacts. This indicated that hydrodynamic damping and non-linearity in tether force-displacement relations have a significant effect on TLP response under high energy collisions.

5 Discussion

The developed time domain analysis can provide solutions for a wide range of conditions that cannot be accounted for by the classical impact theory. Most importantly, the rigorous modelling of the hydrodynamic forces allows a more accurate assessment of their effect, which can be significant particularly in the case of long duration impacts. Consideration of two-dimensional translational and rotational motion allows the assessment of the effects of eccentric impact as well as hydrodynamic force coupling due to possible geometric asymmetries. With regard to TLP, in particular, the non-linear relation of the restoring forces to the platform motion is fully accounted for.

Parametric studies can be performed to investigate the effects of variable local stiffness as a consequence of structural changes or variable impact location; initial movement of the platform at the time of impact; angle of incidence and transverse rigidity. Systematic comparison of predicted responses with respective results by the classical impact theory will establish the extent of influence of the hydrodynamic damping and its dependence on the various collision parameters and structural characteristics.

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