Three examples of flutter analysis of cable-stayed bridges

J. Á. Jurado, S. Hernández, F. Nieto & A. Baldomir
School of Civil Engineering, University of Coruña, Spain

Abstract

There are multiple configurations permitted for a cable stayed bridge such as the number of spans, tower shapes, and cable arrangements. Therefore it is always important to study the behaviour of these types of structures against strong winds so that aeroelastic instabilities do not appear. This paper contains detailed aeroelastic analyses of three cable stayed bridges of very different configurations. Talavera Bridge over the Tagus River in Spain has only a single span of 315 m with a single pylon. The bridge has a central stay cable plane that supports the deck and two skewed planes of rear cables that allow the overall balance. The project of Miradoiros Bridge over La Coruña estuary in Spain has a classical configuration with a main span of 658 m and two secondary spans of 270 m each. The bridge has two lateral stay cable planes and an aerodynamic box girder. Finally, the future cable stayed bridge over the Forth in Scotland is studied. It has three mono-towers with a symmetrical arrangement resulting in two 650 m main spans and 325 m side spans. The deck superstructure is a single cell box girder with the stay cables anchored along the centre, so the bridge has two corridors. The software that analysed the aeroelastic instabilities of these bridges has been developed by the School of Civil Engineering at the University of La Coruña and it is based on hybrid methods. The hybrid methods are computational, although they need some coefficients and functions which must be obtained experimentally in an aerodynamic wind tunnel working with a deck sectional model. These experimental data permit the analysis of the complete structure considering the fluid-structure interaction between the wind flow and the deck.
Keywords: aeroelastic analysis, cable stayed bridges, wind tunnel tests, flutter, modal analysis.

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1 Hybrid flutter aeroelastic analysis

It is well known that one of the most demanding conditions in the design of long span suspension bridges is their stability under wind loads. Particularly, fatal aeroelastic phenomena as flutter must be avoided. To avoid experimental tests of complete bridge models in large wind tunnels that are complicated and expensive, it is necessary to use a hybrid methods which are computational based but need experimental parameters. Sectional models of the deck are initially tested in an aerodynamic wind tunnel of smaller dimensions (figure 1) to obtain some parameters that permit to model the fluid structure interaction between the deck and the wind flow. These data are then used in the computational analysis of the aeroelastic behaviour of the complete bridge. Same examples of this working method can be found in Jurado and Hernández [1]. The flutter condition on long-span bridges can be computationally evaluated working with a structural computational model to calculate natural frequencies and mode shapes, and using 18 experimental functions called flutter derivatives obtained in a wind tunnel.

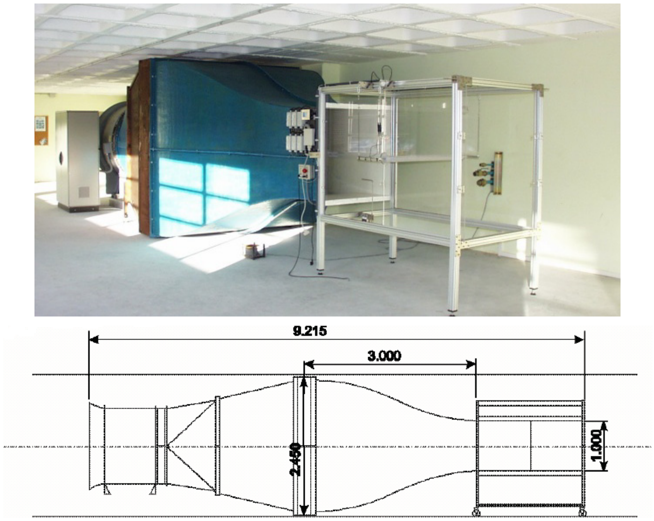

Figure 1: Wind tunnel of the school of civil engineering of the University of La Coruña.

1.1 Experimental identification of flutter derivatives

The set of eighteen flutter derivatives provides important information required to carry out aeroelastic analysis aimed to identify the safety level of long span bridges against wind induced phenomena. In the case of flutter, those functions are the components of the aeroelastic damping and stiffness matrices that relate the lift, drag and moment forces to the vector of displacements and velocities of bridge deck. So far flutter derivatives are usually obtained by testing a reduced model of a deck segment in a wind tunnel. There exist two different alternatives: a) the Forced vibration approach (Diana et al. [2]); b) the Free vibration approach (Jurado et al. [3]). In this occasion the free vibration approach with the sectional model of the deck supported by eight vertical and four horizontal springs was used and test were carried out under different flow speed. Figure 2 shows the three forces acting on a deck. According to (Simiu and Scanlan [4]) formulation, these actions are linealized as functions of the displacements and velocities of the system for vertical w, lateral v and torsional rotation φ_x degrees of freedom. The expressions can be written as

$$\mathbf{f}_a = \mathbf{C}_a \dot{\mathbf{u}} + \mathbf{K}_a \mathbf{u} =$$

$$\begin{Bmatrix} D_a \\ L_a \\ M_a \end{Bmatrix} = \frac{1}{2}\rho U^2 KB \cdot \begin{pmatrix} P_1^* & -P_5^* & -BP_2^* \\ -H_5^* & H_1^* & BH_2^* \\ -BA_5^* & BA_1^* & B^2 A_2^* \end{pmatrix} \begin{Bmatrix} \dot{v} \\ \dot{w} \\ \dot{\varphi}_x \end{Bmatrix} \tag{2}$$

$$+\frac{1}{2}\rho U^2 K^2 \cdot \begin{pmatrix} P_4^* & -P_6^* & -BP_3^* \\ -H_6^* & H_4^* & BH_3^* \\ -BA_6^* & BA_4^* & B^2 A_3^* \end{pmatrix} \begin{Bmatrix} v \\ w \\ \varphi_x \end{Bmatrix}$$

where B is the deck width, ρ is the air density, U is the mean wind speed, $K = B\omega/U$ is the reduced frequency with ω the frequency of the response, and $P^*_i(K)$, $H^*i(K)$, $A^*_i(K)$ $i = 1...6$ are the flutter derivatives which are functions of K. $\mathbf{K}_a$ and $\mathbf{C}_a$ are called aeroelastic matrices. As figure 3 shows, the support system is set up by means of vertical and horizontal springs which permit the three considered degrees of freedom v, w, φ_x. The frequency similarity is not necessary to evaluate flutter derivatives because they are functions of the reduced velocity $U^* = 2\pi U / \omega B$. By changing the wind speed in the tunnel and the stiffness constants of the springs, a wide range of reduced velocities can be simulated.

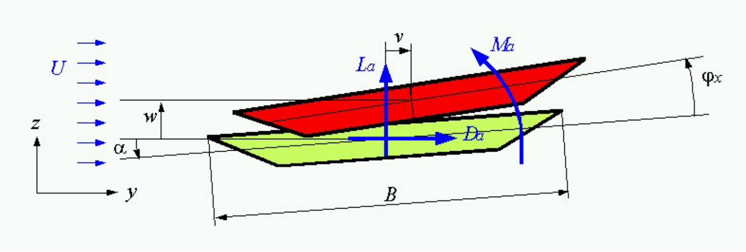

Figure 2: Aeroelastic forces and displacements.

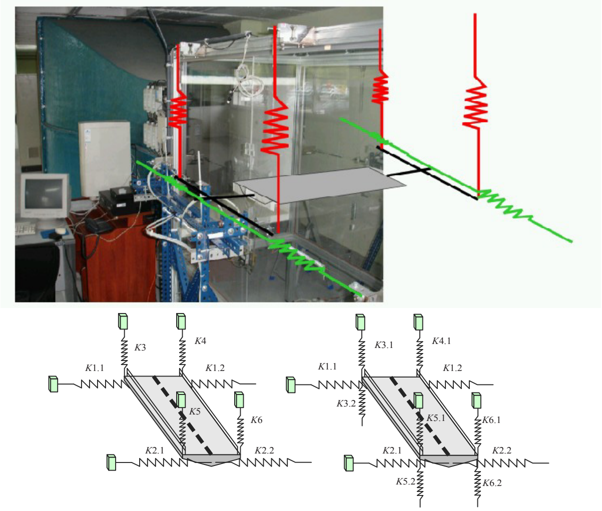

Figure 3: Spring support system of a sectional model.

The dynamic balance equation for the sectional model is

$$\mathbf{M}\ddot{\mathbf{u}}+\left(\mathbf{C}-\mathbf{C}_a\right)\dot{\mathbf{u}}+\left(\mathbf{K}-\mathbf{K}_a\right)\mathbf{u}=\mathbf{0} \tag{3}$$

where $\mathbf{u} = (v,w,\varphi_x)^T$. Multiplying by $\mathbf{M}^{-1}$ and denoting $\mathbf{C}_m = \mathbf{M}^{-1}(\mathbf{C}\text{-}\mathbf{C}_a)$ and $\mathbf{K}_m = \mathbf{M}^{-1}(\mathbf{K}\text{-}\mathbf{K}_a)$ becomes

$$\ddot{\mathbf{u}}+\mathbf{C}_m\dot{\mathbf{u}}+\mathbf{K}_m\mathbf{u}=\mathbf{0} \tag{4}$$

To obtain the flutter derivatives, all terms of $\mathbf{C}_m$ and $\mathbf{K}_m$ matrices are evaluated from the time histories of the model displacements at free vibration. It is necessary to identify the natural frequency ω and the damping ratio ξ for each degree of freedom. Then, the terms of $\mathbf{K}_m$ and $\mathbf{C}_m$ are calculated. Denoting K^U_{ij} and C^U_{ij} the terms for a wind speed U in the tunnel and denoting K^0_{ij} and C^0_{ij} the terms for zero speed $U = 0$, which correspond to null aeroelastic matrices ($\mathbf{K}_a = \mathbf{C}_a = \mathbf{0}$), any flutter derivative can be evaluated by subtraction. For example, A^*_2 is obtained as

$$A_2^*\left(K\right)=-\frac{2I}{\rho B^4\omega}\left(C_{22}^0-C_{22}^U\right) \tag{5}$$

where I is the polar inertia of the deck which appears in the mass matrix. The model is elastically sustained using eight to twelve springs: four or eight vertical and four horizontal ones. The stiffness of the springs determines the vibration frequencies ($2\pi f = \omega$) of the system that together with the wind velocity in the tunnel, U and the model width, B, determines the range of reduced velocities, U^*, in order to be able to obtain flutter functions. Several spring sets are used in order to vary the natural frequencies of the model and include a wider range of reduced velocities. To build the matrices it is necessary to obtain the frequency and damping properties of a free vibration system. In this case the Sarkar *et al.* [5] identification method based on Ibrahim and Mikulcik [6] time domain method has been used.

1.2 Flutter analysis

A coherent matrix formulation has been used for the computational phase of hybrid flutter analysis. Jurado and Hernandez [7] explain that this formulation stems from the equation (3) ensambling the matrices and vector for the full structural model. Through modal analysis it is possible to approximate the deck displacements by means of a linear combination of the most significant mode shapes. Assembling them in columns into the modal matrix $\mathbf{\Phi}$, the displacement vector can be expressed as $\mathbf{u} = \mathbf{\Phi}\,\mathbf{q}$. Each element of the vector $\mathbf{q}$ represents the participation of each mode shape in the displacement vector $\mathbf{u}$. Premultiplying (2) by $\mathbf{\Phi}^T$ it becomes

$$\mathbf{I}\ddot{\mathbf{q}} + \mathbf{C}_R\dot{\mathbf{q}} + \mathbf{K}_R\mathbf{q} = \mathbf{0} \tag{6}$$

where $\mathbf{C}_R = \mathbf{\Phi}^{\mathrm{T}}\,(\mathbf{C} - \mathbf{C}_a)\,\mathbf{\Phi}$, $\mathbf{K}_R = \mathbf{\Phi}^{\mathrm{T}}\,(\mathbf{K} - \mathbf{K}_a)\,\mathbf{\Phi}$ and $\mathbf{\Phi}^{\mathrm{T}}\,\mathbf{M}\,\mathbf{\Phi} = \mathbf{I}$, using mass normalized modes. Knowing that the solution of this equation has the form $\mathbf{q}(t) = \mathbf{w}e^{\mu t}$, becomes

$$\left(\mu^2\mathbf{I}\mathbf{w} + \mu\mathbf{C}_R\mathbf{w} + \mathbf{K}_R\mathbf{w}\right)e^{\mu t} = \mathbf{0} \tag{7}$$

which can be transformed into an eigenvalue problem by adding the identity $-\mu\mathbf{I}\mathbf{w}+\mu\mathbf{I}\mathbf{w} = \mathbf{0}$:

$$\left[\mu\begin{pmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{pmatrix}\begin{pmatrix}\mu\mathbf{w}\\ \mathbf{w}\end{pmatrix} + \begin{pmatrix}\mathbf{C}_R & \mathbf{K}_R\\ -\mathbf{I} & \mathbf{0}\end{pmatrix}\begin{pmatrix}\mu\mathbf{w}\\ \mathbf{w}\end{pmatrix}\right]e^{\mu t} = \mathbf{0} \tag{8}$$

or in short

$$\left(\mathbf{A} - \mu\mathbf{I}\right)\mathbf{w}_\mu e^{\mu t} = \mathbf{0} \tag{9}$$

The imaginary part β of the eigenvalues μ counts on the frequency ω, while the real part α of the eigenvalues is associated with the damping ratio ξ. The condition of flutter corresponds to the lowest wind speed U_f which gives one eigenvalue with vanished real part (figure 4). However, the problem (9) is non-linear because the matrix $\mathbf{A}$ assembles the aeroelastic matrices $\mathbf{K}_a$ and $\mathbf{C}_a$. These matrices contain the flutter derivatives, which are functions of the reduced frequency $K = B\omega/U_f$, and the frequency for each eigenvalue ω remains unknown until the problem has been solved.

2 Cable stayed bridges properties

Three different cable stayed bridges have been studied in this work. They are very different in terms of the number of spans, cable configuration and tower shapes. Two of them have aerodynamic deck cross sections, while the other has a quite different deck section. A beams structural model is necessary to obtain the natural frequencies and mode shapes because the modal data takes into account the bridge structural behaviour during the aeroelastic analysis. Examples of this kind of analysis can be found in Mendes and Branco [8] and Jurado et al. [9].

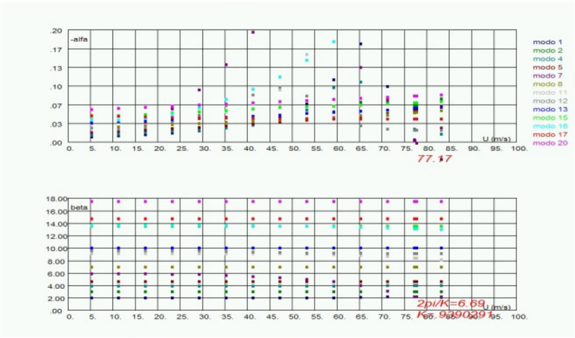

Figure 4: Evolution of eigenvalues with respect to wind velocity *U* until flutter condition.

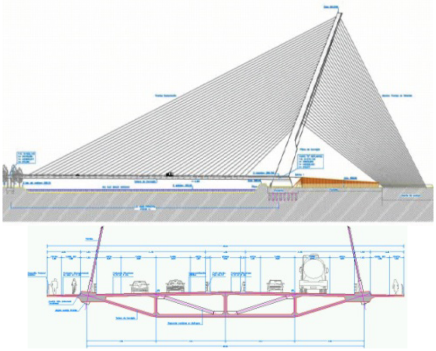

Figure 5: Talavera bridge.

2.1 Talavera bridge

A new cable stayed bridge with significant dimensions has been design at the south ring road of Talavera village, near Toledo in Spain crossing the Tagus River. The bridge has only a single span of 318 m. At one side there is a 164m high pylon. From the pylon two cable planes connect the both sides of the deck and at its back zone there are others two set of cables to balance the pylon by transmitting the forces to the foundation. The ruled surfaces shape of these set of cable results very attractive. As it is shown in figure 5, the deck is a concrete multi-cell box girder with two lateral cantilevers. It measures 36 m wide and 2.77 m deep. The upper slab of the deck resists the traffic of vehicles, and the cantilevers are for the pedestrians.

2.2 Miradoiros bridge

Miradoiros Bridge has been designed as a solution for the traffic congestion problem as the existing bridge over the Ría of La Coruña (Spain) is not able to cope with the current number of vehicles. Its location is shown in figure 6. The proposed alternative is a new cable stayed bridge, upstream of the existing bridge. The projected bridge has a main span of 658 m and two secondary spans of 270 m each. The chosen deck cross-section is a 34 m wide and 3 m deep symmetric aerodynamic box girder. Special attention has been paid to the aesthetic aspects of the bridge. One of the key issues of this proposal is the envision of the bridge receiving visiting pedestrians since it will communicate two populated urban areas and the bridge surroundings will attract a number of visitors due to the scenic landscape. Its balcony zones around the towers at deck level have been dedicated to the recreational use of the structure. In fact, Miradoiros means balcony with beautiful views in Galician language.

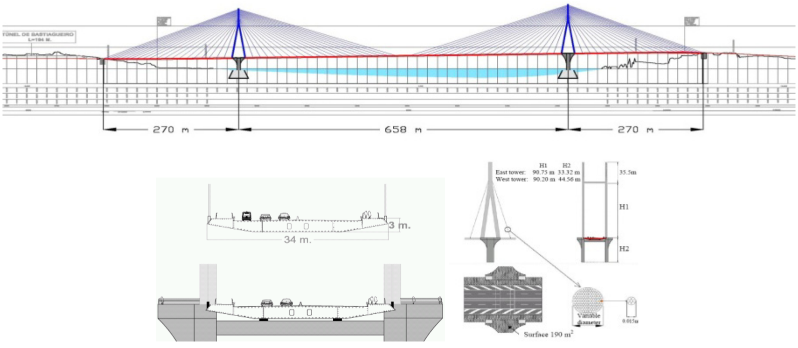

Figure 6: Miradoiros bridge.

2.3 Forth replacement crossing bridge

The third example is the proposed design of the forth replacement bridge (figure 7). The new structure will have three towers with a symmetrical arrangement resulting in two 650 m main spans and 325 m side spans. Each side span includes one additional anchor pier to provide additional stiffness. The deck superstructure is a single cell box girder with the stay cables provided in a fan arrangement and anchored along the centre of the deck. The mono-tower is the slimmest and cheapest design among other studied possibilities as diamond or H shapes. The anchorage of the cables at the centre deck line entails the bridge to have two corridors. The mono-towers and the crossing cables to stabilize the structure will make the bridge unique and instantly recognisable.

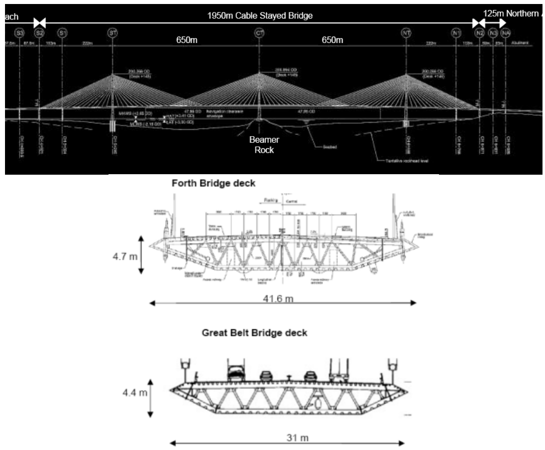

Figure 7: Forth replacement crossing bridge.

2.4 Structural models

A three-dimensional model of each cable stayed bridge structure is essential to obtain the natural frequencies and mode shapes. For these cases the structural beam elements models have been developed using ABAQUS program. Around twenty modes have been obtained in order to evaluate the multimodal flutter response. In figures 8 and 9 the structural models are shown, while and in table 1 the natural frequencies of the mode shapes are presented along with the type of displacements associated. Only the modes which have deck movements are considered in the flutter analysis.

3 Flutter derivatives and critical flutter velocity

According to the experimental method explained in the first point, a sectional model of the Talavera Bridge and another of the Miradoiros Bridge have been tested in the wind tunnel of the School of Civil Engineering of La Coruña. The flutter derivatives for the Forth Bridge deck will be estimated by the eight known values of Great Belt Bridge in Denmark which has also a single box girder of similar shape as it is shown in figure 7.

A sectional test of a reduce model of this bridge will be carried out in a near future to obtain all the functions. Figure 10 shows that Miradoiros and Great Belt have similar functions. However, the flutter derivatives of Talavera present quit high values. A possible explanation is that the top side of this deck is flat, which causes worse aerodynamic behaviour.

The flutter velocity for the Talavera Bridge is 77 m/s, a high enough value for the safety of the bridge, but low for a bridge with 315 m main span. The result for Miradoiros Bridge using 20 mode shapes is 98.11 m/s. Finally, several multimodal analyses have been carried out for the Forth Replacement Crossing Bridge considering a different number of mode shapes. The flutter speed using

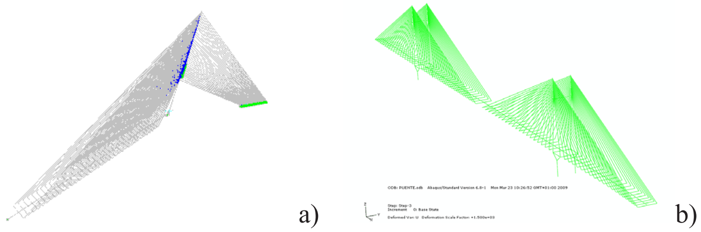

Figure 8: Structural models of a) Talavera, b) Miradoiros.

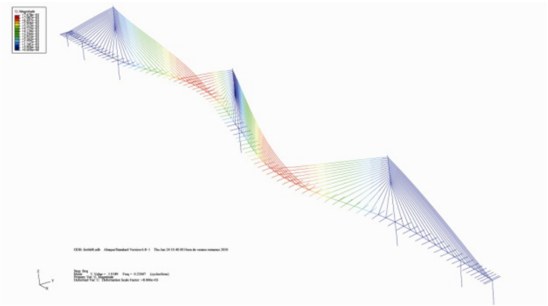

Figure 9: Structural models of the forth bridge.

Table 1: Natural frequencies and mode shapes.

Talavera			Miradoiros			Forth Replacement		
Nº	f (Hz)	Type	Nº	f (Hz)	Type	Nº	f (Hz)	Type
1	0.29	VERTICAL DECK 1	1	0.052	LATERAL DECK 1	1	0.220	VERTICAL V1A
2	0.45	VERTICAL DECK 2	2	0.153	LATERAL DECK 2	2	0.220	LATERAL 1A
3	0.59	LATERAL PYLON 1	3	0.195	LATERAL TOWER 1	3	0.268	LATERAL 1S
4	0.60	LATERAL DECK 1	4	0.196	LATERAL TOWER 2	4	0.329	VERTICAL 1S
5	0.71	VERTICAL DECK 3	5	0.213	LATERAL DECK 3	5	0.339	VERTICAL 2A
6	0.77	LATERAL PYLON 2	6	0.214	LATERAL DECK 4	6	0.484	VERTICAL 2S
7	0.91	TORSION DECK 1	7	0.225	VERTICAL DECK 1	7	0.493	VERTICAL 3A
8	1.08	VERTICAL DECK 4	8	0.280	LATERAL DECK 5	8	0.545	TORSION 1S
9	1.30	VERTICAL PYLON 1	9	0.306	VERTICAL DECK 2	9	0.550	TORSION 1A
10	1.31	LATERAL PYLON 3	10	0.361	VERTICAL DECK 3	10	0.609	VERTICAL 3S
11	1.42	LATERAL DECK 2	11	0.479	TORSION DECK 1	11	0.626	VERTICAL 4A
12	1.49	TORSION DECK 2	12	0.486	VERTICAL DECK 4	12	0.637	LATERAL-TORS 2A
13	1.57	VERTICAL DECK 5	13	0.488	TORSION DECK 2	13	0.669	VERTICAL
14	1.62	LATERAL PYLON 4	14	0.530	VERTICAL DECK 5	14	0.682	LATERAL-TORS 2S
15	2.13	VERTICAL PYLON 2	15	0.576	VERTICAL DECK 6	15	0.694	VERTICAL
16	2.14	VERTICAL DECK 6	16	0.617	VERTICAL DECK 7	16	0.702	VERTICAL
17	2.31	TORSION DECK 3	17	0.655	VERTICAL DECK 8	17	0.864	VERTICAL
18	2.33	LATERAL DECK 3	18	0.662	TORSION DECK 3	18	0.882	VERTICAL
19	2.42	VERTICAL PYLON 3	19	0.685	VERTICAL DECK 9	19	0.992	VERTICAL
20	2.76	VERTICAL DECK 7	20	0.693	LATERAL DECK 6	20	1.076	VERTICAL
			21	0.704	LATERAL TOWER 3	21	1.079	TORSION 2S
			22	0.707	LATERAL TOWER 4	22	1.091	TORSION 2A
			23	0.713	VERTICAL DECK 10			
			24	0.716	TORSION DECK 4			

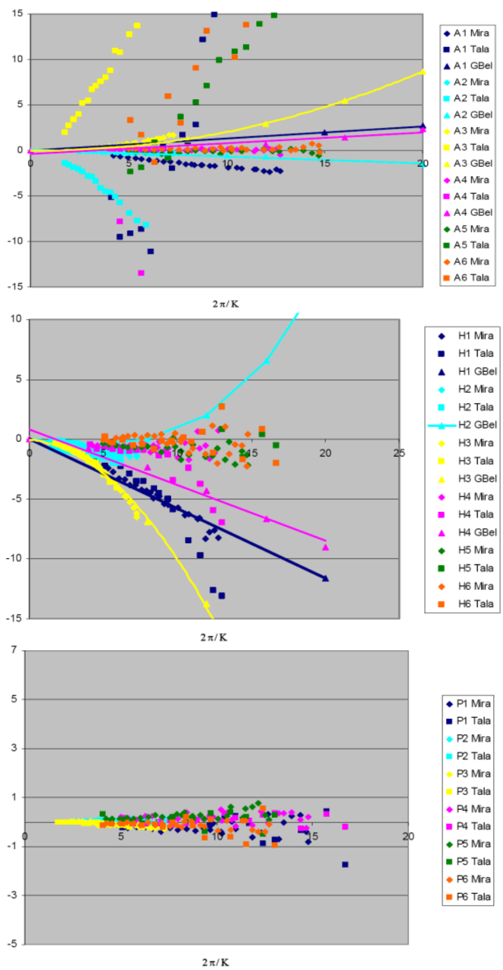

Figure 10: Flutter derivatives of Talavera bridge (Tala), Miradoiros bridge, (Mira) and Great Belt Bridge (GBel).

the first symmetric vertical mode and two first torsional modes is 134 m/s. The critical flutter velocity of the same bridge using 22 mode shapes is 135 m/s, which is very similar to the previous one. According to these results, the flutter phenomenon of this bridge is governed by the vertical and torsional symmetric modes and the flutter appears at very high velocity.

4 Conclusions

Hybrid methods facilitate the calculation of flutter velocity for long-span bridges since it only requires tests of the deck sectional model whose cost is much less than the entire bridge model tests.

The extra flutter derivatives calculated with the lateral degree of freedom in the wind tunnel do not have special influence in the flutter results. Bridges with a main span of more than one kilometre long present much more differences when 18 flutter derivatives are considered. Beside, for those cases, a multimodal analysis with a great number of mode shapes is crucial.

The cost of testing and computation costs using all the flutter derivatives and a multimodal analysis are more or less the same that of the case that does not take into these considerations.

In any case, working with a complete set of flutter derivatives and great number of modes guarantee accurate results.

The flat top part of the deck cross section of the Talavera Bridge produces worse behaviour than the other aerodynamic decks.

The flutter velocities for the three studied cable-stayed bridges are high enough for the safety of the structures.

Acknowledgements

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