

# Nonlinear one degree of freedom dynamic systems with “burst displacement characteristics” and “burst type response”

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## Abstract

The phenomenon of sudden unexpected high frequency large amplitude displacements of offshore structures has not been well understood since it was first identified in wave tank testing about two decades ago. The accelerations associated with the ringing displacements increased the loading by approximately 40%, to the despair of the designers.

We have decided to revisit the phenomena and have used system identification theory to identify mathematical models that could give such “burst” type responses. In the case of negative damping these effects appear. The model has been termed the  $\beta$ -damping term. By comparing predictions of the model with tank test data, we have found good correlations. Furthermore, the comparison has also revealed new insight into the use of the previously used models for the prediction of ringing response.

*Keywords: structural vibration, ringing, burst type response, negative damping, FNV method.*

## 1 Introduction

The problem of ringing response (sudden unexpected high frequency large amplitude displacements) of offshore structures has gained much interest since this type of response was first identified in the wave tank while testing the Draugen concrete mono tower for Northern North Sea wave conditions. Further data from offshore has confirmed that ringing response also exists in the natural



environment, although the largest displacements as seen in the wave tank have not been reported offshore.

Although Faltinsen et al. [1] have proposed a model that gives the ringing response of offshore structures exposed to waves, using the approach of hydrodynamics to identify physical loading terms in steep waves that could cause ringing, there has been a continued search for additional mechanisms that could lead to ringing response.

In this paper we denote the type of response mentioned above for “burst type response”. We report a new approach to identify burst type response: We have studied the displacement response of a single degree slender offshore structure exposed to harmonic loading and have investigated different damping models for the oscillator. The analysis has been purely analytical while applying a Morison type of wave loading including both drag (viscous) and inertia (including added mass) terms. Theoretical modelling of the mathematical problem has given us “burst type” of displacement motions in certain wave conditions in the case of negative damping.

Based on this, we have searched for a physical explanation to the phenomena and identified that a relative velocity formulation of the drag forcing term will lead to said negative damping under certain conditions that are discussed in the paper. The formalism as proposed by Gudmestad and Connor [2] will be referred to when explaining the physics of the mathematical expressions. In order to investigate the goodness of different possible models that could describe the burst type behaviour of offshore structures, we have modelled the Draugen concrete mono tower platform as a one degree of freedom oscillator and applied different loading models:

- the *FNV model* as described by Faltinsen et al. [1];
- the *FNV model with viscous terms* incorporated;
- a “ *$\beta$  damping model*” as described by the above.

The response to the physical model of the Draugen platform in waves as generated in the Marintek wave tank in Trondheim has then been calculated and compared with the actual wave tank measurements of the platform displacement. The results of the analysis have confirmed the goodness of the FNV model and documented that the viscous effects should be implemented. Furthermore, the displacements found by using the  $\beta$  damping model have also proven to give results close to the wave tank measurements, a result that may indicate that negative damping terms might also be incorporated in an extended FNV model.

We underline that we have obtained what we set out to investigate: we have found additional analytical and physical mechanisms that may generate “burst type” response effects. In view of this we will express optimism that we can better understand the ringing phenomena.

## 2 The physical problem: ringing response of offshore structures

The *response* of single degree of freedom oscillators subject to wave and current loadings has been extensively studied, both theoretically and numerically over the years.



For a Morison type of wave loading (Morison et al. [3]), linearization of the *drag forcing term* was discussed by Gudmestad and Connor [2] and the effects of considering different deterministic wave theories and integrating the total loading up to the *free surface* were discussed by Gudmestad and Pombouras [4]. An attempt to extend the analysis of the effects of *wave kinematics* to irregular seas was suggested by Gudmestad [5]. Later, a number of attempts have been suggested to improve on the understanding of wave actions on slender offshore structures, see for example Skjelbreia et al. [6], Stansberg and Gudmestad [7] and Grue et al. [8] regarding wave kinematics.

The response side has been extensively studied in view of so called “*ringing response*” of structures in the sea, Stansberg [9], Jeffreys and Rainey [10], Faltinsen et al. [1] and Krogstad et al. [11]. This ringing response is a transient response that seems to be caused by an impulse type of loading (see for example Kvitrud [12] on structures subjected to either drag forcing terms associated with nonlinear kinematics, free surface effects or quadratic forcing terms (Lighthill [13]).

It has been suggested that ringing mainly occurs in steep waves, see for example Chaplin et al. [14] and Welch et al. [15]. These waves would give a type of loading resembling an impact. Reference is also made to Gurley and Kareem [16], from which Figure 1(a) is copied. Figures 1(b) show a “ringing like response” of the Kvitebjørn jacket installed in the North Sea at 190 m water depth. This is a slender offshore jacket responding dynamically to wave loading. It is, however, agreed that the ringing phenomenon is not well understood and standards, for example Norsok Standard N-003, [17], calls for tank testing whenever ringing may be a feature in the design of large volume offshore structures. For structures dominated by drag type loading, careful dynamic analysis in the time domain is required.

### 3 Equations of motions and forcing function

The general second order ordinary differential equation for the horizontal response  $y(t)$  of a one degree of freedom slender offshore structure when subjected to constant nonlinear *drag loading* (that is *loading generated by the velocity  $U$  of the current*) is according to experiments given by the term  $\frac{1}{2}\rho C'_d DU|U|$  per unit length of the structure; see for example Sarpkaya and Issacsoson [18]:

$$m \frac{d^2 y(t)}{dt^2} + c \frac{dy(t)}{dt} + ky(t) = \frac{1}{2} \rho C'_d DU|U| \quad (1)$$

Here:

- $m$  is the mass of the structure
- $k$  is the linear stiffness of the structure
- $D$  is the diameter of the slender structure
- $\rho$  is the density of the fluid (water)
- $C'_d$  is the drag coefficient for the flow



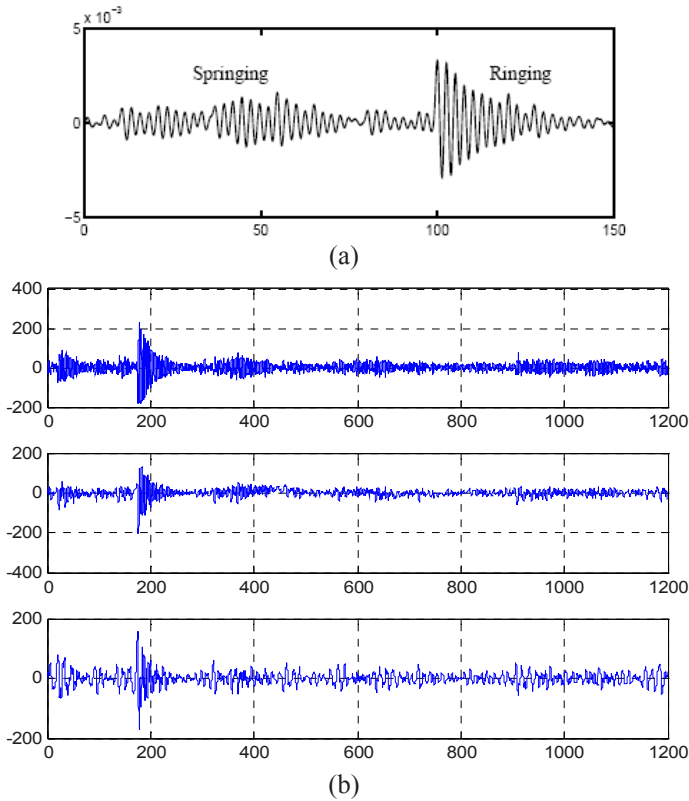


Figure 1: (a) Conceptual diagram showing ringing (and springing) response (in m) as a function of time for an offshore system under viscous load, from Gurley and Kareem [16]. Note that springing is a response phenomenon having less amplitude as compared to ringing. Springing could cause fatigue of members affected by this type of response. (b) Time history of ringing-like response of the Kvitebjørn jacket installed in the North Sea at 190 m water depth. The figure shows from top to bottom the North-South accelerations ( $\text{mm/s}^2$ ), velocities ( $\text{mm/s}$ ) and displacements ( $\text{mm}$ ) as function of time (s) for an event recorded on 01.01.2004 at 2 p.m.

- $U$  is the velocity of the constant flow (current) past the structure.
- $|U|$  Represents the absolute value of the velocity  $U$

This drag type loading is in general attributed to the shedding of vortices in the downstream flow direction of the current. The structural damping is associated with the constant  $c$ , and critical damping is obtained for the case

$$\lambda = \frac{c}{2m\omega} = 1 \text{ while the natural frequency of the non-damped motion is given}$$

by  $\omega = \sqrt{\frac{k}{m}}$ . As is well known in structural analysis, the damping term changes the

natural frequency of motion to  $\omega' = \omega\sqrt{1 - \lambda^2}$

In the case of a combined wave and current loading, the nonlinear drag loading is according to Morison's postulate (Morison et al. [3]) for slender structures ( $D/L \leq 0.2$ , where  $L$  is the wave length) given by:

$$m \frac{d^2 y(t)}{dt^2} + c \frac{dy(t)}{dt} + ky(t) = \frac{1}{2} \rho C_d D \left\{ \frac{du(t)}{dt} + U \right\} \left| \frac{du(t)}{dt} + U \right| \quad (2)$$

Here:

- $u(t)$  is the displacement of the oscillating flow
- $du(t)/dt$  is the velocity of the oscillating flow
- $C_d$  is the modified drag coefficient for the combined flow.  $C_d$  exhibits a significant variation with Reynolds number ( $Re$ ), Keulegan-Carpenter number ( $K$ ) and relative roughness ( $k/D$ ). In this paper we will treat  $C_d$  as a constant.

For the selection of values for  $C_d$ , in accordance with international recommendations, see e.g. Gudmestad and Moe [19]. It should be noted that the influence of the displacement of the structure on the flow is not accounted for in this analysis, see for example Gudmestad and Connor [2] for a discussion of the "relative velocity effects".

In addition we have to include the mass force term (also denoted the inertia term)  $\rho C_m \frac{\pi}{4} D^2 \frac{d^2 u(t)}{dt^2}$ , which for slender structures may be omitted as the drag term is dominating, see for example Sarpkaya and Isaacson [18] for criteria to be fulfilled to omit the mass forcing term from the analysis. It should be noted that the mass term is a linear term proportional to  $du(t)/dt$  and that this term would cause traditional dynamic amplification of the response at the resonance frequency, i.e. when  $\omega = \Omega$ . Further resonances will be triggered when integrating the force contributions to the free surface of the wave(s).

Additional nonlinear loading terms have been suggested, by for example Faltinsen et al. [1] and Newman [23]; the FNV method. These terms are thought to account for the ringing like response of structures and the FNV method represents to day's state of art with respect to understanding ringing response.

In addition to including current in the loading term of the equation of motion, we should also include the effect of the motion of the structure itself on the forcing term, that is, we should consider relative acceleration and velocity terms in (2) as was suggested by Gudmestad and Connor [2]. Equation (2) (without current) would then read:

$$m \frac{d^2 y(t)}{dt^2} + c \frac{dy(t)}{dt} + ky(t) = \frac{1}{2} \rho C_d D \left\{ \frac{du(t)}{dt} - \frac{dy(t)}{dt} \right\} \left| \frac{du(t)}{dt} - \frac{dy(t)}{dt} \right| + \rho (C_m - 1) \frac{\pi}{4} D^2 \left( \frac{d^2 u(t)}{dt^2} - \frac{d^2 y(t)}{dt^2} \right) + \rho \frac{\pi}{4} D^2 \frac{d^2 u(t)}{dt^2} \quad (3)$$



Note that  $u(t)$  here refers to the water particle displacement while  $y(t)$  refers to the displacement of the structure. The right hand side of equation (3) represents the forcing term, taking the relative motion of the structure into account. We will in the further discussion refer repeatedly to this equation.

#### 4 A system identification approach to identifying an equation of motion with “burst type” response

In order to identify systems with “burst type response”, our strategy has been to construct a vector field (time-dependent) in the plane with some of the same properties as observed in Figure 1(a). The work has been reported by Jonassen [20]. We will define three regions  $U_1$ ,  $U_2$  and  $U_3$  in  $\mathbf{R}^2$ , where  $U_1$  is a disk centred in the origin with radius  $r_1$ , the set  $U_2$  is an annulus centred on the origin with inner radius  $r_1$  and outer radius  $r_2$  and the set  $U_3$  is an annulus centred on the origin with inner radius  $r_2$  and outer radius  $r_3$ . Here we have  $0 < r_1 < r_2 < r_3$ . We will call  $U_1$  the fixed-point region,  $U_2$  the gluing region, where bump functions are used to smoothly transform the oscillator in  $U_1$  to the oscillator in  $U_3$ , and  $U_3$  the outer region.

Our aim is to construct a vector field in  $U_1$  with the following (geometrical) properties:

- The vector field should model an oscillator.
- The non-forced field should have a fixed point in the origin.
- The fixed point should periodically or almost periodically change stability, from an attracting fixed point to a repelling fixed point.
- The model should be as simple as possible.

Clearly, there are a large number of candidates for such models, both among linear and nonlinear oscillators. To meet our last aim we start with a linear oscillator of the form:

$$\frac{d^2 x(t)}{dt^2} + a(t) \frac{dx(t)}{dt} + \omega^2 x(t) = b(t) \quad (4)$$

where  $a(t)$  is a periodic or almost periodic function with  $-1 \leq a(t) \leq 1$  and  $\min a(t) = -1$  and  $\max a(t) = 1$ . This choice will clearly fulfil our other aims too ( $b(t)=0$  represents the non-forced field). For simplicity we may choose  $a(t)$  periodic, for example:

$$a(t) = \sin\left(\frac{t}{10}\right) \quad (5)$$

Implying that the stability of the origin is slowly changing compared with a timescale of  $t$  of order 1. We remark that even with this simple equation, one cannot find the exact solution in a closed form, that is, one has to give the solution as a power series in  $t$ . And in order to mimic the model given in Equation (3) we will choose

$$b(t) = \sin(kt) |\sin(kt)| \quad (6)$$



In our numerical simulation in Figures 2(a) and (b), we have used  $k = 5$  and  $\omega = 1$ . Figure 2(a) shows the local behaviour of the model in the extended region of  $U_1$ . Only the  $x$  coordinate is shown here for  $0 \leq t \leq 200$ . Note that this model has “bursts” of the trajectory into the regions  $U_2$  and  $U_3$  if the numbers  $r_i$ ,  $i = 1, 2, 3$  are chosen properly. In region  $U_3$  the full model is chosen to be a Duffing type oscillator, and the bump functions are polynomials. Figure 2(b) shows the full orbit in the phase space. For further discussion on the details of the modelling, reference is made to [20].

## 5 On reanalysis of model tank test data

The numerical prediction of nonlinearly generated, burst-like resonant oscillations corresponding to “ringing” on vertical columns of offshore platforms in steep waves has been addressed in an introductory study by Marintek [21]. Two basically different approaches were investigated and compared through simple models:

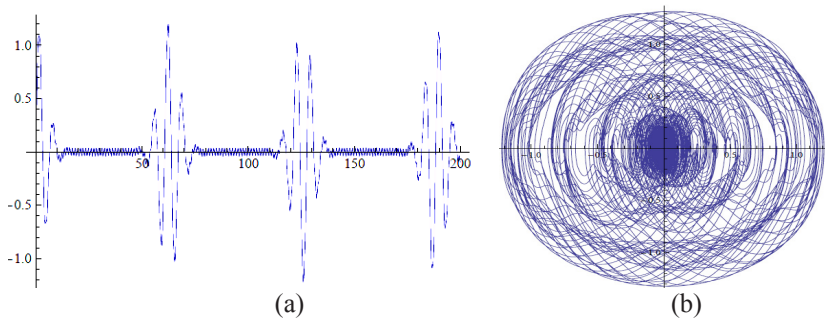


Figure 2: (a) A typical behaviour in the fixed-point region, the  $x$  coordinates only. (b) A typical behaviour in the fixed-point region, here the trajectory in the phase space region.

- Higher-order wave loads in steep waves exciting a linear dynamic system using the FNV load model, and
- Linear excitation combined with a nonlinear, time-varying damping model where the damping factor is correlated with the excitation signal.

The models were also compared to linear modelling and to model test data on a mono tower offshore platform. The model test data used for comparison were made available, by permission from Shell, from the Draugen mono tower model tests carried out in the Marintek Ocean Basin in 1992 [22]. The results showed that both models were capable of generating “burst events” similar to ringing and dynamic characteristics of the events were comparable to those from the measurements. The findings represent a verification of the FNV load model, as well as a confirmation that also a nonlinear damping model can lead to almost the same dynamic events. A one-degree-of-freedom oscillator described by the equation of motion was considered:

$$\frac{d^2 y(t)}{dt^2} + b(t) \frac{dy(t)}{dt} + \omega_0^2 y(t) = f_e(t) \quad (7)$$

Here:

- $f_e(t)$  is the external, time-varying excitation load (a hydrodynamic wave force),
- $\omega_0 \equiv \sqrt{k/m}$  is the natural angular frequency;  $m$  is the oscillator mass,  $K$  is the stiffness,
- $b(t) \equiv b_0 + \Delta b(t)$  is the damping “constant” consisting of one linear term  $b_0$  and one time-varying term  $\Delta b(t)$ .
- All the terms are normalized by dividing with the oscillator’s mass  $m$ .
- The stiffness  $k$  is assumed to be constant, which gives a linear restoring spring force.

Three special versions of the **force formulation** were investigated here:

- *Linear*: The inertia term in Eq. (7) was integrated up to the mean water surface, and only 1<sup>st</sup> order integrated force terms were included. The drag term was excluded.
- *FNV (linear + 3<sup>rd</sup> order)* [1, 23]: The inertia term in Eq. (7) was integrated up to the linear instantaneous wave elevation surface, and third order integrated slender body force terms were included. The drag term was excluded.
- *FNV + viscous*: As FNV above, but also including the drag term.

A second-order inertia force is also present in the FNV formulation [1, 23], while for simplicity here we ignore that term as it has been found previously that the “ringing” load phenomena are mainly connected with the third-order term [11].

We assumed that the mass velocity  $dy(t)/dt$  was small compared to the water velocity  $du(t)/dt$ . We also assumed that in a linear formulation, the cylinder is slender, so that the inertia coefficient,  $C_M$ , can be assumed to be constant and equal to 2.0. For the **damping formulation**, two different types were considered:

- Linear damping,  $\Delta b(t) = 0$
- Nonlinear time-varying damping, with  $\Delta b(t) = \beta \cdot f_e(t)$  (“ $\beta$ -model”), i.e. the nonlinear damping term was assumed to be proportional to the excitation force

For a given level of the relative nonlinear contribution to the dynamic response, the absolute value of the parameter  $\beta$  will depend on the actual motion / velocity level, so it will be different for different systems. In possible future work, one should establish a “normalized” parameter that will not depend on the actual system. Hence, the following four specific combinations have been investigated:

1. A linear wave force  $f_e(t)$  and a linear damping, i.e.  $\beta = 0$
2. A nonlinear potential wave force (FNV model) and a linear damping
3. As 2) but with a viscous wave force added (i.e. incl. the drag term in Morison' Eq.)
4. A linear wave force with a nonlinear damping ( $\beta \neq 0$ ). It should be noted that the Non-Linear damping term could become negative using this model



## 6 Case study with comparison to model test response data

The four different systems described above were applied on a specific physical case. The excitation force is modelled as the integrated wave force on a cylindrical vertical column with radius  $R$ . The water depth  $h$  corresponds to deep water, and the draft  $d$  of the column is equal to  $d$  (or more correctly: ranging over the whole wave zone water column). The column is assumed to oscillate horizontally in the wave direction (one degree of freedom), with a stiffness  $K$ , the total oscillator mass is  $m$ , and the resulting natural period is  $T_0$ . The relative damping level corresponding to the linear damping constant  $b_0$  is  $\kappa$ , and  $C_D$  denotes the drag coefficient of the column.

The system parameter values are chosen as follows:

$R = 8.2\text{m}$  (as for the Draugen mono tower at the waterline [5])

$d = h = 330\text{m}$  (as for the Draugen mono tower)

$K = 154.6\text{MN/m}$  (Draugen)

$m = 100 \cdot 10^6 \text{ kg}$

$T_0 = 5.0\text{s}$  (Draugen [22])

$\kappa = 1.5\%$  (roughly similar to the Draugen model tests)

$C_D = 1.0$ ,  $\beta = 8$

Parameters:  $H_s = 15.5\text{m}$ ,  $T_p = 17.8\text{s}$  (Ultimate Limit State (ULS) condition, no current)

Mathematically, with respect to the excitation and dynamic oscillator system behaviour, this corresponds to the top motions of the Draugen Gravity Base Structure (GBS) mono tower, installed in the Norwegian Sea and tested in scale 1:50 in Marintek's Ocean Basin in 1992 [22]. Thus a comparison to a selected Ultimate Limit State irregular wave test run from the Draugen model test data forms is presented below.

Physically, there is a clear difference between the two set-ups, since the Draugen tower is bottom mounted; its radius increases downwards, and it is dynamically flexible over the full height, while our model is a simplified one-degree-of-freedom oscillator. It should be noted, however, that the purpose of this exercise was simply to see whether or not our simple model is capable of reproducing signals with characteristics similar to those measured, and not to reproduce the model test data in detail.

From the measured wave, a linear wave estimate was extracted according to the procedure in [24]. Simulated oscillator motion responses, were generated. One selected event of the oscillator motion is shown in Figure 3(a). The results are shown in full scale. The corresponding response record is in Figure 3(b). As pointed out earlier, a detailed quantitative comparison cannot be made between the simulations and the measurements, since the physical set-ups are slightly different. But characteristics of the signals, as well as approximate motion values, can be compared.

### 6.1 Discussion regarding the comparison with measured data

From Figures 3(a) and (b) we see that both the FNV load model and the  $\beta$  damping model are clearly capable of generating nonlinear "burst-like" resonant

events similar to ringing. While the ringing events modelled by the FNV model can be reasonably well explained as a physical phenomenon, the results from the  $\beta$ -model will at the present stage be considered as the results from a mathematical exercise based on the Morison equation for load on drag dominated structures [2].

With the actual system parameters used, extreme motion responses are amplified typically by 30%-50%, compared to linear estimates. This has been seen in simulations both with the numerical wave and with the measured Draugen wave.

The values of the damping time-varying parameter  $\beta$  have been chosen empirically, with the purpose to obtain response characteristics that produce “ringing” events in steep waves, and that approximately look similar to the FNV results. With even more tuning and slightly different choices of  $\beta$ , an even closer agreement might have been achieved, while this has not been the ultimate goal of this study, where we are mainly interested in whether or not qualitatively similar characteristics can be obtained.

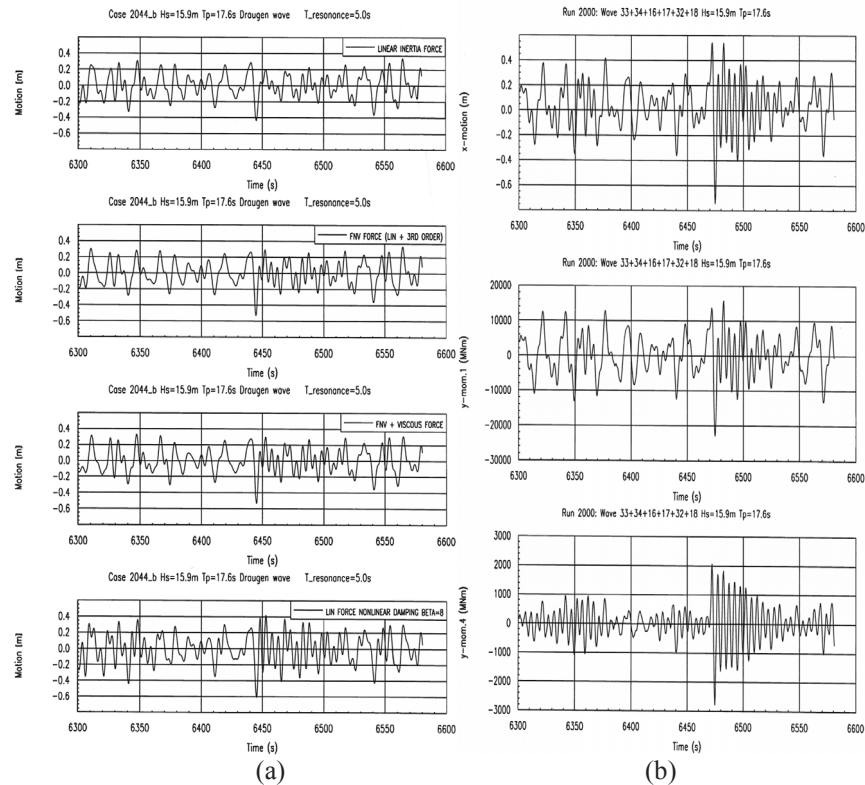


Figure 3: (a) Simulated response using the four system models, Draugen model test. (b) Measured top motion and moment responses from Draugen model tests, same event as in (a). (Note that there is a skewness in the time scale as this has not yet been fully been adjusted in this figure).



It is seen that both the FNV model and the  $\beta$ -model reproduce the characteristics of the “burst-like” behaviour of most of the different measured Draugen events, with a reasonable agreement. This represents an important verification of the FNV load model, as well as a confirmation that the nonlinear damping model is in fact capable of generating the observed nonlinear dynamic events with a good similarity. There are some deviations, which should be expected since the models are quite simplified and cannot be expected to reproduce all complex details. High-frequency oscillations appear to be slightly higher in the measurements than predicted from our present FNV load model. The two numerical models produce slightly different behaviours, depending on the actual event. As an overall observation, however, we consider that taking into account the simplifications made, the observed agreement is in fact quite good.

## 7 Similarities between the $\beta$ damping model [21] and the equation of motion (3) taking the relative motion into account

Let us consider the nonlinear relative velocity drag forcing term of Equation (3):

$$\frac{1}{2} \rho C_d D \left\{ \frac{du(t)}{dt} - \frac{dy(t)}{dt} \right\} \left| \frac{du(t)}{dt} - \frac{dy(t)}{dt} \right| \quad (8)$$

Let us then denote  $\frac{du(t)}{dt} = \dot{u}$  and  $\frac{dy(t)}{dt} = \dot{y}$ , then:

$$\{\dot{u} - \dot{y}\} \left| \dot{u} - \dot{y} \right| = \left| \dot{u} - \dot{y} \right| \dot{u} - \left| \dot{u} - \dot{y} \right| \dot{y} \quad (9)$$

Here  $\frac{dy(t)}{dt} = \dot{y}$  is the velocity of the structural response; in general this is an oscillatory function.

There are two cases:

$$1) \{\dot{u} - \dot{y}\} \geq 0, \text{ then: } \left| \dot{u} - \dot{y} \right| \dot{u} - \left| \dot{u} - \dot{y} \right| \dot{y} = \dot{u}^2 - \dot{u}\dot{y} - \dot{u}\dot{y} + \dot{y}^2 \approx \dot{u}^2 - 2\dot{u}\dot{y} \quad (10)$$

$$2) \{\dot{u} - \dot{y}\} \leq 0 \text{ then: } \left| \dot{u} - \dot{y} \right| \dot{u} - \left| \dot{u} - \dot{y} \right| \dot{y} = -\dot{u}^2 + \dot{u}\dot{y} + \dot{u}\dot{y} - \dot{y}^2 \approx -\dot{u}^2 + 2\dot{u}\dot{y} \quad (11)$$

For both cases we neglect the squared term of the velocity of the structural response. From (10) and (11), the terms proportional to  $2\dot{u}\dot{y}$  represent oscillatory damping and can be transferred to the left hand side of the equation of motion. Thus, the damping term becomes of the form  $b(t) \equiv b_0 + \Delta b(t)$  consisting of one linear term  $b_0$  and one time-varying term  $\Delta b(t)$  as was discussed previously. The damping term may become negative as follows:

Case 1)  $\{u - y\} \geq 0$  and  $\dot{u} \leq 0$  that will be satisfied in case  $\dot{u} \leq 0$  and  $\dot{y} \leq \dot{u} \leq 0$ . Such a situation could arise on the downward-sloping side (backside) of the wave (where:  $\dot{u} \leq 0$ ) when the structural motion has a negative velocity  $\dot{y} \leq \dot{u} \leq 0$ .

Case 2)  $\{u - y\} \leq 0$  and  $\dot{u} \geq 0$  that will be satisfied in case  $\dot{u} \geq 0$  and  $\dot{y} \geq \dot{u} \geq 0$ . Such a situation could arise on the upward-sloping side (front) of the wave (where:  $\dot{u} \geq 0$ ) when the structural motion has a positive velocity  $\dot{y} \geq \dot{u} \geq 0$ .

In case we consider the relative velocity terms of the Morison equation, we see from this that it would be possible to obtain situations where negative damping could occur. The “burst” type of response would be initiated for relatively small values of the water particle velocity, i.e. for small values of  $u$ , and such situations would occur when the wave crosses the waterline. In this position the water particle acceleration is at maximum and the maximum, whereby the effect surely could be interpreted as being caused by the inertia term of the loading.

## 8 Conclusions and suggestions for further work

We have presented a discussion of “ringing response” and have through a system identification approach identified a set of second order nonlinear ordinary differential equations that may give burst type responses. The key to obtaining such response is the occurrence of a damping term that might become negative. With the application of a constant structural damping term plus an oscillatory damping term proportional to the total force on the model, burst type of response that very closely match the response of a physical model in the wave tank has been documented. The new damping model might represent an additional explanation of ringing response in addition to the FNV model [1] that represents the “state of art” in explaining such response.

Further work is recommended to explore the physical properties of the nonlinear phenomena in more detail. Further work to understand the  $\beta$ -model discussed above needs to be carried out, where also time series simulations (based on the Morison formula for load on drag dominated structures) are carried out to investigate how the model is able to generate theoretical bursts similar to those from ringing. In a follow-up, a review of the earlier work in [2] will be useful with attention to implementation of all relevant load contributors. Also the FNV load implementation should be further checked against direct load model test data such as those from [9].

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author was an employee of the company. Shell is furthermore thanked for providing access to the Draugen data that were used to verify the models suggested in the paper.

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