Hydroelastic vibration of a rectangular perforated plate with a simply supported boundary condition

K.-H. Jeong, G.-M. Lee, T.-W. Kim & J.-I. Kim
Mechanical Engineering Division,
Korea Atomic Energy Research Institute, Republic of Korea

Abstract

A theoretical study on the natural frequencies and the mode shapes of a perforated plate in contact with an ideal liquid is presented. In the theory, it is assumed that the plate is simply supported along the edges and the liquid is in contact with the lower surface of the plate. The identical square holes in the plate with a square array are considered. A compatibility requirement on the contact surface between the plate and the liquid is applied for the liquid–structure interaction and the Rayleigh–Ritz method is used to calculate the eigenvalues of the system. The proposed theoretical method for the plate coupled with the liquid is verified by observing a good agreement with the three-dimensional finite element analysis result.
Keywords: hydroelastic vibration, perforated plate, liquid–contacting, Rayleigh–Ritz method, fluid-structure interaction, natural frequency, rectangular holes.

1 Introduction

The perforated plates with a number of holes are used in the commercial nuclear power plants. However, it is very difficult to estimate dynamic characteristics of such a perforated plate. Moreover, the dynamic behavior of the plates in contact with a fluid is very complicated due to the fluid–structure interaction. The powerful numerical tools such as FEM (finite element method) or BEM (boundary element method) make approximate solutions to a simple fluid–structure interaction problem possible. However the use of these methods in the perforated structures requires enormous amounts of time for modeling and computation. The previous study [1] on the perforated plates focused on the

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stress distribution and deformation based on the effective elastic constants such as effective Young's modulus and effective Poisson's ratio. A finite element analysis on the perforated plate with several perforation arrays was carried out to find the modal resonance frequencies [2]. An experimental study was also performed for free vibration of rectangular perforated plates with clamped boundary condition, based on the simple deformation theory and averaged elastic constants [3]. Recently, the dynamic characteristics of submerged plates in water were studied by an experiment and finite element analyses [4]. Jeong and Amabili [5] theoretically obtained the natural frequencies and mode shapes of perforated beams in contact with water. In the theory several types of hole were considered. In this paper, a theoretical formulation will be developed for a free vibration of the rectangular perforated plate. First of all, this study will cover the dynamic properties of the perforated plate in air. So, an analytical method based on the Rayleigh–Ritz method is suggested to estimate the natural frequencies of the perforated plate in air. Finally, the method is extended to the plate in contact with water. In the theoretical formulation, the simply supported boundary condition is assumed for all edges; nevertheless, the theory can be extended to any arbitrary classical boundary condition with an additional formulation. The analytical method will be verified by the three-dimensional finite element analysis.

2 Theoretical formulation

2.1 Rayleigh–Ritz method for a perforated plate in air

A rectangular plate having square identical holes with the same interval is illustrated in Figure 1. The plate with a simply supported boundary condition along all edges has the width *L*, length *H*, thickness *h*, width of square holes *a*. The width of a square cell in the plate is indicated by *d*.

The Rayleigh–Ritz method is introduced to find the natural frequencies and mode shapes of the perforated plate in air or in contact with a liquid. Therefore, each mode shape can be approximated by combination of a finite number of admissible functions and appropriate unknown coefficients, q_{mn}. The dynamic displacement, *w* can be assumed in the form of

$$w(x,y,t) = \sum_{m=1}^{R} \sum_{n=1}^{R} q_{mn} W_{mn}(x,y) \exp(\mathrm{i}\,\omega t) = \sum_{m=1}^{R} \sum_{n=1}^{R} q_{mn} U_m(x) V_n(y) \exp(\mathrm{i}\,\omega t), \tag{1}$$

where, $\mathrm{i} = \sqrt{-1}$, ω is the circular natural frequency of the plate and *R* is the number of expanding terms. When we consider the geometric and natural boundary conditions for the *x*- and the *y*-directions, the bending moment and the displacement at all the support ends must be zero simultaneously for the simply supported boundary conditions. Thus the transverse modal function for the bending vibration can be defined by a multiplication of the admissible functions.

$$W_{mn}(x,y) = U_m(x) V_n(y) = \sin(\alpha_m x) \sin(\beta_n y), \tag{2}$$

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where

$$\alpha_m = m\pi / L, \qquad \beta_n = n\pi / H. \tag{3}$$

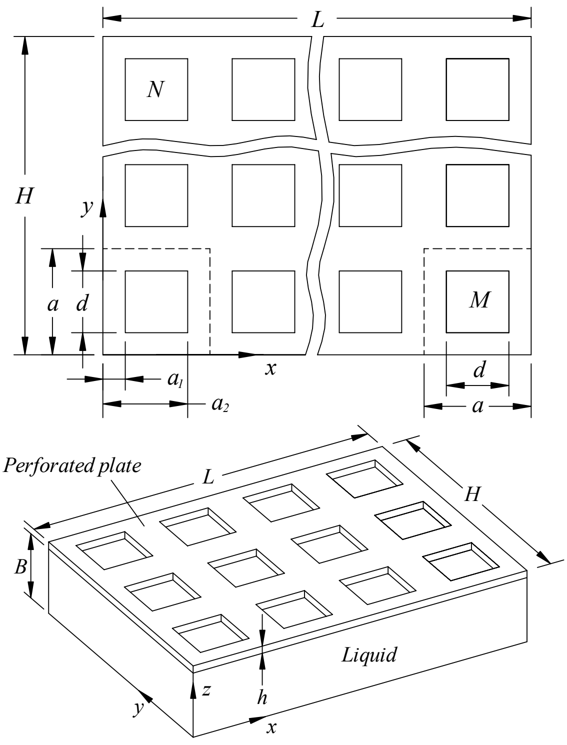

Figure 1: A liquid-contacting perforated rectangular plate with square holes.

Now, it is necessary to establish the reference kinetic energy of the plate to calculate the natural frequencies of the rectangular perforated plate in air. The net reference kinetic energy of the plate can be obtained by a subtraction of the reference kinetic energy of the punched holes from the reference kinetic energy of the solid plate by introducing a vector $\boldsymbol{q}$ of the unknown parameters;

$$\boldsymbol{q} = \left\{ q_1 \quad q_2 \quad q_3 \quad \cdots \quad q_{(R\times R)} \right\}^T, \tag{4}$$

$$T_p^* = \frac{\rho h}{2} \boldsymbol{q}^T \boldsymbol{Z} \boldsymbol{q}, \tag{5}$$

where $\boldsymbol{Z}$ is a $R^2 \times R^2$ non-diagonal matrix and is given as,

$$Z_{ik} = \int_0^L \int_0^H W_i W_k \, dy \, dx - \sum_{s=0}^{N-1} \sum_{r=0}^{M-1} \int_{ra+a_1}^{ra+a_2} \int_{sa+a_1}^{sa+a_2} W_i W_k \, dx \, dy \,. \tag{6}$$

Inserting the modal functions of eqns. (2) and (3) into eqn. (6), the matrix will be rewritten

$$Z_{ik} = \int_0^L U_i U_k \, dx \int_0^H V_i V_k \, dy - \sum_{s=0}^{M-1} \int_{ra+a_1}^{ra+a_2} U_i U_k \, dx \sum_{r=0}^{N-1} \int_{sa+a_1}^{sa+a_2} V_i V_k \, dy \tag{7}$$

The maximum potential energy of the perfotated plate also can be computed by integrations of the derivatives of the admissible modal functions as shown

$$V_p = \frac{D}{2}\int_0^H \int_0^L \boldsymbol{q}^T \left(\begin{aligned} & \frac{\partial^2 \boldsymbol{W}^T}{\partial x^2}\frac{\partial^2 \boldsymbol{W}}{\partial x^2} + \frac{\partial^2 \boldsymbol{W}^T}{\partial y^2}\frac{\partial^2 \boldsymbol{W}}{\partial y^2} \\ & +2\mu \frac{\partial^2 \boldsymbol{W}^T}{\partial x^2}\frac{\partial^2 \boldsymbol{W}}{\partial y^2} + 2(1-\mu)\frac{\partial^2 \boldsymbol{W}^T}{\partial x \partial y}\frac{\partial^2 \boldsymbol{W}}{\partial x \partial y} \end{aligned} \right) \boldsymbol{q} \, dx \, dy$$

$$-\frac{D}{2}\sum_{s=0}^{M-1}\sum_{r=0}^{N-1}\int_{ra+a_1}^{ra+a_2}\int_{sa+a_1}^{sa+a_2} \boldsymbol{q}^T \left(\begin{aligned} & \frac{\partial^2 \boldsymbol{W}^T}{\partial x^2}\frac{\partial^2 \boldsymbol{W}}{\partial x^2} + \frac{\partial^2 \boldsymbol{W}^T}{\partial y^2}\frac{\partial^2 \boldsymbol{W}}{\partial y^2} \\ & +2\mu \frac{\partial^2 \boldsymbol{W}^T}{\partial x^2}\frac{\partial^2 \boldsymbol{W}}{\partial y^2} + 2(1-\mu)\frac{\partial^2 \boldsymbol{W}^T}{\partial x \partial y}\frac{\partial^2 \boldsymbol{W}}{\partial x \partial y} \end{aligned} \right) \boldsymbol{q} \, dx \, dy$$

$$= \frac{D}{2}\boldsymbol{q}^T \left[\boldsymbol{\Gamma 1} - \sum_{s=0}^{M-1}\sum_{r=0}^{N-1} \boldsymbol{\Gamma 2} \right] \boldsymbol{q} \tag{8}$$

where $\boldsymbol{\Gamma 1}$ and $\boldsymbol{\Gamma 2}$ are also a derived $R^2 \times R^2$ matrix obtained by the integrations term by term and D is the flexural rigidity of the solid plate.

$$\begin{aligned} \Gamma 1_{ik} &= \int_0^L U_i'' U_k'' \, dx \int_0^H V_i V_k \, dy + \int_0^L U_i U_k \, dx \int_0^H V_i'' V_k'' \, dy \\ &+ 2\mu \int_0^L U_i'' U_k \, dx \int_0^H V_i V_k'' \, dy + 2(1-\mu)\int_0^L U_i' U_k' \, dx \int_0^H V_i' V_k' \, dy \end{aligned} \tag{9}$$

$$\begin{aligned} \Gamma 2_{ik} &= \int_{sa+a_1}^{sa+a_2} U_i'' U_k'' \, dx \int_{ra+a_1}^{ra+a_2} V_i V_k \, dy + \int_{sa+a_1}^{sa+a_2} U_i U_k \, dx \int_{ra+a_1}^{ra+a_2} V_i'' V_k'' \, dy \\ &+ 2\mu \int_{sa+a_1}^{sa+a_2} U_i'' U_k \, dx \int_{ra+a_1}^{ra+a_2} V_i V_k'' \, dy \\ &+ 2(1-\mu)\int_{sa+a_1}^{sa+a_2} U_i' U_k' \, dx \int_{ra+a_1}^{ra+a_2} V_i' V_k' \, dy \end{aligned} \tag{10}$$

The relationship between the reference kinetic energy of each mode multiplied by its square circular frequency and the maximum potential energy of the same mode is used to establish the natural frequencies of the perforated plate in air. The Rayleigh quotient for the perforated rectangular plate vibration in air is given by V_p / T_p^*. Minimizing the Rayleigh quotient with respect to the unknown parameters $\boldsymbol{q}$, the non-dimensional Galerkin equation can be obtained.

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$$D\left[\boldsymbol{\Gamma 1}-\sum_{s=0}^{M-1}\sum_{r=0}^{N-1}\boldsymbol{\Gamma 2}\right]\boldsymbol{q}-\omega^{2}\rho\,h\,\boldsymbol{Z}\,\boldsymbol{q}=\{0\}\cdot \tag{11}$$

Eqn. (11) gives an eigenvalue problem and the natural frequency ω in air can be calculated.

2.2 Solution for a perforated plate in contact with a liquid

The oscillatory motion of an ideal liquid in contact with a perforated plate can be described by using the velocity potential. It is assumed that the liquid is bounded at the lower bottom by a rigid surface and all lateral ends of the liquid in the x- and y-directions have a zero pressure. The liquid surface at the holes is also assumed to be bounded. This is obviously an approximation for a simple formulation. In an actual condition, the liquid at the holes in the plate has a free surface where a sloshing is possible. The liquid movement induced by the plate vibration should satisfy the Laplace equation:

$$\nabla^{2}\Phi\,(x,y,z,t)=0\,. \tag{12}$$

When the harmonic time function is assumed, the velocity function Φ can be separated with respect to z in terms of the displacement potential function $\phi(x,y)$ as follows:

$$\Phi(x,y,z,t)\;=\mathrm{i}\,\omega\,\phi\,(x,y,z)\,\exp\,(\mathrm{i}\,\omega\,t)=\mathrm{i}\,\omega\,\phi(x)\,\phi(y)\,f(z)\,\exp\,(\mathrm{i}\,\omega\,t)\,. \tag{13}$$

Insertion of eqn. (13) into eqn. (12) gives the two ordinary differential equations:

$$\phi(x,y)_{,xx}\,/\,\phi(x,y)+\phi(x,y)_{,yy}\,/\,\phi(x,y)=-\,f(z)_{,zz}\,/\,f(z)=\gamma_{mn}^{2}\,. \tag{14}$$

When we consider the boundary condition, the velocity potential will require the following equation at the rigid bottom:

$$\partial\phi(x,y,z)\,/\,\partial z\,\Big|_{z=0}=0\,. \tag{15}$$

It is assumed that the liquid is laterally unbounded and has a free surface. This assumption is not realistic except zero gravitational condition but it is often used for simple formulation. So, the displacement potential must be zero along the lateral liquid walls:

$$\phi(x,y,z)\big|_{x=0}=\phi(x,y,z)\big|_{x=L}=0\;, \tag{16}$$

$$\phi(x,y,z)\big|_{y=0}=\phi(x,y,z)\big|_{y=H}=0\,. \tag{17}$$

Hence, the displacement potential $\phi\left(x,y,z\right)$ satisfying the boundary condition of eqns. (15), (16) and (17) can be written as follows.

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$$\phi(x,y,z)=\sum_{m=1}^{R}\sum_{n=1}^{R} C_{mn}\cos\left(\alpha_m x\right)\cos\left(\beta_n y\right)\cosh\left(\gamma_{mn} z\right), \tag{18}$$

where, C_{mn} is the unknown coefficients and $\gamma_{mn}=\sqrt{\alpha_m^2+\beta_n^2}$.

As the liquid displacement and the displacement of the plate must be equal in the transverse direction at the interface between the liquid and the plate, the compatibility condition at the liquid interface with the plate yields:

$$\partial\phi(x,y,z)/\partial z\,\big|_{z=B}=\sum_{m=1}^{R}\sum_{n=1}^{R} q_{mn}W_{mn}(x,y), \tag{19}$$

where B is the liquid depth. Substitution of eqns. (2) and (18) into eqn. (19) gives the following equation,

$$\sum_{n=1}^{R}\sum_{m=1}^{R} q_{mn}\sin(\alpha_m x)\sin(\beta_n y)=\sum_{n=1}^{R}\sum_{m=1}^{R} C_{mn}\,\gamma_{mn}\sin(\alpha_m x)\sin(\beta_n y)\sinh(\gamma_{mn} B) \tag{20}$$

From eqn. (20), the unknown coefficient of the liquid is given as:

$$C_{mn}=\left\{\gamma_{mn}\sinh(\gamma_{mn}B)\right\}^{-1}q_{mn} \tag{21}$$

Now, it is necessary to evaluate the reference kinetic energies of the liquid to calculate the natural frequencies of the perforated plate in contact with the liquid. The reference kinetic energy of the liquid can be evaluated from its boundary motion as follows:

$$T_L^*=-\frac{1}{2}\rho_o\left[\int_0^H\int_0^L q_i\,U_i\,V_k\,\phi_k\left(x,y,B\right)q_k\,dx\,dy\right.$$
$$\left.-\sum_{s=0}^{N-1}\sum_{r=0}^{M-1}\int_{ra+a_1}^{ra+a_2}\int_{sa+a_1}^{sa+a_2} q_i\,U_i\,V_k\,\phi_k\left(x,y,B\right)q_k\,dx\,dy\right]=\frac{1}{2}\boldsymbol{q}^T\boldsymbol{G}\boldsymbol{q}, \tag{22}$$

where ρ_o is the mass density of the liquid and

$$G_{ik}=Z_{ik}\left\{\gamma_{ik}\tanh(\gamma_{ik}B)\right\}^{-1} \tag{23}$$

By minimizing the Rayleigh quotient with respect to the unknown parameters $\boldsymbol{q}$, the following Galerkin equation for the liquid-contacting plate can be obtained:

$$D\left[\boldsymbol{\Gamma 1}-\sum_{s=0}^{M-1}\sum_{r=0}^{N-1}\boldsymbol{\Gamma 2}\right]\boldsymbol{q}-\omega^2\rho\,h\left(\boldsymbol{Z}+\boldsymbol{G}\right)\boldsymbol{q}=\{0\}. \tag{24}$$

The natural frequencies ω of the perforated plate in contact with the liquid can be calculated by eqn. (24) given an eigenvalue problem.

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3 Example and discussion

3.1 Verification of the analytical method

The frequency determinant of eqns. (11) and (24) was numerically solved to estimate the natural frequencies of a perforated plate in air and in contact with water by using a mathematical commercial software MathCAD (2000), respectively. In order to check on the validity and accuracy of the results from the theoretical study, an example was carried out for the liquid–coupled system and the results were compared with three-dimensional finite element analysis result. In the liquid–coupled system, the perforated plate had a length of 480 mm, a width of 360 mm, and a wall thickness of 5 mm. The square holes in the plate had a width of 70 mm and a pitch between the holes of 120 mm. The physical properties of the plate material were as follows: modulus of elasticity = 69.0 GPa, Poisson's ratio = 0.3, and mass density = 2700 kg/m^3. Water was used as the contained liquid with a density of 1000 kg/m^3, and a liquid depth of 40 mm was considered. The simply supported boundary condition along the edges of the plate was taken into account. The expanding terms of the admissible function was 16 for x- and y-directions respectively in the numerical calculation. The calculation of the integrations and the eigenvalue extraction were carried out by the built-in algorithms of MathCAD.

The finite element analyses using a commercial computer code, ANSYS (version 9.0), were performed to verify the theoretical results for the liquid–contacting perforated plate. In the FEM analysis, a three-dimensional model was constructed with three-dimensional liquid elements (FLUID80) and shell elements (SHELL63). The hexahedral liquid region was divided into a number of cubic liquid elements with eight nodes. On the other hand, we modeled the perforated plate as deformable square shell elements with four nodes. The nodes connected entirely by the liquid elements were free to move arbitrarily in a three-dimensional space, with the exception of those, which were restricted to motion in the axial direction along the bottom and top surfaces of the liquid cavity. The finite element model had a total of 59856 elements including 55296 liquid elements and 4560 shell elements.

Tables 1 and 2 will make it easier to check the accuracy of the natural frequencies and compare the theoretical natural frequencies with the corresponding FEM ones for the perforated plate in air or in contact with water. The symbols in the tables, n' and m' represent the number of the nodal lines of the mode shapes for the x- and y-directions, respectively. The largest discrepancies between the theoretical and FEM results were 10.0% for the 11 lowest natural frequencies. The discrepancy of the liquid–contacting solid plate was approximately 6.6% for the lowest natural frequencies. For the case of the perforated plate in contact with water, the discrepancies were in the range of –5.3% ~ 3.8%. As can be seen, the present results for the perforated plate in contact with water agreed quite well with the FEM solutions. Figure 2 illustrates the mode shapes of the perforated plate in air.

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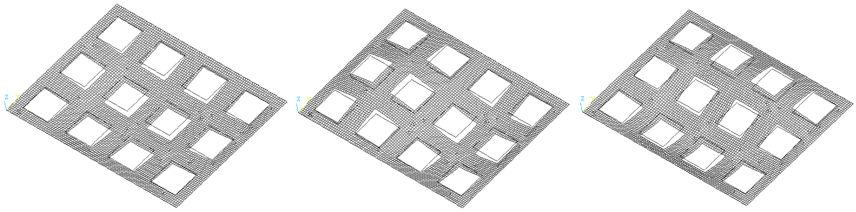
Table 1: Natural frequencies of the rectangular solid plate and perforated plate with the simply supported boundary condition in air.

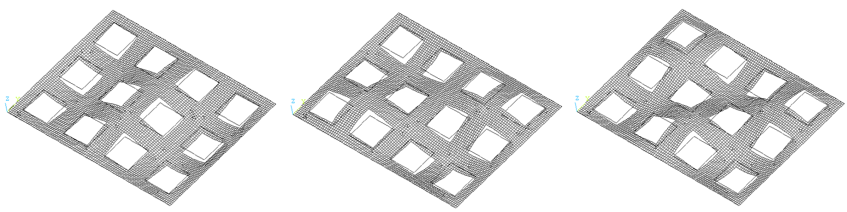
Mode n'	Mode m'	Natural frequency (Hz)				
		Solid plate		Perforated plate (d = 70 mm)		
		Theory	ANSYS	Theory	ANSYS	Error (%)
0	0	144.9	144.9	128.6	122.9	4.6
1	0	301.3	301.3	270.2	255.6	5.7
0	1	423.0	423.0	376.9	355.9	5.8
2	0	562.0	562.0	511.8	473.8	8.0
1	1	579.4	579.3	520.8	491.3	6.0
2	1	840.2	840.0	759.9	707.3	7.4
0	2	886.5	886.4	863.8	826.2	4.6
3	0	927.1	926.9	911.7	876.8	4.0
1	2	1043.0	1042.7	1061.9	1007.2	5.4
3	1	1205.2	1204.8	1214.8	1158.2	4.9
2	2	1303.7	1303.2	1324.6	1204.1	10.0

Table 2: Natural frequencies of the rectangular solid plate and perforated plate with square holes in contact with water (d = 70 mm, B = 40 mm).

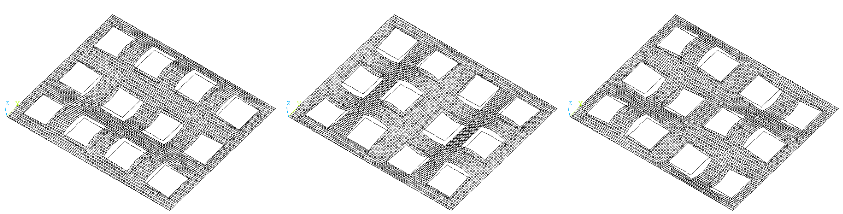
Mode		Natural frequency (Hz)					
		Solid plate			Perforated plate		
n'	m'	Theory	FEM	Error (%)	Theory	FEM	Error (%)
0	0	34.6	32.4	6.8	30.5	32.2	-5.3
1	0	98.0	91.9	6.6	87.2	89.5	-2.6
0	1	156.7	147.1	6.5	137.8	140.1	-1.6
2	0	230.4	216.1	6.6	208.0	203.6	2.2
1	1	239.9	225.0	6.6	213.4	213.1	0.1
2	1	391.5	366.8	6.7	351.8	338.8	3.8
0	2	419.7	393.9	6.5	399.9	421.4	-5.1
3	0	444.6	417.1	6.6	434.5	451.3	-3.7
1	2	517.0	484.4	6.7	516.7	530.0	-2.5
3	1	620.9	580.9	6.9	624.1	623.8	0.0
4	0	746.5	699.2	6.8	698.5	656.6	6.4
2	2	685.2	640.6	7.0	706.0	670.3	5.3



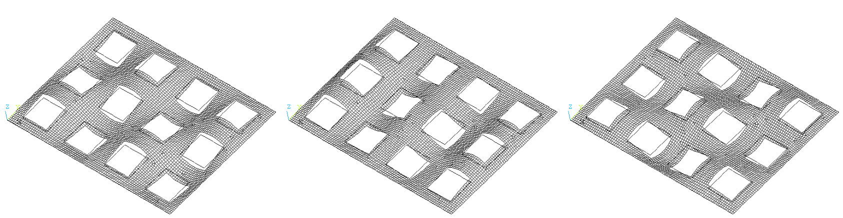
1st mode (122.9 Hz) 2nd mode (255.6 Hz) 3rd mode (355.9 Hz)



4th mode (473.8 Hz) 5th mode (491.3 Hz) 6th mode (707.3 Hz)



7th mode (826.2 Hz) 8th mode (876.9 Hz) 9th mode (1007.2 Hz)



10th mode (1158.2 Hz) 11th mode (1204.1 Hz) 12th mode (1232.1 Hz)

Figure 2: Mode shapes of the perforated plate with square holes in air (FEM results).

4 Conclusions

Free vibration analysis of a rectangular perforated plate in air or in contact with a liquid was studied theoretically. It was assumed that the plate was simply supported along its edges and the square holes in the plate had an identical size and pitch. To estimate the approximate eigenvalues of the perforated plate, the Rayleigh–Ritz method was utilized. In the theory, the beam function of a simply supported solid uniform plate was introduced as the admissible function, and the liquid was assumed as an ideal liquid with a zero pressure along the lateral surfaces. The proposed analytical method for the perforated plate was verified by a three-dimensional finite element analysis with a good agreement.

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