PROPOSITION OF FRACTAL DIMENSION APPLICATION FOR THE ESTIMATION OF CERTAINTY OF WATER DELIVERY

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ABSTRACT

One of the most difficult steps during evaluating of the system safety plan is the evaluation of certainty of water delivery through the distribution network. So far, no universal method of this estimation has been fully elaborated. Most often, the various reliability factors are used. However, these factors does not always include the geometrical structure of distribution network. It can be analysed by methods of minimal cut or efficiency sets paths between water source and outtake or even all nodes. However, such methods are very laborious and their difficulty increases together with the increase of degree of network complexity. The aim of this paper is to present a proposition of an application of two methods: reference nodes and fractal dimension, for estimation of certainty of water delivery though complex distribution networks. Exemplary calculation presented in the paper, were conducted for 6 different models and real water supply networks. The conducted calculations indicated that reference nodes method has a limitation to maximum 6 looped networks. Fractal dimension method doesn't have such limitation. The proved in the paper relationship between fractal dimension value and number of minimal efficiency sets paths indicates that this method can be used for complex networks estimation.
Keywords: water delivery certainty, minimal efficiency paths, fractal dimension, looped networks.

1 INTRODUCTION

The main aim of constructing and functioning of water-pipe networks is to supply water to the users in desired amounts, of adequate quality, of right pressure and at desired times [1]–[3]. The fulfilling of these assumptions requires to evaluate the water delivery certainty both at the stage of designing as well as operating of water supply system. Additionally, such evaluation is a part of system safety plans, recommended by WHO. Most often, safety plans are developed on the basis of operational system reliability analysis, which is also a possible way to estimate the water delivery certainty. This problem is often solved by predicting the number, frequency and location of potential water distribution networks failure in the scope of interest of researchers [4]–[6]. Basing on various models simulating failure occurrence, they are able to estimate necessary network maintenance costs and strategy [7]–[10]. The applied methods are founded on data collected from particular elements of water-pipe system in operation. The analysis of gathered information allows creating characteristics of various failures occurrences, then applied to numerous simulation models. However, described actions do not include the shape of existing or designing of geometrical water supply network structures. Thus, the seconds step of estimating the water delivery certainty is the analysis of possible delivery paths from water source to the customer. While the task is considerably uncomplicated in case of branched networks, in case of looped and mixed networks it becomes more complex. The existing methods of estimating the water delivery certainty in complex network structures may be classified as follows [11]:

1. Networks with node failures only
2. Networks with link failures only
3. Networks with both node and link failures

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Considering properties of each of the aforementioned cases, the estimation methods of water delivery certainty are mainly focused on defining minimal cut sets in s-t networks (s - source, t - sink node). There is a range of algorithms to define minimal cut sets, and they are mainly based on graph theory [12]–[15]. An interesting suggestion of such an algorithm was put forward by Zhibin [16]. His method enables defining minimal cut sets for network categories 2 and 3. The method was tested on networks composed of 4 nodes and 7 links. Yeh [15] designed an algorithm considering stochastic signal flow in the analysed s-t network, which consisted of a single source and sink as well as two additional nodes.

The aforementioned methods, based on minimal cut set analysis, implicate that the more complex the structure of analysed networks, the more cut sets there are, which results in improving of water delivery certainty. Additionally, these methods enable reliability estimation of single s-t network structures, which is an extremely uncommon for water delivery systems. Therefore, Kansal and Devi [17] suggested an algorithm to determine the number of cut sets among all network nodes, which facilitates reliability estimation of multi-sink node structures, such as water delivery networks. The algorithm was tested for a system composed of 18 nodes, allowing link failure only. It was also suggested that in fault diagnosis the number of minimal cut sets alone might be treated as a reliability indicator.

The real existing water delivery systems consist of hundreds or even thousands of nodes and links, in addition, a great number of nodes simultaneously function as sinks. Structures created by these networks are of an individual character, describable better in terms of fractal sets than by means of Euclidean geometry [18]. Application of the aforementioned methods to the estimate the certainty of water delivery of such networks results in growing complications concerning the task itself, as well as interpretation of results.

The solution to these problems might be found in using approximate estimation of geometrical shape of water-pipe networks. Such analyses might be beneficial in the water delivery network designing stage. As this stage consists in analysing alternative variants of network modelling, immediate comparison of versions would allow the engineers to opt for the best solution.

This paper presents two methods of approximate estimation of certainty of water delivery. The first one bases on determining probability of supplying water to the most inconveniently located (reference) nodes. The other presented method draws on the connection between the number of minimal cut sets, describing nodes supply, and network fractal dimension. The methods were applied to model looped structures along with three existing water supply networks.

2 ASSUMPTIONS AND NOTATIONS

2.1 Assumptions

In the paper two cases are considered. In one case, analysis are limited to selected (reference) nodes, which are the most inconveniently located nodes, with relation to water source. The other method draws from the relation between minimal cut sets number, describing nodes supply, and fractal dimension of the network geometrical structure.

2.2 Notations

S	water source
A,B,..F, R	sink nodes, reference nodes
1,2,... 7	links

r	number of minimal cut sets
m	number of elements in minimal cut set
$P_l(t)$	link 1 efficiency probability
$P_{i,j}(t)$	operational probability of i-th element in j-th minimal cut set
$dim_B\Gamma$	fractal dimension (box counting method)
δ	linear size of single box used in method above
Γ	fractal geometric set
$N_\delta(\Gamma)$	the number of δ sized boxes needed to completely cover the Γ set

3 METHODS

3.1 Reference nodes method

The minimal cut set defining algorithms, presented in the literature review, bases on the assumption of a single s-t network. As mentioned, such a situation is quite rare in existing water distribution systems. Therefore, their estimation of water delivery certainty should be conducted individually for each sink node, which would consequently make the research extremely labour-consuming. The authors argue that it is possible to reduce labour intensity of these methods by limiting the analysis to selected nodes, which will be henceforth referred to as “reference nodes”. The approximate estimation of water delivery certainty network established on reference nodes is characterised by the following assumptions:

- link failures only,
- known efficiency probability of each pipe,
- single-input water source,
- reference node is the most inconveniently located node, with relation to water source,
- the water delivery certainty estimated by the reliability indicator is the probability of water supply to the referential node from the source,
- computations include minimal cut sets method.

The analysis of potential application of this method was based on model looped structure, presented in Fig. 1. The structure enumerates sink nodes (A, B,...F) and links (1, 2,...). The water is supplied with links, the assumed efficiency probability of which is identical – 0.99.

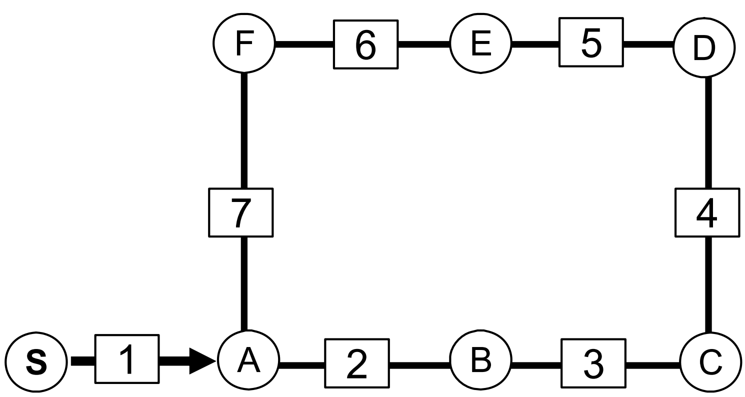

Figure 1: Analysed looped network. S – water source; A,B,…F – nodes; 1,2,...7 – links.

Reliability analysis assessed the water supply to all nodes is presented in Fig. 1., assuming that the node marked as S is the source. The probability of supplying water to node A is equal to link 1 efficiency probability. For other nodes it is possible to distinguish two minimal cut sets. The supply probability of these nodes results from the eqn (1) [1], [4]:

$$P_s(t) = P_1(t) \cdot \left\{ 1 - \prod_{j=1}^{r} \left[1 - \prod_{i=1}^{m} P_{i,j}(t) \right] \right\}, \tag{1}$$

where: $P_1(t)$ – link 1 efficiency probability, r – number of minimal cut sets, m – number of elements in minimal cut set, $P_{i,j}(t)$ – operational probability of i-th element in j-th minimal cut set.

Table 1 presents the collation of possible minimal cut sets enabling water supply from link 1 to individual nodes, as well as supply probability.

The lowest water supply probability was evaluated for node D. Consequently, this will be the reference node (the most inconvenient). In this particular situation, this is the node most distanced from the water source. The determined value, in light of assumption made, is the approximate reliability measure of the looped network in question.

Identical methodology was applied to slightly more complex looped networks shown in Fig. 2. The computations used the same assumptions as the previous example. Nodes most distant from the water source are the reference nodes. The results of analysis are collated in Table 2.

The value highlighted in the table, 0.99, stands for efficiency probability of a link supplying water to all structures; the link is marked as 1 in Fig. 2. Interestingly, in case of variant D, a structure composed of 6 loops, the probability of delivering water to the reference node, with the exclusion of link 1, amounts to 1-1.12685E-27. With further expanding number of loops, the limit of possible accuracy of computer calculations has been reached. With 6 loop structures the probability of supplying water to the reference point is invariably 1.0. That is why, the suggested method cannot be applied to reliability analysis of structures more complex than 6 loops.

3.2 Fractal dimension method

The aforementioned problem of numerical computations inadequacy emphasises the need to search for a reliability indicator in addition to the probability of sink nodes water supply efficiency method. The method of approximate estimation of water distribution networks, suggested by the authors of this article, bases predominantly on two premises. The first draws from Kansal and Devi [17], who stated that, in reliability analysis, minimal cut set of all

Table 1: Minimal cut sets and probability of water supply to nodes.

Node	Minimal cut set	Supply probability
A	1,	0.99
B	1, 2, 7-6-5-4-3	0.98951480149401
C	1, 2-3, 7-6-5-4	0.98922370199301
D	1, 2-3-4, 7-6-5	0.98912667209301– minimum
E	1, 2-3-4-5, 7-6	0.98922370199301
F	1, 2-3-4-5-6, 7	0.98951480149401

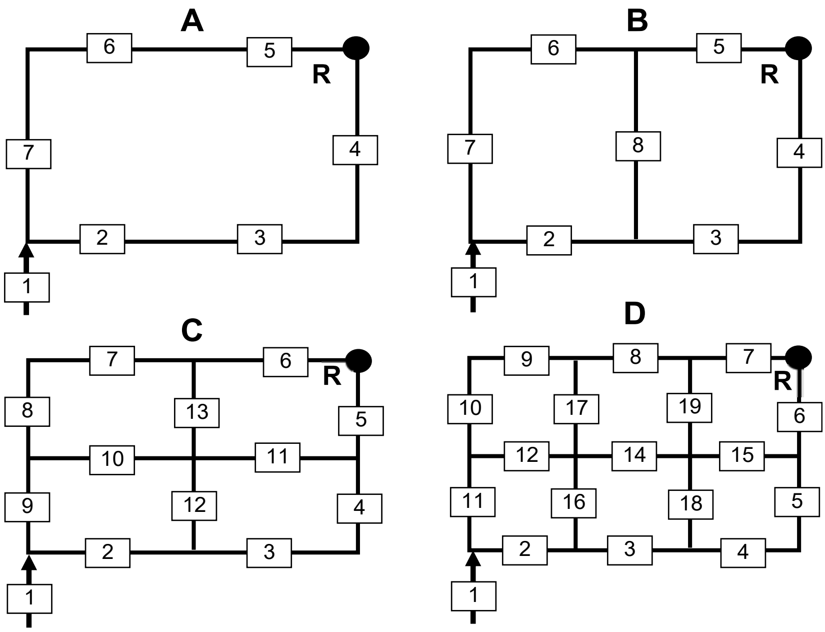

Figure 2: Model looped networks. A, B, C, D – structure variants; 1,2,…,19 – links; R – reference nodes.

Table 2: Results of reliability analysis of structures in Fig. 2.

Variant	Minimal cut sets number	Supply probability
A	2	**0.99** · 0.999117850599 = 0.98912667209301
B	4	**0.99** · 0.999996419663803 = 0.989996455467165
C	9	**0.99** · 0.999999999994469 = 0.989999999994524
D	24	**0.99** · (1-1.12685E-27) ≅ 0.99

links alone, might be treated as a reliability indicator. The second is supported by numerous researchers [1], [4], [13]–[16], and argues that with increased complexity of networks the reliability of structures and certainty of water delivery do customers increases.

Proposed method of approximate estimation of water-pipe distribution network is characterised as follows:

- link failures only,
- single-input water source,
- reference node is determined from all sink nodes,
- the number of minimal cut sets between the source and the reference point is treated as a reliability indicator, characteristic of a given network,
- determining the degree of network complexity is sufficient to estimate the number of minimal cut sets,
- the degree of network complexity, given in number, may approximately reflect the reliability of its structure.

Defining network degree of complexity may be carried out in terms of fractal sets. Assuming that water distribution system creates structures of a fractal character [18] the degree of its complexity is describable by its fractal dimension.

Fractal dimension, for elements covering a fractal plane, stands for the degree of it being covered by a given set (points, lines, figures). Its application to describe fractal sets was first introduced by Mandelbrot [19], who suggested using Hausdorff typological dimension. More recent researchers have proven that in practical application covering dimensions comes as much more convenient [20]. Calculations presented in this article engage box counting method. The intuitive base of box counting method is a measure in δ scale (Fig. 3).

The fractal dimension of *F* set is given by eqn (2) [21]:

$$\dim_B F = \lim_{\delta \to 0} \frac{\log N_\delta(F)}{-\log \delta}, \tag{2}$$

where: $N_\delta(F)$ – the number of δ sized boxes needed to completely cover the *F* set.

In practice, this dimension is often described as a slope of the regression line based on logarithmic dependence, log Nδ (F)= f(log 1/δ) [20], [21].Considering structures in Fig. 2, the calculated fractal dimensions were A - 1,229, B - 1,404, C - 1,453, D - 1,500 respectively. Combining these values with minimal cut sets presented in Table 2, a diagram presenting the interdependency has been drawn – Fig. 4.

The resulting curve suggests the relation between the number of minimal cut sets and fractal dimension. The increase of this dimension above 1.4 was reflected by quick increase in the number of minimal cut sets in the analysed loop structures.

4 APPLICATION TO REAL WATER DISTRIBUTION NETWORKS

Using suggested minimal cut sets number and fractal dimension of model fractal networks relation, the authors moved to test its validity for three existing water-pipe networks.

First network, composed of 27 nodes and 17 approximately 3.5 km long links, providing water for estimated 600 users is shown in Fig. 5. The next network, composed of 199 nodes and 213 approximately 24 km long links, providing water for estimated 4,000 users is presented in Fig. 6. The last network is composed of 562 nodes and 645 approximately 250 km long links, providing water for more than 50,000 users and is presented in Fig. 7. The first is a typical branched network, typical of small settlements. The other two networks consist of both looped and branched structures, of various complexity degrees. For each network reference nodes, located in the areas furthest from the water source, have been determined.

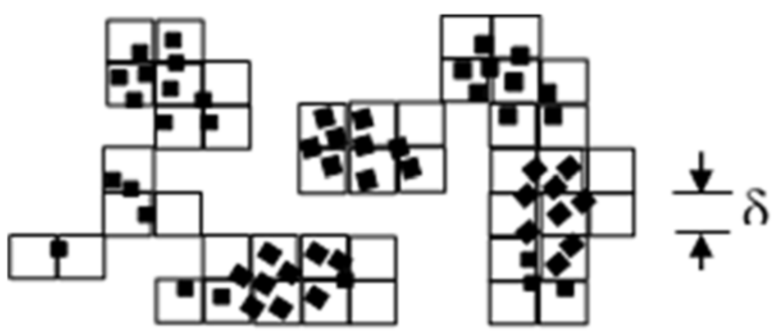

Figure 3: Box counting method for F points set on the plane [21]. δ – single box linear size.

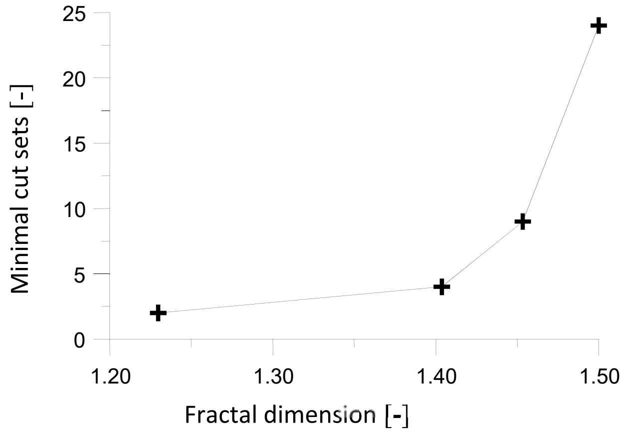

Figure 4: Interdependency between minimal cut sets and fractal dimension of structures in Fig. 2.

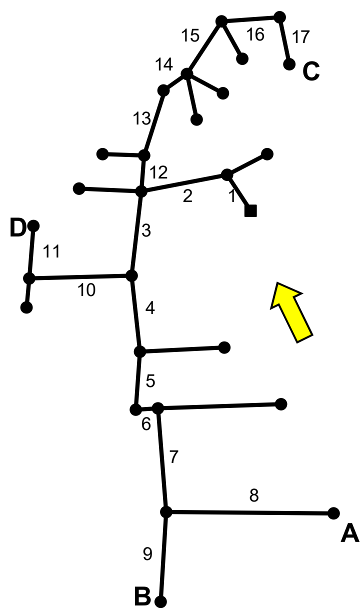

Figure 5: First analysed network. A, B, C, D – determined reference nodes. 1,2,…,17 – links numbers. The arrow represents the water source.

For the analysed three water distribution networks, the minimal cut set supplying water to marked reference points was determined with heuristic method. The computations results of these, along with the results of fractal dimensions calculated using box counting are collated in Table 3.

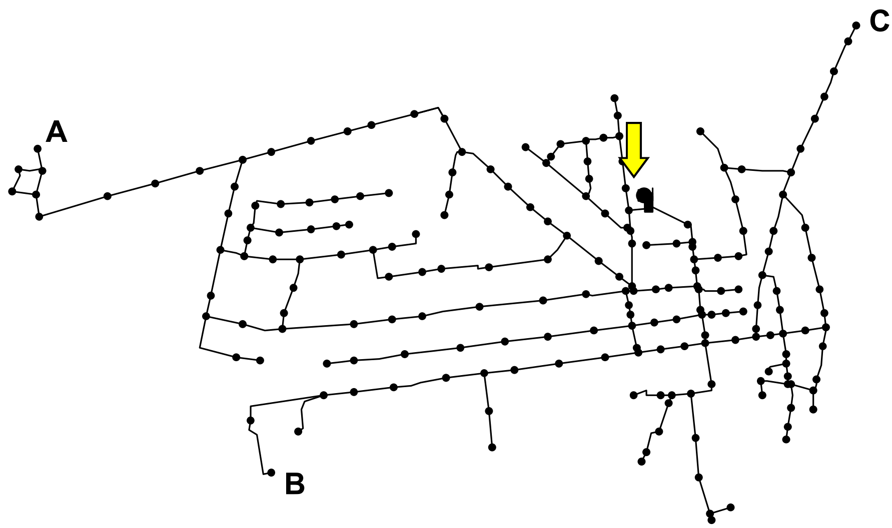

Figure 6: Second analysed network. (Symbols as in Fig. 5.)

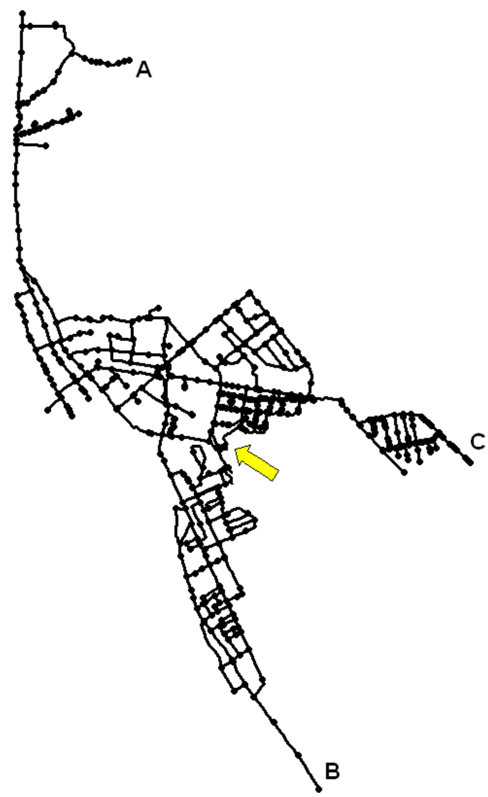

Figure 7: Third analysed network. (Symbols as in Fig. 5.)

Table 3: Collation of calculations results of minimal cut set with fractal dimensions analysed on existing water-pipe networks.

Network	Reference node	Minimal cut sets number	Fractal dimension
1	A	1*	1.3207
	B	1	
	C	1	
	D	1	
2	A	156	1.4020
	B	24*	
	C	136	
3	A	627	1.6045
	B	821	
	C	416*	

* Minimal value – analysed network structures reliability indicator.

Using data from Table 3, the diagram of dependency between the number of minimal cut sets and fractal dimensions, characterising analysed networks, was drawn – Fig. 8.

Although the three analysed networks are distinctly different in terms of structure and size, certain regularities might be observed. In comparable manner to formerly analysed model looped structures, the increase of fractal dimension is reflected by the growing number of minimal cut sets characterising analysed networks. Comparison of results presented in Figs 3 and 7 indicates that, similarly to model looped networks case (Fig. 4), above 1.4 fractal dimension value a sharp increase in the number of minimal cut sets is observed. Insufficient research material made it impossible to analyse functional dependency between fractal dimension and the number of minimal cut sets connecting reference nodes to the source.

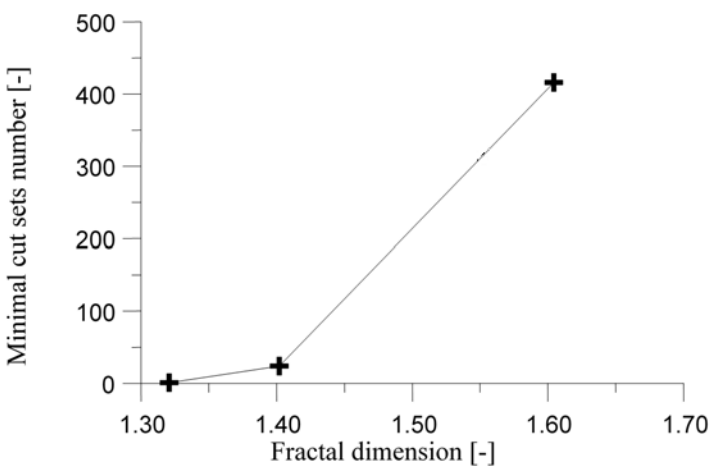

Figure 8: Minimal cut sets and fractal dimension of analysed existing water delivery system dependency graph.

5 CONCLUSIONS

Even though the development of reliability theory and its applications to describe water-pipe networks is highly dynamic, there still is vast research field concerning the evaluation of water delivery certainty by water supply networks.

The proposed reference nodes method enables approximate estimation of water delivery certainty, but the limited numerical computation accuracy can be used for networks containing maximum 6 loops.

The second method, basing on fractal dimension, doesn't contain this limitation. The conducted calculations proved the relationship between fractal dimension value and number of efficiency path sets in every analysed model and the real network. Thus, fractal dimension value of network geometrical structure may be treated as an indicator allowing approximate evaluation of estimation of certainty of water delivery.

The realised investigations show that below 1.4 fractal dimension value the number of efficiency path sets is very low. It indicates the low level of water delivery certainty. Above value 1.4 the number of efficiency path sets and simultaneously water deliver certainty essentially increase. Nevertheless, the number of analysed in the paper networks was insufficient to ascertain the functional dependence between these parameters.

The practical usage of fractal dimension method requires the elaboration specialized classification, include low, medium and high level of a water deliver certainty and connected with them fractal dimension values.

For the analysis of more complex structures, authors suggest using their fractal dimension, its numerical value may be treated as an indicator allowing approximate evaluation of estimation of certainty of water delivery.

Basing on comparative analyses of four models and three existing water distribution networks it was concluded that there exists a relation between the fractal dimension and minimal cut set linking, water source with sink reference node. Nevertheless, the number of analysed networks was insufficient to ascertain the functional dependence between these parameters. It appears however, that in light of obtained results, the research should be continued in the future.

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