

# Allocation of initial attack resources

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## Abstract

Increased scrutiny of federally funded programs combined with changes in fire management reflects a demand for new fire program analysis tools. We formulated an integer linear programming (ILP) model for initial attack resource allocation that operates in a performance-based, cost-effectiveness analysis (CEA) environment. The model optimizes the deployment of initial attack resources for a user-defined set of fires that a manager would like to be prepared for across alternative budget levels. The model also incorporates fire spread, multiple ignitions, simultaneous ignitions and monitoring resources on a landscape. It also evaluates the cost effectiveness of alternate fire fighting resources and alternative pre-positioning locations. Fires that escape initial attack are costly during the extended attack phase of fire management. To address this within the scope of initial attack, we constructed and analyzed alternative objective functions that incorporate a proxy for internalizing the cost of fires that escape initial attack. This type of model can provide the basis for a wider scale formulation with the potential to measure an organization's performance and promote a higher level of accountability and efficiency in fire programs.

*Keywords:* integer programming, initial attack, wildland fire, optimal deployment.

## 1 Introduction

Wildland fire organizations, including US federal land management agencies, customarily organize the suppression of unwanted fires into the three stages of suppression: initial attack (IA), extended attack (EA) and large fire management. Compartmentalizing this problem allows organizations to focus on the functioning and funding of different stages of fire management. This enables the analyst to focus with depth on the part of the problem of primary interest, but it introduces the problem of potential "spill over" effects that can be costly. For



example, initial attack fires that are not contained will spill over into extended attack or even to large fire management. The potential cost of such spill-over is a necessary consideration in a proper benefit and cost calculus of initial attack.

We build on previous optimization literature to address the issues of multiple fires, simultaneous fires, monitoring resources, with special attention to potential spillover effects from IA to EA. A demonstrative example shows how an ILP model can be used to identify and optimize the dispatch of initial response resources in a performance-based and cost-effectiveness analysis (CEA) framework. The analysis includes four important features that have not been previously demonstrated: 1) use of an integer linear program (ILP) to model a functional relationship between cost and performance, 2) inclusion of multiple fires and optimal dispatch locations, with the potential to address a season of fires 3) the capability of including simultaneous ignitions, and 4) because fires that escape initial attack can be costly, we address alternative means of including a proxy for the cost of fires that escape initial attack. The remainder of the paper is structured as follows: in the next section we present a description and a mathematical formulation of the ILP with alternative objective functions to address the cost of escaped fires, this is followed by a demonstrative numerical example to illustrate the capabilities and relationships of the model. The last section provides discussion and conclusions including model limitations and potential extensions of the formulation.

## 2 A performance-based fire preparedness ILP

We make the customary assertion of minimizing damage for a given level of expenditure consistent with the least cost plus loss expressions (Rideout and Omi [3]). Consistent with this assertion, we compare the effectiveness of alternative initial attack organizations by minimizing expected damage (loss) of unwanted wildland fires for any specified budget level where a range of budget levels are modelled. We recognize that to the extent that firefighting resources are scarce, not all fires are of equal importance to contain because not all resources that could be damaged by fire are of equal consequence. Wildland fires that occur in the wildland urban interface threaten life and property are typically of greater importance to aggressively manage than are fires occurring in remote areas such as wilderness. Because acres differ in their importance to protect from wildfire, our formulation provides the ability to proportionally weight acres that might be differentially affected by the damaging effects of wildfire (Rideout et al. [4]). The calculation of natural resource loss for a given budget level involves multiplying the area burned from each fire by its per acre weight to calculate the per acre loss. The weight reflects the marginal rate of substitution of resource disimprovement. The ILP optimization allows us to focus on cost effective solutions while avoiding interior (inferior) solutions.

A set of fires is provided as input to the ILP and each fire includes information on its initial size and its change in perimeter and area by time period. Perimeter is directly related to suppression cost through resource production rates and the area burned is directly related to performance through expected



loss. We use the free burning fire containment rule from previous deployment models (for example, NWCG [5]) stating that a fire is contained when the total fireline produced by firefighting resources overtakes the fire perimeter. A fire is defined as having “escaped” if it is not contained during the initial attack period due to a lack of funds to apply to fire fighting resources resulting in a lack of sufficient fireline production capability.

A pool of potential firefighting resources is established for evaluation where each resource can be selected and optimally allocated to a set of candidate dispatch locations and fire events. Each firefighting resource is defined by a fireline production rate and by its fixed and variable costs. Fireline production is modelled by a cumulative value that is input for each time step of each fire. An advantage of the discrete time step approach is that the production function does not have to be constant or linear. Thus, production rates can reflect fatigue and other disruptions in production such as water and fuel refills. Arrival times and travel delays can also be reflected in these production values by entering zero chains of fireline production during travel periods. The model uses the production information along with other factors to solve for the optimal deployment.

The costs of initial response resources and of fire escapes are important considerations in preparedness modelling that directly impact the preparedness budget. This ILP model inputs fixed and variable costs of firefighting resources that directly impact optimal deployment. The fixed cost is modelled as a one-time charge that is incurred if the resource is deployed to any fire during the season. Each resource’s variable cost is modelled as an hourly cost that reflects its operating expenses on each fire including maintenance, fuel, regular hourly wages, overtime and hazard pay. Also during the IA period, we deploy a monitoring resource to escaped fires to reflect the concept that every fire, contained or not, will receive some monitoring efforts during initial attack. The full cost of escapes is addressed in the section “Incorporating a Proxy for the Cost for Escaped Fires”.

Formulating the ILP for fire suppression requires developing a set of equations to track containment on each fire. The ILP optimized firefighting resource allocation to a single fire to minimize the total suppression cost plus net value change. They used a separate set of constraints at each time period to track whether the targeted fire would be contained during that period. This formulation expands their approach to support IA firefighting resource allocations across multiple and simultaneous ignitions. Although firefighting resources can be dispatched to multiple fires, they often cannot be dispatched to simultaneous fires, and this introduces heightened competition for firefighting resources. To model simultaneous ignitions, we forced each resource to choose one of the simultaneous ignitions to attack and we assumed that resources would not be redeployed to other simultaneous ignitions. This restriction reflects the pragmatic consideration that ground based resources often lack the mobility to address simultaneous fires. We also introduced constraints across time to track the time that each fire was contained.



$$Loss = \sum_i \sum_{d \in (1 \text{ to } D_e)} (W_{id} * f_{id} * A_{id}) \quad (1)$$

$i$  index of a fire in the set  $I$  of all fires,

$d$  index of fire durations in the set  $D$  and  $D_e$ .  $D$  is the set of periods before a fire escape.  $D_e$  is the period at which fire is considered to have escaped,

$f_{id}$  binary variable,  $f_{id} = 1$  if fire ( $i$ ) burns for a duration of ( $d$ ) time periods, otherwise  $f_{id} = 0$ ,

$W_{id}$  predicted fire losses for each unit of area burned by fire ( $i$ ) after a duration of ( $d$ ) time periods,

$A_{id}$  total area burned by fire ( $i$ ) for the duration of ( $d$ ) time periods,

The objective function (1) minimizes the expected fire loss for a given budget and each firefighting resource is restricted to a single location. This expands the model to consider alternative locations for any particular resource. Each suppression resource can only be deployed to each fire for a fixed duration and each fire lasts for a single duration. Additional restrictions are available from the authors. For each contained fire, the total length of fireline produced by all suppression resources from different dispatch points must equal or exceed the fire perimeter at the period it is contained. We also ensure that fireline will be effective only during the containment period of any fire. The index of fire duration  $d$  is used to make this assumption valid in the model. For example, if there is a single suppression resource  $r$  available of constructing fire line to contain a fire within an 8-hour IA period, this constraint will take a simplified form of:

$$f_{r1} + 2f_{r2} + \dots + 8f_{r8} \geq x_{r1} + 2x_{r2} + \dots + 8x_{r8} \quad (2)$$

## 2.1 Incorporating a proxy for the cost of escaped fires

While compartmentalizing suppression into IA and EA provides managerial clarity for planning, budgeting and operations, it introduces a classic externality problem if not properly addressed. In the IA preparedness planning context, such an externality can be generated if the costs of fires that escape IA are not considered in the IA model or decision process. A correct approach, consistent with the Coase Theorem (Coase [2]), would be to maximize the sum of the net benefits across both program components (IA and EA) when considering resource allocations to IA preparedness planning. Simultaneously modelling both would, in principle, provide the correct set of costs to the IA analysis. In this way we could solve for the optimal number of escaped fires. The problem is that there is no precedent for modelling large fires in this context or for modelling IA and EA simultaneously.

In lieu of a credible simultaneous solution, we tested three potentially practical proxies for the cost of escaped fires by using three alternative objective functions. These were: A) using a large per escaped fire penalty, B) increasing the penalty for escapes in proportion to estimated loss at the time of escape and

C) combining approaches one and two. The objective function is separated into two parts where the first part represents the loss during IA and the second part represents a penalty for escapes. Of particular interest is how modifying the second part of the objective function will influence the allocation of IA resources and fire containment.

Objective function A) penalizes each escaped fire by using a large constant penalty “M”. As M becomes large, this objective function effectively maximizes the number of fires contained, regardless of their importance. This is also known as initial attack success rate: a common performance metric. In objective function B), escaped fires are penalized by a value proportional to their loss right before escape.

The penalty increases linearly with respect to loss and the term  $K \geq 1$  enables us to increase the magnitude of the penalty. The rationale for penalizing escapes based upon the estimated loss at the time of escape is that it reflects the last information known to the IA model regarding the potential resource damage from an escape. It also reflects the restriction of the scope of the problem to IA preparedness.

Objective function C) combines A) and B) to penalize escapes by using a constant penalty combined with the estimated loss prior to escape. The rationale for adding the per fire escape cost is that escaped fires can be costly to manage even if there is little potential for resource loss at the time of escape.

With the loss minimizing ILP formulated and expressed through three alternative objective functions to address the cost of escapes, we apply the model to a demonstrative example that is designed to show how the model addresses optimal placement and dispatch of resources in a CEA context at different budget levels.

## 2.2 Demonstrative example

We begin by defining a fire scenario that includes 10 fires where two occur simultaneously. For simultaneous ignitions we make the simplifying assumption that no single suppression resources can be assigned to both. This assumption can be relaxed to allow some resources to serve simultaneous fires, but such relaxation does not add to the substance of our findings or formulation. We also assume eight time periods where each period is one hour. The duration can take any time step and the time steps are not required to be uniform. The initial perimeter of each fire represents the size of each fire when at discovery and the perimeter of each fire will grow as defined by the user during the eight hour IA period.

Our list of fire fighting resources was selected to illustrate key model features of optimal allocation and dispatch while recognizing that agency planning units would be considerably more complex. For demonstration we model three kinds of resources: resources that are relatively inexpensive and have relatively low production rates such as handcrews, resources that are moderately expensive but produce greater line production such as engines, and we also included dozers as an expensive and highly productive resource. Resource production rates were based on the National Wildfire Coordination Group Fireline Handbook



(NWCG [5]). To demonstrate the model's ability to evaluate optimal resource placement, we allowed the model to choose from three possibilities: dispatch from location HC1.A, dispatch from location HC1.B, and no dispatch. The difference in dispatch locations is represented by differences in arrival times and by the subsequent fireline production on each fire. The cost and productivity of each kind of resource was used. By using the firefighting data on fire growth, expected loss and firefighting production, we generated the following results.

### 3 Results

The results of the model formulation using the demonstrative example are discussed in two parts: 3.1) model formulation on resource allocation and fire containment with the effects of simultaneous fire events including the use of monitoring resources, and 3.2) effects of the alternative objective functions reflecting different proxies for the cost of escaped fires.

#### 3.1 Resources and fires

The detailed containment period for each fire and the allocation and dispatch schedule for each resource are based on a budget level of \$21M. At this budget level all fires can be contained and there was no difference among the alternative objective functions for escaped fire cost. In addition, the deployment duration of any resource is less than or equal to the duration of each corresponding fire. The necessary and sufficient condition of containing fire  $i$  at period  $d$  is that the total length of fireline produced for fire  $i$  at or before period  $d$  has to be equal to or longer than the perimeter of fire  $i$  at period  $d$ .

All fires, except for the simultaneous fires were contained within either the first or the second hour. The key advantage to containing fires earlier is to reduce potential loss. Keeping fires small also means that less fireline is needed and this should not imply a lower suppression cost because minimizing fire size implies an intensive effort that could employ the most expensive and productive equipment and labour. The results also show that handcrew 1 would be allocated to dispatch point B at this budget level and that handcrews 2 and 3 and engine 3 were also dispatched. The expensive and technically superior dozer was not dispatched at this budget level.

The ILP was required to make "tough" choices in resource deployment on the simultaneous fires. Simultaneous fire F10 used all of the hand crew resources while simultaneous fire F9 relied entirely upon engine three. The opportunity cost of deploying all of the handcrews to F9, in terms of reduced effectiveness on F10, is apparent as it took longer to contain F9 (five time periods). The cost of deployment includes both the variable cost of deploying the resource plus the opportunity cost incurred by not allowing that resource to attack the competing simultaneous fire. Additional tests showed that after removing the assumption of simultaneity for fires F9 and F10, a 100% IA success rate was achieved at a lower budget level of \$18M.

### 3.2 Alternative proxies for the cost of escaped fire

Results from the model at the 11 different budget levels were analyzed. Our tests of the 11 budget levels show that all 10 fires can be contained at budgets of \$21M or above within the 8-hour initial attack period. Fire containment schedules are insensitive to the choice of objective function above this budget level. Reducing the budget increased scarcity and the model allowed some fires to escape. The number of escapes was influenced by the budget level and the objective function. For a given budget level, which fires escaped was sensitive to how the proxy cost of escapes was modelled.

Analysis of objective function A) showed that the model would contain as many fires as possible. That is, a simple per fire proxy for the cost of escapes maximized initial attack success rate. This objective function will always maintain or increase the number of contained fires with increases in the budget. However, using this kind of objective function produces dispatch schedules with higher fire losses during IA. Because its constant penalty treats all fires with equal importance for containment, it fails to recognize the relative importance between fires.

Objective function B) penalized each escape proportionate to its loss at the time of escape. Given the IA scope of the analysis, this might reflect the best, albeit imperfect, information available to the model. Weighted size reflects the last known information from IA regarding values at risk, the size of the fire, and the likely cost of managing fire in an EA setting. Here, with a budget level that is insufficient to contain all the fires, containment decisions reflect the relative importance of fires at escape. Test results, with  $K = 1$ , show that as the budget increased from \$15M to \$16M, the number of escaped fires increased from three to five while the loss decreased from 1,771 to 1,429. With a \$1,000 budget increase, the model shifted from containing a group of five less important fires to a group of three more important fires. This local result reflects the possibility of encountering the economically inferior fire (fires that would not be contained at higher budget levels). Globally, however, as the budget increases so will the number of contained fires. In B) the value of  $K$  can be increased in an attempt to reduce the number of escapes, but this is nearly always futile because increasing the value of  $K$  does not change the relative importance between escaped fires. Increasing  $K$  had no effect on containment decisions in our example.

Objective function C) combines the costs from objective functions A) (cost per fire) and B) (loss at escape). By using a large constant penalty  $M$  the model will contain as many fires as possible, thus maximizing initial attack success rate. If there are multiple ways of containing the same number of fires, the model will select the most important fires. Model results also show that by using this objective function, as the budget level increased from \$ 15M to \$16M the initial attack success rate did not decline. This suggests that using objective function B) suffers from the same problem as objective function A), where five less important fires were contained but three more important fires are allowed to escape.



## 4 Conclusion

The ILP model developed in this paper includes several innovations while demonstrating key economic principles of optimal initial attack. The ILP expanded on previous work to address the planning principles for a set of fires. It shows how scarce firefighting resources would be allocated to alternative fires to minimize loss at any given budget or appropriation level. By addressing the allocation of resources across a set of fires, we enabled the model to identify which fires to fight and how aggressively to fight them. In this way, the model also demonstrated how optimal dispatch locations can be scheduled and how different kinds of firefighting resources might be utilized. Altering firefighting resource scarcity through budget levels also demonstrates how optimal results and their locations are dependent upon the level of the budget. Increases in the available budget allow for greater loss reduction and usage of more effective resources, but changes in the available budget can affect optimal location decisions. The management of scarcity is particularly important when simultaneous fire events are considered. While all fires compete for scarce resources across a planning season, simultaneous fires compete more intensively by effectively precluding the simultaneous use of individual firefighting resources. Our example showed how two simultaneous fires were managed differently by different kinds of resources to minimize overall loss.

Optimal resource use for initial attack requires that key cost elements are included in the model. These include the cost of having firefighting resources available (fixed cost), deployment costs (variable costs) and the cost of fires escaping IA. Managing the cost of escapes within the initial attack scope is inherently problematic because, by definition, they are external to the scope of analysis. Therefore, they can pose the classic externality problem if not properly analyzed. Because expanding the scope of analysis to extended attack (and potentially beyond), is currently infeasible, we analyzed three alternative approaches to include a proxy for this cost. The first proxy effectively maximized initial attack success rate by including a large per fire cost where all fires escape costs were treated equally. This resulted in important fires escaping under the constrained budget while relatively unimportant fires were contained. The second proxy introduced a cost based upon the loss at the time of escape. While this approach distinguishes between important and unimportant fires, a local consequence is that fewer fires may be contained as the budget increases. The principle applied is intended to reflect the potential cost and especially cost differences of fires that would escape. The technology applied to make these cost estimates could be greatly expanded through predictive fire behaviour modelling and GIS mapping to generate a reasonable estimate of escaped fire cost. However, improving the technology does not alter the principles of in this demonstration. The third proxy includes both costs modelled simultaneously. Since the priority of this proxy is to maximize the IA success rate, it could also allow important fires to escape while containing fires of lesser importance.

While the ILP was intended to demonstrate managerial principles of optimal resource use in preparedness planning, especially in initial attack, it serves





several other purposes. First, it is a useful demonstration of key economic elements of optimal resource allocation across a set of fires or initial attack. Such a model can also serve as a framework for thinking about how decisions can be made in ways that are consistent with principles economic efficiency. Secondly, an ILP model can be augmented or modified in many ways. For instance, instead of using a single fire scenario, as we did here, multiple scenarios could be used. Other enhancements could include a stochastic analysis of modelling the uncertainties in size and cost of escaped fires, and the variations existed in fire line productivities. Optimal deployment models, such as the approach illustrated here provide potentially useful insights for understanding and illustrating the efficient use of scarce resources. While optimization models have strengths and weaknesses, capitalizing on the strengths may be best realized by combining optimization with other complementary approaches such as simulation.

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