Estimation of the maximum allowable drift at the top of a shear wall (within elastic limits)

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Abstract

There is a considerable uncertainty regarding the evaluation of the maximum allowable story drift as well as the maximum allowable lateral drift at the top of a building. Various seismic codes suggest values that range from h/50 to h/2000 where h is the height of a building. This paper reviews the maximum allowable drift presented in various seismic codes and uses structural dynamics along with the finite element method to suggest a formula that can be used to determine the maximum allowable drift at the top of a shear wall based on the maximum allowable strain values in that shear wall. This formula provides an elastic limit for the drift after which the designer knows that his reinforced concrete section is going into the plastic region. In comparing the results generated by the suggested formula with the drift values suggested by other codes, it can be observed that the results are very close to the values obtained by the use of the French code PS92 and far from the values suggested by UBC and IBC.

1 Introduction

A shear wall is one of the main structural elements in a reinforced concrete building. It is constructed to support mainly the lateral forces due to wind or earthquakes. Under these loads, the structure will have a lateral displacement (what is known nowadays as Drift) the magnitude of which is defined by the movement of the lateral load resisting elements (shear walls). The question remains: what is the allowable drift at the top of a given shear wall? And consequently what is the maximum allowable drift at the top of a building?

Many investigators have suggested values for maximum displacement at the top of buildings. These values range from taking h/50 where h is the height of the building as suggested by The Uniform Building Code (UBC97) [1] and

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International Building Code (IBC2006) [2] to choosing about h/2000 as in The Lebanese Code [3].

This study makes use of the finite element method [4,5] to evaluate the displacement at the top of a shear wall and suggests an equation that can be followed in determining the maximum allowable drift. It is divided into five parts:

* In the first part, the stiffness matrix of a shear wall is determined by making the assumption that a shear wall is a vertical beam in flexure with constant stiffness throughout its height.
* In the second part, a relation between maximum displacement and the base shear is found using the force-displacement relationship.
* In the third part, expressions for displacement and relative displacement at any level are determined.
* In the fourth part, a general formula for the strain at the ith story along the shear wall is computed and maximum strain values are found.
* In the last part, a combination of the results is used to obtain a relation between maximum displacement at the top of the shear wall and maximum strain at the bottom of the shear wall. Consequently, an expression for the allowable drift at the top of the building is determined and represented in the graphs later. Comparison between the suggested formula and various seismic codes is then done.

Table 1: Inter-story drift limits for various seismic codes.

UBC97[1]	International IBC-2006[2]	Lebanese Code[3]	UBC91[6]
if $T < 0.7$ sec $\Delta_a \leq 0.025\,h_s$ if $T \geq 0.7$ sec $\Delta_a \leq 0.020\,h_s$	Over four stories: $\Delta \leq \Delta_a = 0.010 h_{sx}$, group III $\Delta \leq \Delta_a = 0.015 h_{sx}$, group II $\Delta \leq \Delta_a = 0.020 h_{sx}$, group I	$\Delta \leq h_s / 200\ 0$	if $T < 0.7$ sec $\Delta \leq 0.005\,h_s$ if $T \geq 0.7$sec $\Delta \leq 0.004 h_s$

Algerian RPA[7]	PS92[8]	Japanese SSCECJ[9]	New Zealand NZS[10]
$\Delta_a \leq 0.00$	$\Delta \leq \frac{\sqrt{\alpha}}{1000} h_s$ Z Zone I: a = 0.5 Zone II: a = 1 Zone III: a = 1.5	$\Delta_a \leq h_s/200$	Elsewhere $\Delta \leq 0.010\,Z_f\,h_s$ $Z_f = 1$ Zn A $Z_f = 5/6$ Zn B $Z_f = 2/3$ Zn C

2 Background

Many investigators and seismic codes suggest values for maximum allowable story drift or maximum allowable displacement at the top of a building but these values differ significantly. A comparison of these values is presented in Table 1.

As can be seen from this table, drift values vary between h/50 and h/2000 and the questions that can be asked are: why are these differences in the estimation of the lateral drift? On what basis these estimations are made? How can we use actual structural behavior to determine maximum allowable drift? The following article explains a procedure to estimate the maximum allowable drift between stories and consequently determines the same at the top of a building or a shear wall.

3 Assembly of matrices (shear building)

Consider a regular inclined beam as in Figure 1. The objective is to generate the stiffness matrix for a shear building.

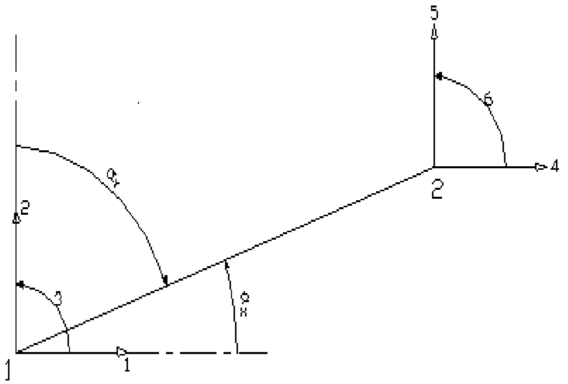

Figure 1: Inclined beam with α_x and α_y angles. (Three displacements at each node: horizontal, vertical and rotational.)

To represent a shear wall, a vertical beam in flexure is assumed which means that only the flexural effects of the element are considered and the horizontal displacements are only due to flexure, where the rotation of the nodes are taken equal to zero.

$$k = \frac{EI}{L^3}\begin{bmatrix} 12\mu^2 & -12\lambda^2 & -6L\mu & -12\mu^2 & 12\lambda^2 & -6L\mu \\ -12\lambda^2 & 12\lambda^2 & 6L\lambda & 12\lambda^2 & -12\lambda^2 & 6L\lambda \\ -6L\mu & 6L\lambda & 4L^2 & 6L\mu & -6L\lambda & 2L^2 \\ -12\mu^2 & 12\lambda^2 & 6L\mu & 12\mu^2 & -12\lambda^2 & 6L\mu \\ 12\lambda^2 & -12\lambda^2 & -6L\lambda & -12\lambda^2 & 12\lambda^2 & -6L\lambda \\ -6L\mu & 6L\lambda & 2L^2 & 6L\mu & -6L\lambda & 4L^2 \end{bmatrix} \quad (1)$$

Where:

$\lambda = \cos\alpha_x$ and $\mu = \cos\alpha_y$,
L is the length of the beam-column,
E is the modulus of elasticity,
I is the moment of inertia of the beam-column.
$\lambda = \cos\alpha_x = \cos(\pi/2) = 0$, and $\mu = \cos\alpha_y = \cos(0) = 1$.

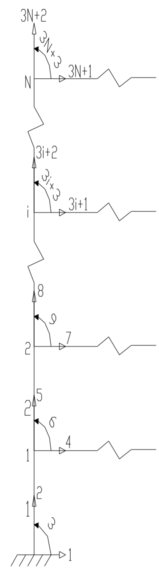

Figure 2: Representation of an N-story building.

Substituting λ and μ by their values in eqn (1), and using boundary conditions q3=q6=0 to assume no rotation, and F2=F5=0 to assume no axial force, and replacing in {F}=[K]{q}, the following matrix can be obtained,

$$\begin{Bmatrix} F_1 \\ F_4 \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & -12 \\ -12 & 12 \end{bmatrix}\begin{Bmatrix} q_1 \\ q_4 \end{Bmatrix} \tag{2}$$

From Figure 2, eqn (3) can be assembled relative to degrees of freedom 4, 7, …,3j+1… 3(N-1)+1, 3N+1. Also constant stiffness is assumed in all stories where $k^{(1)} = k^{(2)} = \ldots = k^{(N)} = k$.

The system of N degrees of freedom will be of the following form:

$$\begin{Bmatrix} F_4 \\ F_7 \\ \vdots \\ \vdots \\ F_{3j+1} \\ \vdots \\ \vdots \\ F_{3(N-1)+1} \\ F_{3N+1} \end{Bmatrix} = k \begin{bmatrix} 2 & -1 & 0 & \dots & \dots & \dots & \dots & 0 & 0 \\ -1 & 2 & -1 & 0 & \dots & \dots & \dots & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & 0 & \dots & \dots & 0 \\ 0 & \dots & 0 & -1 & 2 & -1 & 0 & \dots & 0 \\ 0 & \dots & \dots & 0 & \ddots & \ddots & \ddots & 0 & 0 \\ 0 & \dots & \dots & \dots & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & \dots & \dots & 0 & -1 & 2 & -1 \\ 0 & 0 & \dots & \dots & \dots & \dots & 0 & -1 & 1 \end{bmatrix} \cdot \begin{Bmatrix} q_4 \\ q_7 \\ \vdots \\ \vdots \\ q_{3j+1} \\ \vdots \\ \vdots \\ q_{3(N-1)+1} \\ q_{3N+1} \end{Bmatrix} \tag{3}$$

where, k is the stiffness of one story $= \sum_{j=1}^{n} 12\frac{E_j I_j}{L^3}$

4 Relation between maximum drift and base shear

To find a relation between the total base shear V, stiffness k and maximum displacement Δ for N stories structure, a triangular distribution of V is assumed [11]. Please note that the base shear can be found by any procedure or any seismic code. This distribution of V gives a formula of the applied force F_i at every level i of the structure as a function of N,

$$F_i = \frac{2iV}{N(N+1)} \qquad \text{and} \qquad F = k.q \Rightarrow q = k^{-1}.F \text{ , where}$$

$$q = \begin{Bmatrix} q_4 \\ q_7 \\ \vdots \\ \vdots \\ q_{3i+1} \\ \vdots \\ \vdots \\ q_{3N+1} = \Delta \end{Bmatrix} = \frac{2V}{N(N+1)}\frac{1}{k} \cdot \begin{bmatrix} 1 & 1 & \cdots & \cdots & 1 & \cdots & \cdots & 1 \\ 1 & 2 & \cdots & \cdots & 2 & \cdots & \cdots & 2 \\ \vdots & \vdots & \ddots & & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & \cdots & i & & & i \\ \vdots & \vdots & \ddots & & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & \cdots & i & \cdots & \cdots & N \end{bmatrix} \cdot \begin{Bmatrix} 1 \\ 2 \\ \vdots \\ \vdots \\ i \\ \vdots \\ \vdots \\ N \end{Bmatrix} \tag{4}$$

$$q_{3N+1} = \Delta = \frac{2V}{N(N+1)}\frac{1}{k}(1\times 1 + 2\times 2 + 3\times 3 + \ldots + i\times i + \ldots + N\times N),$$

$$\Delta = \frac{2V}{N(N+1)}\frac{1}{k}\sum_{i=1}^{i=N} i^2 \text{ , but } \sum_{i=1}^{i=N} i^2 = \frac{N(N+1)(2N+1)}{6},$$

Therefore, $\Delta = \frac{(2N+1)V}{3}\frac{1}{k}$,

The formula for the base shear becomes $V = \frac{3}{2N+1} k\,\Delta$ (5)

5 Displacement and relative displacement values

The displacement $q_i = (V/k)\,X_i$, and let $X = \sum_{i=1}^{N} i = \frac{N(N+1)}{2}$, =>

$$q = k^{-1}.\,F = \begin{bmatrix} 1 & 1 & \cdots & \cdots & 1 & \cdots & \cdots & 1 \\ 1 & 2 & \cdots & \cdots & 2 & \cdots & \cdots & 2 \\ \vdots & \vdots & \ddots & & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & \cdots & i & & & i \\ \vdots & \vdots & \ddots & & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & \cdots & i & \cdots & \cdots & N \end{bmatrix} \cdot \begin{bmatrix} 1/X \\ 2/X \\ \vdots \\ \vdots \\ i/X \\ \vdots \\ \vdots \\ N/X \end{bmatrix} = \left(\frac{V}{k}\right) \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ \vdots \\ X_i \\ \vdots \\ \vdots \\ X_N \end{bmatrix} \tag{6}$$

$$X_1=\sum_{i=1}^{N}\frac{i}{X}=\left(\frac{1}{X}\right)\frac{N(N+1)}{2}=1,\ \text{and}\ X_2=\frac{1}{X}+2.\sum_{i=2}^{N}\frac{i}{X}=\frac{1}{X}+2.\left[1-\frac{1}{X}\right]$$

and the formula of X_i can be computed as follows:

$$X_i=i+\frac{1}{X}\sum_{j=1}^{i-1}j^2-\frac{i}{X}\sum_{j=1}^{i-1}j\ \Rightarrow X_i=i+\frac{1}{X}\sum_{j=1}^{i-1}(j^2-ij),\ \text{Where,}$$

$$\sum_{j=1}^{i-1}j=\frac{1}{2}(i-1)(i)\qquad \text{and}\qquad \sum_{j=1}^{i-1}j^2=\frac{1}{6}(i-1)(i)(2i-1)$$

$$X_i=i+\frac{1}{X}\left[\frac{1}{6}i(i-1)(2i-1)-\frac{i^2}{2}(i-1)\right]\Rightarrow X_i=i\left[1-\frac{(i^2-1)}{6X}\right]$$

Therefore, the formula that gives the displacement q_i at the i^{th} story is,

$$q_i=i\left[1-\frac{(i^2-1)}{6X}\right]\times\frac{V}{k} \tag{7}$$

$$q_i-q_{i-1}=\left\{i\left[1-\frac{i^2-1}{6X}\right]-(i-1)\left[1-\frac{((i-1)^2-1)}{6X}\right]\right\}.\frac{V}{k}$$

$$q_i-q_{i-1}=\left(\frac{2X-i^2+i}{2X}\right)\left(\frac{V}{k}\right) \tag{8}$$

6 Computing the strain using the finite element method

U(x) represents the deflection at x. At x=0, $U=q_1$ and $U'=q_2$ and at x=L, $U=q_3$ and $U'=q_4$.

If f represents the shape function, $f=[f_1\ \ f_2\ \ f_3\ \ f_4]$, the deflection function is, $U = f_1q_1 + f_2q_2 + f_3q_3 + f_4q_4$, Where,
$f_1 = 1 - 3\varphi^2 + 2\varphi^3$, $f_2 = L\varphi(1-\varphi)^2$, $f_3 = \varphi^2(3-2\varphi)$, and $f_4 = L\varphi^2(\varphi-1)$.
Now, $\varphi = x/L$ (L is the length of the element, height of one story on the shear wall.) If we have a Shear building $\Rightarrow\ q_2 = q_4 = 0\ \Rightarrow$
$U = f_1q_1 + f_3q_3$, $\Rightarrow\ U = (1 - 3\varphi^2 + 2\varphi^3)\,q_1 + \varphi^2(3-2\varphi)\,q_3$,

$\varepsilon_x=\frac{\partial u}{\partial x}$, and $U'=\frac{dU}{dx}=\theta$, Where:

u is the displacement in the x direction, u = - y dU/dx.

- y is the algebraic distance measured from the neutral axis to the extreme fiber of the shear wall. (N.B. maximum ε_i is at maximum y)
- x is the abscissa along the shear wall between the levels (i-1) and i.
- ε_x is the strain in x direction.
- U' is the angle of rotation of the section of the shear wall at level x.

The strain ε can be computed as, $\varepsilon_x=\frac{\partial u}{\partial x}=-y\frac{\partial^2 U}{\partial x^2}$

If y is considered to be positive in the opposite direction of U, => $\varepsilon_x = y\dfrac{\partial^2 U}{\partial x^2}$,

$B = y.\dfrac{\partial^2}{\partial x^2}\begin{bmatrix} f_1 & f_2 & f_3 & f_4 \end{bmatrix}$, and $\varepsilon = B.q$.

B is a row matrix: $B = \dfrac{y}{L^3}\begin{bmatrix}(12x\text{-}6L) & (6Lx\text{-}4L^2) & (\text{-}12x\text{+}6L) & (6Lx\text{-}2L^2)\end{bmatrix}$, and q is a column matrix q(4,1), with q2 = q4 = 0

$$=> \varepsilon = \frac{y}{L^3}(12x - 6L)(q_1 - q_3)$$

The strain ε_i at the i[th] story in terms of displacement factor $(q_i - q_{i-1})$ is,

$$\varepsilon_i = -\frac{y}{L^3}(12x - 6L)(q_i - q_{i-1}) \tag{9}$$

By substituting the values of $(q_i - q_{i-1})$ from eqn.(8) into eqn.(9), then

$$\varepsilon_i = -\frac{y}{L^3}(12x - 6L)\left(\frac{2X - i^2 + i}{2X}\right)\left(\frac{V}{k}\right) \tag{10}$$

Also, the maximum value of the strain ε_i between the levels (i-1) and i, i.e. (x €[0,L]).

$$\frac{\partial}{\partial x}[\varepsilon_i] = -12\frac{y}{L^3}\left(\frac{2X - i^2 + i}{2X}\right)\left(\frac{V}{k}\right) < 0$$

The function ε_i is decreasing which means that the maximum strain in a shear wall, between levels i-1 and i, occurs at the bottom of the shear wall (at x = 0), and this maximum strain from eqn(10) is equal to:

$$\varepsilon_{(x=0)} = \frac{6y}{L^2}\left(\frac{2X - i^2 + i}{2X}\right)\left(\frac{V}{k}\right)$$

The maximum strain ε_i in the entire shear wall is calculated such that,

$$\frac{\partial}{\partial i}\left[\varepsilon_{(x=0)}\right] = 0 \Rightarrow \frac{\partial}{\partial i}\left[6\left(\frac{2X - i^2 + i}{2X}\right)\right] = 0 \quad \Rightarrow i = 1/2\,;$$

So, the function $\varepsilon_{(x=0)}$ is decreasing for i > ½, (i_{min} =1), which means that the maximum value of strain ε_i occurs at the lowest point in the first story $\Rightarrow$

$$\varepsilon_{max} = 6\frac{y}{L^2}\frac{V}{k} \tag{11}$$

7 Maximum drift at the top of a shear wall

The maximum displacement at the top of the shear wall is reached when the reinforcement strain in the tension zone at the lowest section of the shear wall is equal to ε_{st} (maximum allowable strain in steel), and the strain in the extreme fiber of the compression zone in the same section is equal to ε_c = maximum strain limit of concrete in compression = 0.003.

So the lowest section in the shear wall, which is the most critical section, is considered to have a triangular distribution.

From eqn(11), $\varepsilon_{max} = 6\frac{y}{L^2}\frac{V}{k}$,

and from eqn(5) $V = \frac{3}{2N+1}k\Delta \Rightarrow \frac{V}{k} = \frac{3}{2N+1}\Delta$,

$$\Rightarrow \varepsilon_{max} = 6\frac{y}{L^2} * \frac{3}{2N+1}\Delta = \frac{18y}{L^2(2N+1)}\Delta, \qquad (12)$$

where D is the maximum displacement at the top of the shear wall. From similar triangles of the section:

$$\frac{y}{\varepsilon_{st}} = \frac{x}{\varepsilon_c} \Rightarrow y = \frac{\varepsilon_{st}}{\varepsilon_{st} + \varepsilon_c}d. \qquad (13)$$

The maximum strain in the shear wall presented in eqn(12) at the level of steel should be smaller than ε_{st}:

$\varepsilon_{max} = \frac{18y}{L^2(2N+1)}\Delta \le \varepsilon_{st}$, replace y from eqn(13) $\Rightarrow$

$$\boxed{\Delta \le \frac{L^2(2N+1)(\varepsilon_c + \varepsilon_{st})}{18.(d)}} \qquad (14)$$

If the strain in the steel is to stay below ε_y (yielding strain), the maximum allowable drift at the top of the shear wall is,

$$\boxed{\Delta \le \frac{L^2(2N+1)(\varepsilon_c + \varepsilon_y)}{18.(d)}} \qquad (15)$$

In this case, the lowest section of the shear wall behaves as a balanced section; the limits are reached in the reinforcement in tension and in the concrete in compression at the same time, and at that point, the maximum allowable drift at the top of the shear wall is reached.

Notice that, in formula (15) above, as d increases the maximum allowable displacement decreases since any small movement tends to cause larger strain at the bottom section of the shear wall. On the other hand, and as far as maximum displacement is concerned and disregarding economical and architectural issues, it is better to use more number of shear walls with small d than to use fewer shear walls with large d; keeping in mind that the inertia of a shear wall is increased cubically as a function of d, and a bigger d will increase the stiffness significantly in a certain direction.

8 Comparison with different seismic codes and investigators

The formula proposed by the author is: $\Delta \le \frac{L^2(2N+1)(\varepsilon_c + \varepsilon_{st})}{18.d}$. If the effective depth of the shear wall d = 2m, the inter-story length L = 3m; also from Figure 3, if $\varepsilon_{st} = \varepsilon_y = 0.00207$ and $\varepsilon_c = 0.003$, the formula becomes $\Delta \le \frac{(H_t + 1.5)}{1183.432}$, where H_t is the total height of the building. See Figure 4 for the comparison of this formula with the maximum allowable displacement suggested by other investigators.

The allowable displacement (drift) suggested by IBC and UBC are much greater than the others. Also Fintel [12], Searer [13] and Searer and Freeman [14] suggest relatively large value as compared to that given by PS92, and the Lebanese Code. Also, the formula proposed in this work gives comparative results to the PS92 and the Lebanese Code.

Note that most codes give the allowable drift at the top of the building or at the top of a story as a function of the total height or the story height only, while the formula of eqn (15) suggested by the author gives the maximum allowable drift as a function of the height of one story, the number of stories, the effective depth of the shear walls used, and the elastic properties of the materials used in the shear wall (steel and concrete).

The conservative results of the suggested formula is due to the fact that the building analyzed is purely shear building and the compression effects due to vertical loads were disregarded; the vertical compressive force applied on the shear wall helps in the lateral resisting capacity. In addition, constant stiffness is assumed in all the stories, which usually is not the case since shear walls tend to have variable stiffness (stiffer at the bottom and less stiff going up the building).

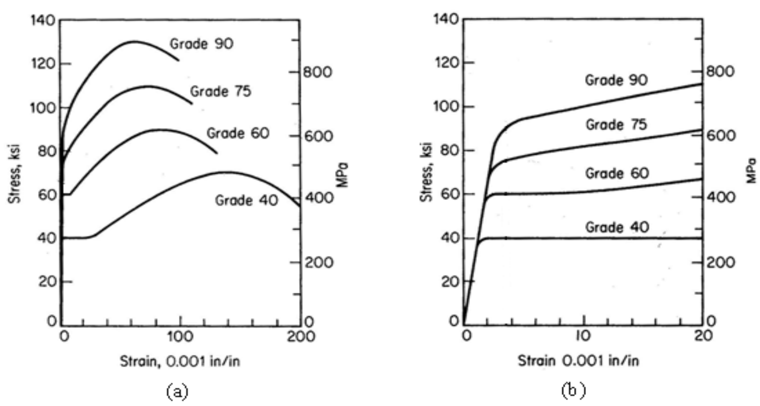

Figure 3: Typical Stress-Strain Curves for Reinforced Bars [15].

In addition, if the allowable strain in steel ε_{st} is considered to be larger than ε_y (which means that yielding of steel is permitted, or in other words the strain is beyond yield on the yield plateau as in Figure 3 (b)), a larger allowable inter story and overall drift will be permitted. This may be the reason why some codes suggest a larger allowable drift than eqn (15); the structure is allowed to pass the elastic limits and displace within the plastic region taking into consideration dynamic reversals under wind or earthquake loading.

It is also important to note that the formula suggested by the author also provides a good approximation of the drift values for strains that go beyond the elastic limits in the reinforcing steel.

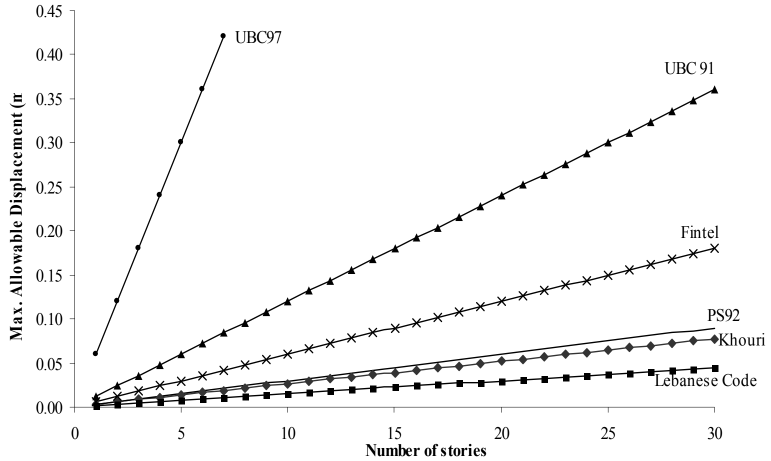

Figure 4: Maximum allowable displacement at the top of a shear wall vs. number of stories for various investigators.

9 Conclusion

In this study, shear building was analyzed using the finite element method. The shear was obtained as a function of the displacement. A value for the displacement at any story was obtained, and from which a function for the relative displacement between two stories was then determined. Using the above, an equation for the maximum strain was resolved. A limiting value for the maximum displacement within the elastic limits was obtained as a function of the height of a story, number of stories, depth of tension steel d in a shear wall, the strain of steel ε_{st} and maximum allowable concrete strain. Note that shear building was analyzed like a beam with ignoring vertical loads and assuming constant lateral stiffness in all stories. The author now is in the process of developing a formula that calculates the maximum allowable elastic strain with variable lateral stiffness in all stories.

Comparison between the results of the formula suggested in this work by the author and the maximum allowable drift values suggested by others show that most codes tend to suggest a high maximum allowable displacement at the top of the building or high story drift value; while the formula suggested in this paper tends to give conservative values as compared to most other codes with the exception of the French (PS92) which in some case gives more conservative values than this formula. Also, the Lebanese code gives more conservative values than the suggested formula.

On the other hand, the value h/50 suggested by UBC97 and IBC 2006 can be considered an aggressive suggestion for a shear building in the sense that they generate large strains at the bottom of a shear wall; it is important to note that even though the high drift values correspond to a flexible structure thereby lower lateral forces, however such large displacements may be dangerous.

It is now left for the designing engineer to evaluate his structure and decide/choose a maximum allowable strain limit for concrete and for steel, and determine the corresponding maximum allowable drift values.

Finally, the formula suggested by the author can serve as a starting point after which the designing engineer would know that the shear wall in question has passed the elastic limit in a shear building.

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