Kinematic interaction of a single pile in heterogeneous soil

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Abstract

The behaviour of deep foundation under static loads has been widely investigated and the available calculation procedures can be considered suitable for the current engineering applications. However, pile behaviour under seismic loading is more complex and less known, because one can find the contemporary action of inertial forces rising from the over-structure (inertial interaction) and of the soil deformations rising from the seismic waves (kinematic interaction). Italian code DM 14/01/2008 requires the dynamic soil-structure interaction in seismic foundation design to be taken into account, but it does not give any information about kinematic interaction strains evaluation criteria. Experimental evidences and theoretical considerations of many authors show that simply the kinematic interaction may induce high stresses on piles, especially near an alternation between a soft and a rigid soil layer interface. In this work pile behaviour due to kinematic interaction will be examined. An approach based on the differential equation proposed by Kavvadas and Gazetas (Kinematic seismic response and bending of free-head piles in layered soil. Géotechnique, 43, N.2, 207-222, 1993) will be used. The analysis is focused on the response of a single pile in a heterogeneous three-layer soil profile.
Keywords: deep foundations, seismic loads, dynamic soil-structure interaction, pile behaviour, numerical model.

1 Introduction

Pile seismic response results from a complex soil–pile–overstructure interaction affected by non-linear phenomena, which takes place in the soil near piles and by kinematic effects linked to ground shaking. Dynamic pile–soil interaction is

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remarkable when piles are embedded in soil layers with strong discontinuities in strength and stiffness.

In 1977, Tajimi [14] admitted that pile design should take into account dynamic behaviour. During the last 30 years a deep improvement in knowledge has been established, as many experimental observations during real earthquakes are now available and dynamic simulations on physical models and numerical analysis in various load conditions have been developed. The results of those studies can be found in technical literature (Dennehy et al. [2]; Flores-Berrones et al. [5]; Gazetas [6]; Kagawa and Kraft [7]; Kainya and Kausel [8]; Kobori and Shoichi [9]; Masayuki et al. [10]; Penzien [12]; Tajimi [13, 14]; Wolf and Von Arx [15]).

To analyse the dynamic behaviour, the application of rigorous analytical tools would be desirable, but in design practise this is too onerous, especially when a frequency domain seismic analysis is brought about, as pile response should be evaluated with such a high frequencies number (thousands) that it would be enough to cover the seismic signal frequency content. Nowadays, the practise level is lower than theoretical knowledge and theoretical developments have not yet been converted to simple calculation methods. In professional practise, dynamic effects are usually neglected, because easy analysis methods are missing or they are considered neglectable. So, piles are designed taking into account only the loads applied at the pile head. In this work a simplified analytical model that allows the kinematic interaction to be taken into account in piles embedded in layered soils design is presented. The results of this study are shown through dimensionless plots for preliminary design estimations.

2 Mathematical model

The proposed simplified model is based on the Beam on Dynamic Winkler Foundation (BDWF) method, in which soil behaviour is represented by springs and dampers distributed along the pile (Figure 1). Soil around piles is hypotised in free-field conditions, so seismic S-waves propagate vertically and are not influenced by pile presence. Based on the model proposed by Kavvadas and Gazetas [1, and on their application on a two layered soil deposit, in this work an application on a three layered soil profile is carried out.

The analysis assigns free-field displacement of undisturbed soil $u_{ff}(z,t)$ and applies it to the pile through springs and dampers (representing the pile–soil interface) to set seismic actions on the pile. It is necessary to take into account both the pile with the involved soil layers parameters, and the interface parameters, that are, however, functions of both soil and pile geometrical and physical parameters. One of the most critical aspects in modelling the soil–pile system is the determination of the springs and dashpot mechanical parameters (stiffness k_x and viscosity c_x), functions of frequency ω.

To determine the soil free-field displacement, a one-dimensional S-wave propagation can be used, assuming a linear hysteretic soil behaviour:

$$u_{ff}(z,t) = U_{ff}(z) \cdot \exp\left[i \cdot \left(\omega \cdot t + \alpha_{ff}\right)\right] = \hat{U}_{ff} \exp(i \cdot \omega \cdot t) \tag{1}$$

where $U_{ff}(z)$ is the free-field soil displacement modulus; $\alpha_{ff} = \arctan\left[(2 \cdot \xi \cdot \beta)/\left(1-\beta^2\right)\right]$ is the phase displacement between seismic input at the bedrock and the soil answer; $\beta = \omega_f / \omega_s$ is the ratio of the excitation frequency to the fundamental natural frequency of the "free" (i.e. without piles) soil deposit in vertical S-waves; ξ is the hysteretic damping ratio.

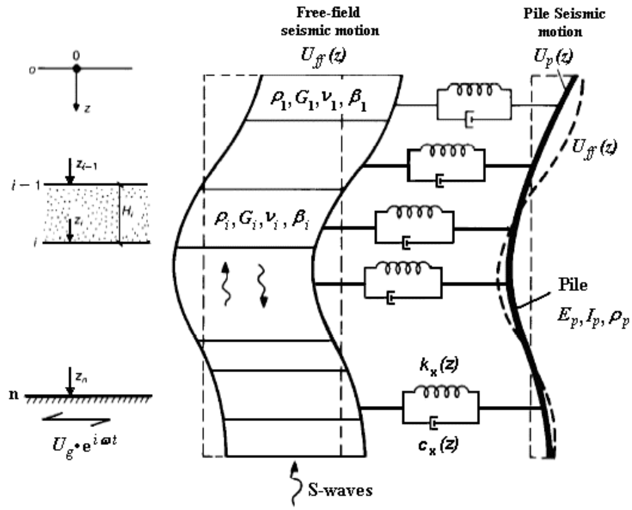

Figure 1: BDFW model for a layered soil and a free head pile (from Kavvadas and Gazetas [1]).

Each layer is characterized by a complex shear wave velocity:

$$V_s^* = V_s \cdot \sqrt{1 + 2 \cdot i \cdot \xi} \tag{2}$$

where $V_s = \sqrt{G/\rho}$ is the actual shear wave velocity. An acceleration with a frequency equal to the fundamental natural frequency of a homogeneous soil layer thick as the pile length has been used as a simplified seismic input. To determine the soil fundamental natural frequency Rayleigh method can be used. The differential equation governing pile answer is the following:

$$E_p \cdot I_p \cdot \frac{\partial^4 u_p}{\partial z^4} + m_p \cdot \frac{\partial^2 u_p}{\partial t^2} = S_x \cdot \left(u_{ff} - u_p\right) \tag{3}$$

where $E_p \cdot I_p$ is the bending stiffness, m_p is the unit length mass, u_p is pile displacement; S_x represents features of the interface by which free-field ground displacement transmits strains to pile:

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$$S_x = k_x + i \cdot \omega \cdot c_x \tag{4}$$

As a first approximation, the spring stiffness k_x can be considered approximately frequency independent and expressed as multiple of the local soil Young's modulus E_s:

$$k_x \approx \delta \cdot E_s \tag{5}$$

where δ a frequency independent coefficient assumed to be constant (i.e. the same for all layers and independent of depth), that will be called "pile–soil interaction coefficient". δ has been determined by Finite Elements method by Kavaddas and Gazetas [1].

The stiffness parameter c_x, in eqn. (4) represents both radiation and material damping; the former arises from waves originating at the pile perimeter and spreading laterally outward and the latter from hysteretically-dissipated energy in the soil.

Solving eqn. (3), pile deformations (displacements and rotations), bending moment and shear will be determined as functions of both depth z and time t.

Horizontal pile displacements can be determined with the following equation:

$$u_p(z,t) = U_{pp}(z) \cdot \exp\left[i \cdot \left(\omega \cdot t + \alpha_p\right)\right] = \hat{U}_{pp}(z) \exp(i\omega t) \tag{6}$$

where $U_{pp}(z)$ is the pile displacement modulus; α_p is the phase difference between free-field displacement and pile answer in terms of displacement.

Eqn. (3) can be alternatively written as follows:

$$\hat{U}_{pp}^{IV} - \lambda^4 \cdot \hat{U}_{pp} = \alpha \cdot \hat{U}_{ff} \tag{7}$$

where ω is the excitation round frequency and:

$$\lambda^4 = \frac{m_p \cdot \omega^2 - S_x}{E_p \cdot I_p} \tag{8}$$

$$\alpha = \frac{S_x}{E_p \cdot I_p} \tag{9}$$

Eqn (7) has the following general solution:

$$\hat{U}_{pp}(z) = \left[e^{-\lambda \cdot z} \cdot e^{-\lambda \cdot z} \cdot e^{-\lambda \cdot z} \cdot e^{-\lambda \cdot z}\right] \cdot \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix} + s \cdot \hat{U}_{ff}(z) \tag{10}$$

where D_1, D_2, D_3, D_4 are arbitrary constants to evaluate basing on the compatibility equations and the boundary conditions, while

$$s = \frac{\alpha}{q^4 - \lambda^4} \tag{11}$$

$$q = \frac{\omega}{V_s} \tag{12}$$

By eqn. (10) the following equation can be obtained:

$$\begin{Bmatrix} \hat{U}_{pp}(z) \\ \hat{U}'_{pp}(z) \\ \hat{U}''_{pp}(z) \\ \hat{U}'''_{pp}(z) \end{Bmatrix} = \begin{bmatrix} e^{-\lambda\cdot z} & e^{\lambda\cdot z} & e^{-i\cdot\lambda\cdot z} & e^{i\cdot\lambda\cdot z} \\ -\lambda\cdot e^{-\lambda\cdot z} & \lambda\cdot e^{\lambda\cdot z} & -i\cdot\lambda\cdot e^{-i\cdot\lambda\cdot z} & i\cdot\lambda\cdot e^{i\cdot\lambda\cdot z} \\ \lambda^2\cdot e^{-\lambda\cdot z} & \lambda^2\cdot e^{\lambda\cdot z} & -\lambda^2\cdot e^{-i\cdot\lambda\cdot z} & -\lambda^2\cdot e^{i\cdot\lambda\cdot z} \\ -\lambda^3\cdot e^{-\lambda\cdot z} & \lambda^3\cdot e^{\lambda\cdot z} & i\cdot\lambda^3\cdot e^{-i\cdot\lambda\cdot z} & -i\cdot\lambda^3\cdot e^{i\cdot\lambda\cdot z} \end{bmatrix} \cdot \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix} + s\cdot \begin{Bmatrix} \hat{U}_{ff}(z) \\ \hat{U}'_{ff}(z) \\ \hat{U}''_{ff}(z) \\ \hat{U}'''_{ff}(z) \end{Bmatrix} \tag{13}$$

or concisely, for a pile element in the domain of the soil layer $\boldsymbol{j}$:

$$\widetilde{U}_{pj}(z) = \widetilde{F}_j(z)\cdot \widetilde{D}_j + s_j \cdot \widetilde{U}_j(z) \tag{14}$$

The vector $\widetilde{U}_j(z)$ can be determined from the free-field displacement solution.

In the case of a multi-layered soil profile with N layers ($j = 1, 2, ..., \mathrm{N}$), eqn. (13) will be a system of $4N$ equations with $4N$ arbitrary constants $\widetilde{D}_1$, $\widetilde{D}_2$, $\widetilde{D}_3$, $\widetilde{D}_4$ that could be evaluated from the compatibility equations between pile and soil and the boundary conditions.

Compatibility equations express that at the (N-1) soil layer and pile interfaces, the pile deflection u_p, rotation ϑ, bending moment M, and shear force Q must be continuous: these compatibility requirement can be expressed by the following $4\cdot(N-4)$ equations (for a arbitrary interface j)

$$\widetilde{U}_{pj}(z_j) = \widetilde{U}_{p(j+1)}(z_j) \tag{15}$$

For as regards boundary conditions it can be observed that at the pile top, in the case of a free head pile,

$$\Theta(0,t) = 0 \tag{16}$$

$$Q(0,t) = 0 \tag{17}$$

At the pile end, in the case of a pile hinged at the bedrock:

$$M(z_N,t) = 0 \tag{18}$$

$$u_p(z_N,t) = u_g(t) \tag{19}$$

while in the case of a floating pile:

$$M(z_N, t) = 0 \tag{20}$$

$$Q(z_N, t) = 0 \tag{21}$$

being M and Q pile bending moment and shear.

Thus a set of 4N equations can be obtained and they can be solved for the constants $\widetilde{D}_1$, $\widetilde{D}_2$, …., $\widetilde{D}_N$. Once these constants are evaluated, pile displacements, bending moments, shear forces, etc. can be obtained directly from eqn. (13), since:

Pile displacement: $$u_{pp}(z,t) \tag{22}$$

Pile rotation: $$\Theta(z,t) = u'_{pp}(z,t) \tag{23}$$

Pile bending moment: $$M(z,t) = -E_p \cdot I_p \cdot u''_{pp}(z,t) \tag{24}$$

Pile shear: $$Q(z,t) = -E_p \cdot I_p \cdot u'''_{pp}(z,t) \tag{25}$$

3 Numerical analysis

A system made of a fixed head single pile, with a length L and a diameter d, embedded in a three layer soil deposit, h_1, h_2 and h_3 thick respectively (Figure 2(a) and (b) have been studied. The soil deposit lies on a rigid bedrock and is subjected to vertically propagating S-waves, that produce a horizontal harmonic motion of this kind:

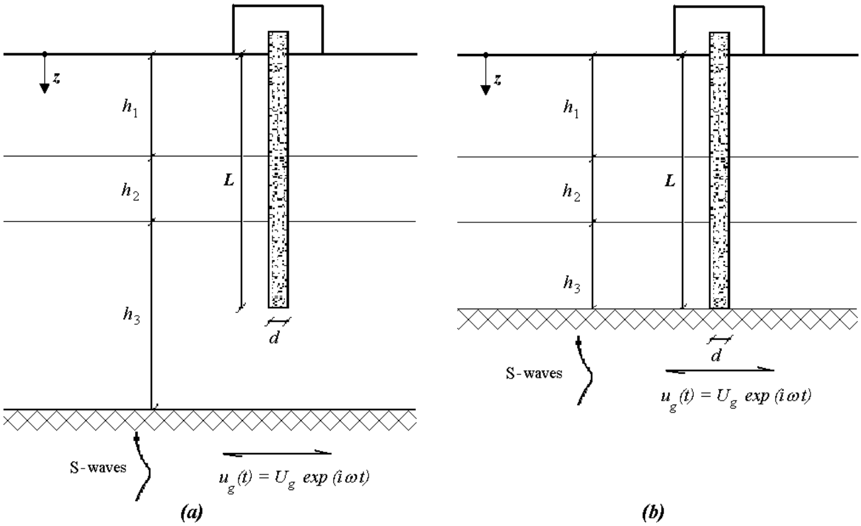

Figure 2: The calculation model. (a) Floating Pile. (b) Pile hinged at the bedrock.

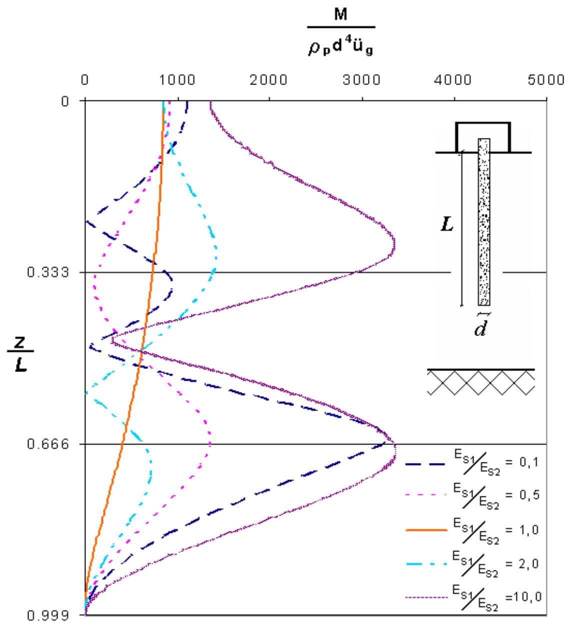

Figure 3: Dimensionless bending moment for different soil layer strains for a floating pile.

$$u_g(t) = U_g \cdot \exp(i \cdot \omega \cdot t) \tag{26}$$

The case of a floating pile and that of a pile hinged at the bedrock have been analysed. In the first case $M = 0$ and $Q = 0$ are the boundary conditions at the pile end; in the second case the conditions $u_p = u_g$ and $M=0$ have been imposed.

In the case of floating pile $h_1 = h_2 = h_3 / 2$, $d = L/30$ e $L = 3 \cdot h_1$ has been assigned. In the second case $h_1 = h_2 = h_3$ and once more $d = L/30$ and $L = 3 \cdot h_1$ has been assigned.

As regards the frequency of the bedrock displacement, the value of fundamental natural frequency calculated in the case of a homogeneous soil layer thick as pile length with the same mechanical features of layer 1 has been assumed.

Soil has been hypotised as a linearly hysteretic solid made of three layers, respectively with Young's modulus E_{s1}, E_{s2}, E_{s3}, damping ratio $\xi_1 = \xi_2 = \xi_3 = 5\%$, mass density $\rho_1 = \rho_2 = \rho_3$ and Poisson's ratio $\nu_1 = \nu_2 = \nu_3 = 0.40$. Pile has been represented as a linearly elastic beam with mass density $\rho_p = 1.60 \cdot \rho_1$. For soil elastic features $E_{s3} / E_{s1} = 1$ has been assigned and the cases $E_{s1} / E_{s2} =$ 0.1, 0.5, 1.0, 2.0 and 10.0 have been analysed.

4 Preliminary results and conclusions

According to Kavvadas and Gazetas [1] results have been exposed in dimensionless terms. In particular, dimensionless bending moment has been defined as $M^* = \dfrac{M}{\rho_p \cdot d^4 \cdot \ddot{u}_g}$, where $\ddot{u}_g = \omega^2 \cdot u_g$ is the maximum seismic acceleration at the bedrock.

In Figure 3 single pile response, in the case of floating pile, is shown in terms of bending moment modulus for E_{s1}/E_{s2} = 0.1, 0.5, 1.0, 2.0, 10.0. Results show that a peak of moment is recorded near mechanical discontinuities.

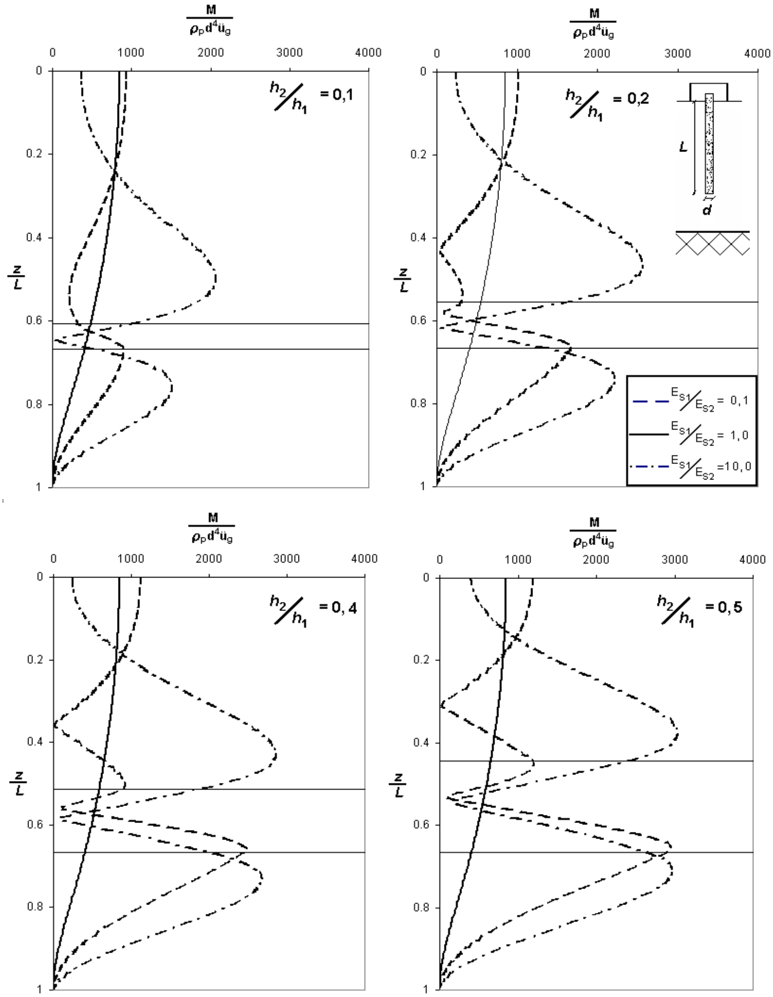

Figure 4: Dimensionless bending moments for various intermediate layer thicknesses.

For floating piles, the maximum bending moment value is recorded where the difference between mechanical features is higher (E_{s1}/E_{s2} =0.1; 10.0). For piles hinged at the bedrock the maximum moment along pile seems to be always lower than that observed at the pile top in the case of homogeneous soil layer. However, this can be attributed always to the input frequency adopted, coinciding with the fundamental natural frequency of the case of a homogeneous soil layer. In Figure 4, for the only case of floating pile dimensionless moment modulus versus depth is shown, for various intermediate soil layer thickness, for h_2/h_1 = 0.1, 0.2, 0.3, 0.5. Regardless of the various layers stiffness ratio, for increasing intermediate layer thickness a raise in the maximum solicitation has been observed. This is even more evident in Figure 5, where, for the case of floating pile, the ratio $M_{het}/M_{\hom}$, of the maximum bending moment along the pile in a heterogeneous soil profile and that of the case of a homogeneous soil layer, with varying intermediate layer thickness is shown.

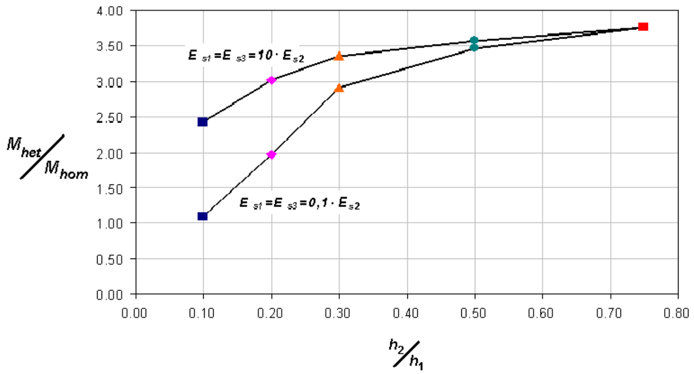

Figure 5: Floating pile: the dimensionless ratio of maximum bending moments M_{het} (in a heterogeneous soil profile) to $M_{\hom}$ (in a homogeneous soil profile) versus the ratio h_2/h_1 for various intermediate layer thicknesses.

Some final remarks of this preliminary study can be resumed as follows.

As observed by other authors, in this study it has also been highlighted that in the presence of mechanical discontinuities due to stratigraphical discontinuities peak in bending moment values are recorded.

Among the analysed cases and in the case of floating pile, the maximum solicitations in the presence of mechanical discontinuities appeared higher than in the case of homogeneous soil, regardless of higher or lower rigidity of the intermediate layer.

Moreover, as regards floating piles, an effect of bending moment amplification, in comparison to the maximum bending moment of the pile embedded in a homogeneous soil deposit, with the intermediate layer thickness arising has been observed. This result seems to be independent of the higher or lower rigidity of the intermediate layer in comparison to the external ones.

Finally, it must been underlined that the excitation frequency can sensitively condition results, depending on being near or far from the fundamental natural frequency. However, this study is still at its preliminary phase, so it is not possible to derive final general remarks.

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