

# Controlling nonlinear vibrations in steel structures using an evolutionary gain formulation to optimally satisfy performance objectives

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## Abstract

An evolutionary gain formulation is implemented within a nonlinear quadratic control algorithm framework used to minimize the performance index of a structural steel system that is subjected to various earthquake ground motions. The control architecture is formulated using a numerical integration scheme that solves the nonlinear responses of a degrading system and formulates an optimal gain matrix that is used to control building displacement demands by satisfying the desired performance-objectives per time-step. The performance-objectives are defined for various 'damage-safe' and elastic demands to show the versatility of the proposed control solution. The results of the evolutionary gain approach are compared to more conventional LQR techniques. Towards this end, a Control NONlinear time-history analysis (CONON) program was developed to simulate the responses of kinematically strain-hardened systems and to compute the optimal semi-active device output forces per time-step as part of the control solution that implements the proposed evolutionary gain. The minimization of the cost function is independent of the weighing matrices of the system, thus alleviating any need to compute these terms per time step. Instead, an iterative Riccati matrix is determined per time-step and used to generate the evolutionary gain. The results are compared by examining several hysteresis plots of the steel system against other feedback-based methods. The proposed system implementation shows a marked increase in the ability to control the desired target response and meet acceptable performance goals.

*Keywords: performance-based analysis, evolutionary control, state-space analysis, nonlinear analysis.*



## 1 Introduction

Research in performance-based engineering continues to be active wherein the design and implementation of various damage-mitigation systems for seismic protection is investigated [1]. Through structural control, the lateral force-resisting system in a building is enhanced by using performance objectives to define a *permissible and safe* response level – such as not allowing a particular amount of lateral deflection to be exceeded on a given story. One means of satisfying performance criteria has been through the implementation of semi-active devices [2, 3] that generate *reaction* forces in response to imposed demands. The objective of these systems is to offset responses in buildings that exceed pre-defined performance levels by dissipating imparted earthquake energy that would otherwise be absorbed by the structure.

Passive base isolation systems have been adopted by agencies, such as FEMA [4], and building codes officials, such as the IBC [5], for the seismic protection of buildings. Except for certain near-field, pulse-type earthquake motions with low frequencies [6, 7], base isolation systems have shown very adequate ability to dissipate the low energy associated with high-frequency far-field motions especially in low-rise ‘stiff’ buildings [8]. For low-energy motions, passive base isolation uses a soft and flexible base to decrease a building’s natural frequency and filter out the high frequency ground motion components. However, one added advantage that semi-active systems provide over passive systems [9] is that vibrations can be controlled in real-time (through a closed-loop control scheme) that enables the structure to respond in a ‘safe-mode’ of vibration at all times by dissipating the large magnitudes of earthquake energy often observed in near-field excitations. In fact, various semi-active stiffness devices have been developed to overcome limitations of passive systems, including variable stiffness systems wherein braces engage and disengage during an earthquake resulting in a change in the vibration properties of a building [10]. To overcome the suddenness of the engaging/ disengaging nature of the stiffening system, a semi-active device was developed that transitioned smoothly between states thus avoiding high-frequency resonance [11]. Variable damping devices have also been developed and applied as part of a ‘hybrid’ system [8] that uses passive base isolation to avoid structural resonance and dissipate high-energy from near-field excitations. MR fluid dampers [12] have been used as variable damping devices and also in combination with variable stiffness devices in smart base-isolated systems that are able to vary their stiffness and damping continuously and remain in a low energy non-resonant state during an earthquake. In this light, the control solution used in this study is applied through a semi-active system that is controlled using an optimal nonlinear centralized algorithm that is based on linear quadratic theory [13]. The control solution uses an evolutionary gain matrix that adapts the Ricatti matrix per time-step, or in a steady-state snapshot. The procedure has been integrated in a fully-automated program called CONON – Control NONlinear Time-History Analysis – that was developed to simulate



the controlled responses of buildings under earthquake events through the feedback mechanism in semi-active systems.

## 2 Nonlinear control and inelastic performance-objectives

A simulated random plastic analysis of single- and multi-story buildings under stationary earthquake input by Attard and Mignolet [14] expectedly revealed that the induced structural damage could not exceed certain thresholds even under near-resonant conditions unless an increase in the amount of energy was supplied through the ground motion. Conversely, only minimal damage was observed in non-resonant buildings where there were high levels of earthquake-energy imparted to the structures. While passive base isolation and (passive) viscous damping provide viable and relatively inexpensive solutions toward controlling such potential damages, near-field earthquakes (Northridge in 1994, Landers in 1992, Chi-Chi in 1992, and Turkey in 1999) in the direction of the fault-plane rupture have tremendous damage-inducing potential because of the high-energy content contained in the ground motion. The implementation of a hybrid control system that is composed of semi-active dampers that dissipate this high energy and variable stiffening elements that smoothly decrease the stiffness (and frequency) of the building to avoid earthquake resonance seems to provide a 'best of both worlds' solution.

The objectives of this study are to control a building's responses consistent with elastic and marginally-inelastic performance objectives using a semi-active control architecture to reduce damages and large inter-story drifts in the main structural components.

### 2.1 Optimal nonlinear control through an evolutionary gain

The equation of motion for a multi-degree of freedom shear frame excited by a ground acceleration is given in equation (1).

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{F}_R(t) = -\mathbf{M}\ddot{\mathbf{x}}_g(t) \quad (1)$$

The mass and damping matrices are defined as  $\mathbf{M}$  and  $\mathbf{C}$ , respectively, where  $\mathbf{x}(t)$  is the relative displacement vector of the building. The horizontal ground acceleration applied to the base of the building is given as  $\ddot{\mathbf{x}}_g(t)$ . The function  $\mathbf{F}_R(t)$  is used to define the stiffness of the system, which changes when any member of the building responds inelastically – such as when a member reaches its elastic limit during loading, unloading, or re-loading on the force vs. deflection hysteresis curve. This will occur when a member's cross section just starts to yield. The simulation of the plastic excursions that are experienced by members provides information on the extent of damages in an uncontrolled system, which justifies the need to implement a control system [1]. In satisfying performance objectives that are (minimally) greater than a member's yield capacity, system responses that venture into the inelastic state because of



incurred time-delays can be accurately analyzed, and the semi-active devices can be designed to be able to generate any additional output reaction forces in that event. The generating output control force that semi-active devices are required to generate for inelastic systems increases because of the dissipated hysteretic energy of the structural member that the device now needs to compensate for to control the response. Therefore, while it is undesirable to utilize semi-active devices in inelastic systems, time-delays or inter-modal coupling in multi-story buildings can realistically cause some members to become inelastic. To compensate for such limitations, an effective control solution is developed herein using an evolutionary gain to maximize the force output of the semi-active system such that it is not restricted by typical 'weighing matrices.'

The control algorithm that is developed for this purpose utilizes an optimal Ricatti matrix that is computed during *each* time-step and is subsequently used to determine an evolutionary gain that relates the control force output. As such, a new control force is computed based on a new gain during each time step, wherein subsequent changes to the overall stiffness and damping properties of the system can potentially cause one or more structural components to surpass their elastic yield limits. Therefore, the algorithm checks all the degrees of freedom of the building during each time step to verify that each performance is satisfied. In order to incur inelastic behavior in structural members because of time-delays that would not allow a device to react quickly enough to elastically control a response, inelastic target performance levels are assumed. As such, the stiffness and damping of the structure can alter the state-space terms used to minimize the cost function that is given in equation (2).

$$J_k = \int_{t_k}^{t_k + \Delta t} \frac{1}{2} [z_k^T(t) Q z_k^T(t) + f_{c,k}^T(t) R f_{c,k}^T(t)] dt \quad (2)$$

Here,  $J_k$  is the cost function that is minimized over each time-step,  $k$ , over the time interval  $\Delta t$  using the computed state-space responses,  $z(t)$ , and control forces,  $f_{c,k}(t)$ , at each time-step. The state-space responses are defined in continuous form as

$$z(t) = \begin{Bmatrix} x(t) \\ \dot{x}(t) \end{Bmatrix}_{(2 \times \text{DOF}) \times 1} \quad (3)$$

where  $x(t)$  is the elastic displacement relative to the ground, and  $DOF$  is the system's number of degrees of freedom. The matrices  $\mathbf{Q}$  and  $\mathbf{R}$  are weighing matrices that monitor the performance of the system in terms of the desired controlled responses and the applied control force, respectively. Including a control force component on the right side of equation (1) and separating the elastic and inelastic components, the equation of motion of the system can be expressed as



$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}\ddot{\mathbf{x}}_g(t) - \sum_{i=1}^{plex} \alpha_i \mathbf{K}\mathbf{x}_{ie,i}(t) + \mathbf{D}\mathbf{f}_c(t) \quad (4)$$

where  $\mathbf{D}$  is a location vector of the control forces in the building, and  $\mathbf{K}$  is the elastic stiffness of the system. The factor  $\alpha$  is a proportionality term used to assess the (discretized) inelastic stiffness over the time span of each plastic excursion, where *plex* is the total number of excursions. The corresponding inelastic displacement over each plastic excursion is defined as  $\mathbf{x}_{ie,i}(t)$ , which is calculated using equation (5) or (6) as

$$\mathbf{x}_{ie,i}(t) = \mathbf{x}_{tot}(t) - \mathbf{x}(t) - \sum_{j=1}^{i-1} \mathbf{x}_{ie,j}(t) \quad \text{where } i \geq 2 \quad (5)$$

$$\text{and } \mathbf{x}_{ie,1}(t) = \mathbf{x}_{tot}(t) - \mathbf{x}(t) \quad (6)$$

where  $\mathbf{x}(t)$  was previously defined as the elastic displacement. In state-space form, equation (4) can be expressed as

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{H}\ddot{\mathbf{x}}_g(t) + \sum_{i=1}^{plex} \mathbf{F}_{c,ie} \mathbf{x}_{ie,i}(t) + \mathbf{B}\mathbf{f}_c(t) \quad (7)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}_{(2 \times \text{DOF}) \times (2 \times \text{DOF})} \quad (8)$$

$$\mathbf{H} = \begin{Bmatrix} \mathbf{0} \\ -\mathbf{I} \end{Bmatrix}_{(2 \times \text{DOF}) \times 1} \quad (9)$$

$$\mathbf{F}_{c,ie} = \begin{Bmatrix} \mathbf{0} \\ -\alpha_i \mathbf{M}^{-1}\mathbf{K} \end{Bmatrix}_{(2 \times \text{DOF}) \times 1} \quad (10)$$

$$\mathbf{B} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{D} \end{Bmatrix}_{(2 \times \text{DOF}) \times 1} \quad (11)$$

In equations (8) – (11),  $\mathbf{I}$  is the identity matrix,  $\mathbf{0}$  is a vector of zeros,  $\mathbf{H}$  is a location vector that excites all the lateral *DOFs* using the ground accelerations, and  $\mathbf{B}$  is a location vector of the control forces used to control the inelastic

responses which are distinguished by the components  $\mathbf{F}_{c,ie}$ . When the structure displaces inelastically (and nonlinearly), the reduced stiffness,  $\alpha\mathbf{K}$ , reflects the incurred state of damage that is to be controlled. Equation (2) reflects the minimization of the cost function with respect to  $\mathbf{z}(\mathbf{t})$  at each discrete time step by using a non-steady-state Ricatti matrix solution where

$$P_{k+1} = Q + \left(e^{A\Delta t}\right)^T P_k \left(I + GR^{-1}G^T P_k\right)^{-1} e^{A\Delta t} \quad (12)$$

where the convergence of  $\mathbf{P}$  per time step occurs when  $(\mathbf{P}_{k+1} - \mathbf{P}_k \rightarrow 0)$ . In equation (12),  $e^{A\Delta t}$  is defined as the continuous state-space equation of motion and  $\mathbf{G}$  is defined as

$$G = A^{-1} \left( e^{A\Delta t} - I \right) B \quad (13)$$

Finally, the evolutionary gain matrix can be expressed per time step as

$$Gain(ev)_{k+1} = - \left( G^T P_{k+1} G + R \right)^{-1} G^T P_{k+1} e^{A\Delta t} \quad (14)$$

where the control force,  $f_{c,k+1}$ , is a function of the changing states-space response and the evolutionary gain,  $Gain(ev)$ , at each time step.

$$f_{c,k+1}(t) = [Gain(ev)] z_{k+1} \quad (15)$$

Equation (15) assumes a zero time delay. The minimization of the *total* state-space response,  $\mathbf{z}(\mathbf{t})$ , using equation (2) is then given in equation (16) as

$$J = \sum_{k=1}^{TimeCount} \int_{t_k}^{t_k + \Delta t} \frac{1}{2} \left[ z_k^T(t) Q z_k^T(t) + f_{c,k}^T(t) R f_{c,k}^T(t) \right] dt \quad (16)$$

where *TimeCount* is used in CONON to indicate the number of discrete time steps used in the numerical analysis.

### 3 Numerical example

The procedure using the evolutionary gain,  $Gain(ev)$ , for controlling inelastic responses as discussed above was applied to a single-degree-of-freedom model consisting of a mass of 87.5kN-s<sup>2</sup>/m supported by two – 3.65m columns (SI, W310x74 - W12x50) with a total elastic stiffness of 160.8kN/cm ( $\sigma_y = 248\text{MPa}$ ; modulus,  $E = 200\text{GPa}$ ). The elastic natural frequency of this system was found to be 2.16Hz, and a 5% damping ratio was assumed to be present. The stiffness



degradation was characterized by  $\alpha_1 = 0.25$  in equations (4), (7), and (10) where the yield deflection,  $\Delta_y$ , was calculated as 1.79cm and a value of 1 was used for  $p_{lex}$  in equations (4) and (7). Further, this system was subjected to the El Centro excitation (S00E component) of the 1940 Imperial Valley Earthquake - (EC). The response of the SDOF system was marched in time in CONON starting from zero initial conditions using the Newmark-Beta scheme with linear acceleration and a time step,  $\Delta t$ , of 0.02s.

Shown in Fig. 1(a) is the controlled force-displacement hysteresis for the inelastic SDOF system described above using the evolutionary gain to control demands and satisfy a performance of  $1.5^x\Delta_y$ , or 2.68cm. Fig. 1(a) shows the uncontrolled response (no control is applied), and the SDOF system in both cases is excited using the EC ground record. Fig. 1(b) shows the controlled hysteresis determined using a 'Constant Gain' formulation where  $z(t)$  was calculated over the entire time history. The Ricatti and Gain formulations in equations (12) and (14), respectively, remained constant in the minimization of equation (17).

$$J = \int_0^{\text{TimeCount}} \frac{1}{2} [z(t)Qz(t) + f_c(t)Rf_c^T(t)] dt \quad (17)$$

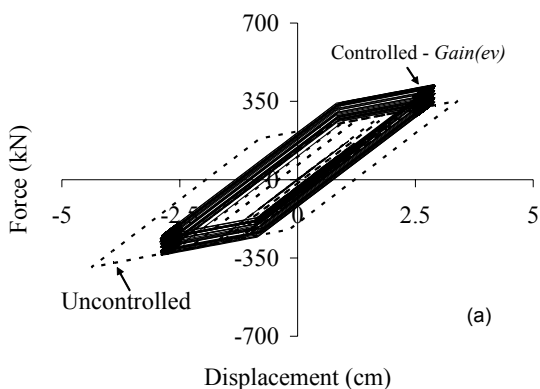
A comparison of the 'Controlled' and the 'Constant Gain' plots in Figs. 1(a) and 1(b), respectively, reveals a distinct likeness. An advantage of the proposed evolutionary gain formulation is that it does not directly utilize the weighing  $Q$  and  $R$  matrices to arrive at the final hysteresis. A change in  $Q$  and  $R$  generates the hysteresis shown in Fig. 1(b) that is labeled as 'Constant Gain ( $Q$  and  $R$ ).' Not only did this control system fail to meet the desired performance of 2.68cm, but it also necessitated a control force output that was nearly 3-times as large as that used in the evolutionary gain formulation. This was a result of the additional inelastic energy that the SDOF system lost as it became further damaged. Consequently, the semi-active device tried to restore this loss with a large output. This was not the case with the controller that utilized the evolutionary gain since the displacement converged to the desired performance on *each* time step if the performance was exceeded. A hysteresis using a 'Constant Gain - Delta Z' formulation is also shown in Fig. 1(b), which was computed using  $z(t) - z(t)_{target}$  in equation (2) where  $z(t)_{target}$  was the desired performance of  $1.5^x\Delta_y$ . For this formulation, the semi-active device provides a small amount energy and control force output to the structure because of the small net value of  $z(t) - z(t)_{target}$ .

Shown in Fig. 2(a) are four hysteresis plots when a target displacement demand of  $1.0^x\Delta_y$  is used. These plots indicate 1) that some damage occurs in the system (possibly) because of a reaction time-delay of the semi-active device in responding to the imposed demands, 2) that the evolutionary gain and constant gain are once again nearly identical – see the two dark center hysteresis plots in Fig. 2(a) – with similar control force outputs, and 3) that a change in  $Q$  and  $R$  causes the 'Constant Gain' system to deviate from the desired performance, which also results in larger control force outputs. In comparing the control force



outputs that were calculated in the inelastic range, the 'Constant Gain (Q & R)' system generates about 4-times as much control force as the control system that was formulated using the evolutionary gain; in the elastic range, the former generates about 3-times as much force, which could be costly.

SDOF Controlled and Uncontrolled Hysteresis  
for an Inelastic Performance ( $1.5^* \Delta_y$ ) - EC Excitation



SDOF Controlled Hysteresis using Constant  
Gains for an Inelastic Performance ( $1.5^* \Delta_y$ ) - EC Excitation

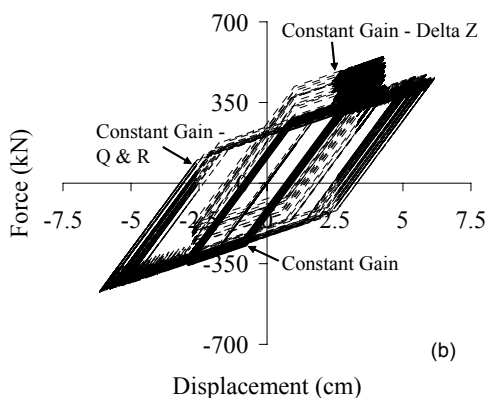
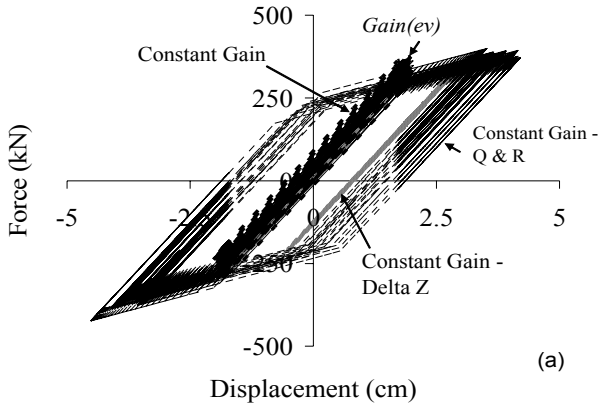


Figure 1: (a) Controlled (evolutionary gain) and uncontrolled hysteresis; (b) controlled hysteresis using a constant gain based on  $\Delta z(t)$ , a constant gain based on  $z(t)$ , and a constant gain that uses another set of **Q** and **R** matrices.



SDOF Controlled Hysteresis for an Elastic Performance  
( $1.0^x\Delta_y$ ) - EC Excitation



SDOF Controlled Hysteresis for an Elastic Performance  
( $0.8^x\Delta_y$ ) - EC Excitation

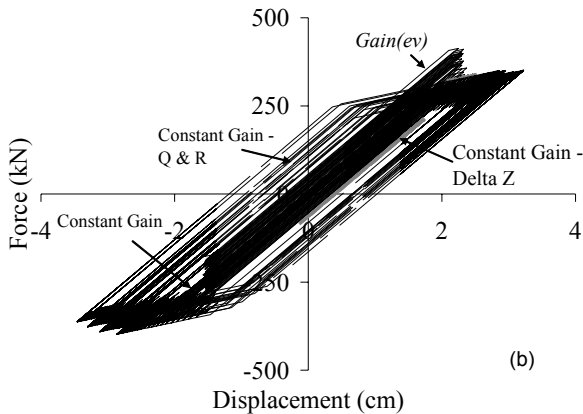


Figure 2: Plots showing the evolutionary gain hysteresis, a constant gain (with delta Z), a constant gain, and another constant gain using different  $\mathbf{Q}$  and  $\mathbf{R}$  matrices. The hysteresis are developed in the elastic range for (a)  $1.0^x\Delta_y$  and (b)  $0.8^x\Delta_y$ .

The results in Fig. 2(b) are similar to those in Fig. 2(a), and show that even for a performance 20% below that of yield, the system still behaves inelastically. This magnifies the importance of developing inelastic control algorithms in structural control frameworks. As a comparison, the control model for evolutionary gain was analyzed using two other records as input: 1) a scaled

1994 Northridge record (Alhambra – Fremont School), near-field excitation – sNR; and 2) modulated (non-stationary) Gaussian white noise with Kanai-Tajimi spectra of parameters  $\omega_g = 3\text{Hz}$ ,  $\xi_g = 0.6$ , and  $G_0 = 0.07$ , nonstationary record – (mKT). Figs. 3(a) and (b) show the controlled hysteresis loops for a ‘minimal’ inelastic performance objective (sNR) and for an elastic objective (mKT) where there was some damage to the system caused by delays. In both cases, the controller using the evolutionary gain satisfactorily met the performance objectives.

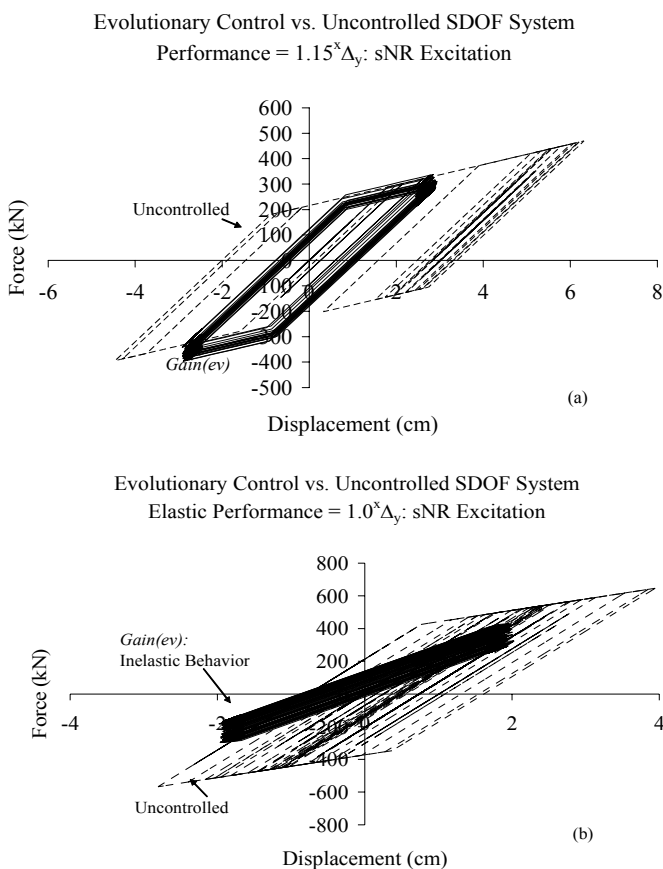


Figure 3: (a) Northridge excitation on a minimally inelastic performance; (b) modulated white noise applied to an elastic performance.

## 4 Conclusions

An optimal nonlinear control solution is proposed for inelastic and elastic systems using an evolutionary gain formulation to compute the required force output generated by semi-active devices per time step. The procedure discretizes



the structural system's stiffness and separates the elastic and inelastic components based on any plastic excursions experienced by the kinematically strain-hardened system. The algorithm, CONON – Control NONlinear Time-History Analysis – was developed to simulate and control inelastic responses in buildings using the proposed evolutionary gain. The code uses an efficient subroutine to expeditiously converge to the desired performance objectives. It was found that the procedure uses a minimal amount of output control force to adequately converge to the desired elastic and inelastic performance conditions (time-delays notwithstanding) for various near-field, non-stationary, and far-field earthquake excitations without relying on weighing matrices that might inconsistently meet performance criteria and result in excessive output forces.

## References

- [1] Ohtori, Y., Christenson, R.E., and Spencer, Jr., B.F. (2004). "Benchmark control problems for seismically excited nonlinear buildings," *Journal of Engineering Mechanics*, 130(4), 366-385.
- [2] Spencer, Jr., B.F., and Nagarajaiah, S. (2003). "State of the art of structural control," *Journal of Structural Engineering*, 129(7), 845-856.
- [3] Yang, G. (2001). "Large-scale magnetorheological fluid damper for vibration mitigation: modeling, testing, and control," Ph.D. Dissertation, Department of Civil Engineering and Geological Sciences, University of Notre Dame, Notre Dame, IN.
- [4] Federal Emergency Management Agency (FEMA) (2001). "NEHRP recommended provisions for seismic regulations for new buildings and other structures. Part 1: Provisions," *FEMA 368*, Washington, DC.
- [5] International Code Council, *2000 International Building Code*, Falls Church, VA., 2000.
- [6] Makris, N. (1997). "Rigidity-plasticity-viscosity: can electrorheological dampers protect base-isolated structures from near-source ground motions?" *Earthquake Engineering & Structural Dynamics*, 26, 571-591.
- [7] Kelly, J.M., (1999), "The current state of base isolation in the United States," *Proc., 2<sup>nd</sup> World Conference on Structural Control*, Kyoto, Japan, Vol. 1, 1043-1052.
- [8] Madden, G., Symans, M., and Wongprasert, N. (2002). "Experimental verification of seismic response of building frame with adaptive sliding base-isolation system," *Journal of Structural Engineering*, 128(8), 1037 – 1045.
- [9] Attard, T. (2007). "Controlling all Inter-Story Displacements in Highly-Nonlinear-Moment-Resisting Frames Using Optimal Passive Damping," *Journal of Structural Engineering, ASCE*. Accepted for Publication.
- [10] Spencer, B.F. and Nagarajaiah, S. (2003). "State of the art of structural control," *Journal of Structural Engineering*, 129(7), 845-856.
- [11] Varadarajan, N. and Nagarajaiah, S. (2004). "Wind response control of building with variable stiffness tuned mass damper using empirical mode



- decomposition / hilbert transform,” *Journal of Engineering Mechanics*, **130**(4), 451-458.
- [12] Gavin, H.P., Alhan, C. and Oka, N. (2003). “Fault Tolerance of Semi-Active Seismic Isolation,” *Journal of Structural Engineering*, **129**(7), 922-930.
- [13] Franklin, G.F., powell, J.D., and Emami-Naeini, A. (2002). “Feedback Control of Dynamic Systems,” Prentice-Hall, Upper Saddle River, New Jersey.
- [14] Attard, T., and Mignolet, M. (2005). “Evolutionary Model for Random Plastic Analyses of Shear-Frame Buildings Using a Detailed Degradation Model,” *9<sup>th</sup> International Conference on Structural Safety and Reliability*, ICOSSAR2005, Rome, Italy

