

# Modelling Stirling engines by means of an electrical analogy

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## Abstract

The increase of energy efficiency in pyro-metallurgical plants has been extensively analysed in the last decades. Since the energy demand for these plants is rising, one way to improve the energy-efficiency is to recover any eventual thermal wastes. This task is challenging because the waste heat has a low thermodynamic potential (its temperature does not exceed 120°C). Hence, the energy conversion of these wastes becomes difficult, because few technologies can work under low temperature differentials. One of them is the Stirling engine, which is gaining importance in the scientific community. This interest is further justified by the following reasons: the Stirling ideal cycle is inherently efficient (it reaches the Carnot efficiency limit); the working fluids are not hazardous for pyro-metallurgical plants and the technology is reliable at relatively low cost. For these reasons, the need of a trustworthy model to predict the engine behaviour is of primary importance. A part of such a model is presented in this work. It describes thermodynamic and part of the dynamic effects occurring in the expansion and compression chambers, the heater, the cooler and the regenerator of a Stirling machine. A model based on an electrical analogy between the pressure/voltage and flow-rate/current is developed. A preliminary validation of the approach is discussed by comparing the predictions of the model with the data collected for the RE1000 free piston Stirling engine.

*Keywords: free piston Stirling engines, conservation equations, electrical analogy, equivalent network.*

# 1 Introduction

Many metallurgical plants are energy intensive. This aspect has brought many engineers to enhance the industrial energy efficiency; however, this improvement cannot be reached by simply reducing the energy consumption, as the latter is directly proportional to the production of output goods (i.e., the aluminium production in a smelter is linearly proportional to its energy consumption). Therefore, in order to achieve the aforementioned goal, the industries can improve the energy efficiency by recovering thermal wastes, which are rejected to the environment. They can be used for heating purposes or converted into useful work, by means of a proper technology (by using Organic Rankine Cycle engines, Stirling engines or thermoelectric modules). Amongst all these technologies, Stirling engines are gaining a particular interest [11]. Several reasons can justify this concern. An analysis of the ideal Stirling cycle shows that it reaches the Carnot-efficiency limit, since it undergoes two isothermal and two isochoric transformations [2]. Another important aspect of Stirling engines is their ability of converting low temperature heat into useful work; several patents of Stirling engines powered by solar energy [9], as well as by fossil fuels [11], are available in the open literature. For instance, Formosa *et al.* [4] developed an engine that works under the conditions of 150°C for the hot source and 25°C for the cold one. Moreover, the Stirling engine technology is reliable at low costs and it satisfies the industrial safety requirements, because the working fluid is not hazardous for metallurgic plants (i.e., the working fluid may be air).

Therefore, the need of trustworthy models able to predict the dynamical behaviour of these engines is necessary. One of the most known approaches for modelling is the Schmidt analysis [21]; however, it describes the thermodynamic effects of the engine, excluding the mechanical dynamics (which proves to be fundamental when the behaviour of the whole engine is evaluated). Hence, researchers have tried to extend the model. A thermoacoustic approach can be used by developing an analogy between the working-gas pressure/volumetric flow rate and the current/voltage of the engine. In this way, an equivalent electrical network of the engine is obtained; the dynamical effects are analysed by means of the Newton equation of motion applied to engine moving parts (power piston and displacer), while the continuity and the momentum conservation equations take into account the effects occurring in the expansion and compression chambers, heater, cooler and regenerator [3].

The electrical analogy proposed by Formosa *et al.* [4] is not the only analogy available in the literature; amongst others we can mention the works of Olson [13], and Herrmann and Schmid [7]. The use of an electrical analogy is very common in thermoacoustic theory, too. One of its milestone has been proposed by Swift [17], who developed an analogy (known as five-element impedance model) from continuity, momentum and energy conservation equations. Thus, he was able to describe the behaviour of both heat engines (thermoacoustic Stirling and standing-wave engines) and refrigerators (orifice-pulse tube, standing-wave refrigerators) by applying the conservation equations to the working gas [17]. Swift's electrical



analogy has been further improved, in particular by Huang and Chuang [8] who added more mathematical rigour to the Swift's theoretical approach.

The present paper aims to accomplish two tasks. The first one is to present a theoretical approach based on an electrical analogy similar to the one proposed by Huang and Chuang [8]. This analogy relies on the conservation equations (continuity, momentum and energy) and their linearisation for the harmonic evolution in the vicinity of an operation point. The solution of the system of differential equations, different from the one given by Huang and Chuang, is suggested. The second one is to test the electrical analogy to predict the pressure drops in three components of a conventional Stirling engine, i.e., the heater, the cooler and the regenerator. Five experimental correlations are used to estimate the friction factor for the regenerator (which is the most critical component of Stirling engines). Their validity will be discussed by comparing the predictions of the electrical-analogy model with data collected at the NASA Lewis Research Centre [16] for a RE1000 free piston Stirling engine.

## 2 Description of the model

As mentioned above, the proposed model relies on the conservation equations. Since the working gas may be air, argon or helium [4, 11, 16], the perfect gas law is introduced. In order to write the conservation equations, the convective term in the momentum equation is neglected, as the spatial derivatives of the velocities are an order of magnitude lower than the temporal derivatives [17]. Moreover, we will introduce the Rott's hypothesis [15], which states that the variables under analysis (mass flow rate, gas pressure and temperature) can be written as the sum of a mean and an oscillatory term, where only the latter depends on the temporal coordinate. Therefore, a generic variable  $\psi(x, t)$  can be written as:

$$\psi(x, t) = \bar{\psi}(x) + \psi'(x, t) \quad (1)$$

The oscillatory term is written in complex notation, hence,  $\psi'(x, t) = \tilde{\psi}'(x) e^{i\omega t}$ . We assume that the Laplace transform of  $\psi(x, t)$  exists:

$$\mathcal{L}\{\psi(x, t)\} = \Psi(x, s) \quad (2)$$

This means that  $\psi(x, t)$  has the following properties:  $\psi(x, t)$  is piecewise continuous on the interval  $0 \leq t \leq \tau$  for any  $\tau \in \mathbb{R}_0^+$  and  $|\psi(x, t)| \leq M e^{-kt}$  for some constants  $k$  and  $M$  (that is, the exponential function is an upper bound of  $\psi$ ) [10]. With these hypotheses, an electrical analogy model is obtained.

### 2.1 Model for the expansion and the compression chamber

In both the expansion and the compression chambers, the gas pressure and temperature are assumed to be spatially uniform; therefore, only the continuity



equation is required [4, 8]. Introducing the state equation yields to:

$$\frac{1}{RT_i} \frac{\partial p_i(x, t)}{\partial t} = -\frac{1}{A_p} \nabla \dot{m}_i(x, t) \quad (3)$$

where  $R$  is the gas constant for the working gas ( $J kg^{-1} K^{-1}$ ),  $T_i$  is the temperature ( $K$ ), the index  $i$  stands for both expansion,  $e$ , and compression,  $c$ ,  $p_i$  is the gas pressure ( $Pa$ ),  $t$  is the temporal variable ( $s$ ),  $x$  is the spatial one ( $m$ ),  $A_p$  is the surface of the piston ( $m^2$ ) and  $\dot{m}_i$  is the mass flow rate ( $kg s^{-1}$ ). Introducing the Rott's hypothesis and neglecting high order terms, eqn (3) becomes:

$$\frac{1}{RT_i} \frac{\partial p'_i(x, t)}{\partial t} = -\frac{1}{A_p} \nabla \dot{m}'_i(x, t) \quad (4)$$

The linearisation of the spatial term of eqn (4) in the complex domain gives the variations of the mass flow rate in the expansion (or compression) chamber; thus,

$$\Delta \dot{M}'_i(x, s) = \dot{M}'_p(x, s) - \dot{M}'_c(x, s) = C_i s P'_i(x, s) \quad (5)$$

with  $C_i = \frac{A_p}{RT_i}$  ( $s^2$ ). Therefore, a capacitance  $C_i$  models the variations of the mass flow rate in the expansion (or compression) chamber. The equivalent electrical circuit is shown in Figure 1.

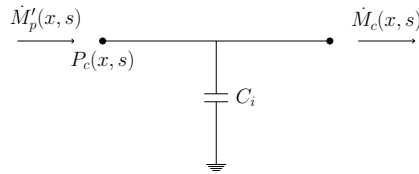


Figure 1: Equivalent circuit for the compression/expansion chamber.

## 2.2 Model for the heater and the cooler

The spatial variations of the gas pressure in both heater and cooler cannot be neglected; therefore, the momentum equation must be solved. However, it can be considered that the gas temperature is constant and equal to the heat exchanger temperature; thus, it is unnecessary to solve the energy conservation equation for the gas. Nevertheless, eqn (3) must be coupled to the momentum equation. Introducing the state equation and recalling the hypotheses listed at the beginning of this section, the momentum equation can be written as:

$$\frac{1}{A_t} \frac{\partial \dot{m}_q(x, t)}{\partial t} + \frac{\partial p_q(x, t)}{\partial x} - \mu \frac{\partial^2 u}{\partial x^2} = 0 \quad (6)$$

In this equation, the index  $q$  stands for the heater ( $h$ ) and for the cooler ( $k$ ), while  $\mu$  is the gas viscosity ( $Pa \cdot s$ ) and  $u$  is the gas velocity ( $ms^{-1}$ ). Assuming that

the gas velocity can be written as eqn (1) and that its oscillatory part is sinusoidal we have  $u(x, t) = \bar{u}(x) + U \sin(kx) \sin(\omega t)$  and a simpler equation is found. Following the procedure shown in the previous subsection, the Rott's hypothesis is applied and the highest order terms are neglected. The simplified momentum conservation equation is written as:

$$\frac{1}{A_t} \frac{\partial \dot{m}'_q(x, t)}{\partial t} + \frac{\partial p'_q(x, t)}{\partial x} + \frac{\kappa}{A_t} \dot{m}'_q(x, t) = 0 \quad (7)$$

where  $\kappa \text{ (s}^{-1}\text{)}$  is an experimental coefficient given by Roach and Bell [14]:

$$\kappa = 0.1556 Re_{max}^{-0.201} \frac{u_{max}}{d_h} \quad (8)$$

where  $Re_{max}$  is the Reynolds number evaluated at the maximal velocity of the oscillating flow ( $u_{max}$ ) and  $d_h$  is the hydraulic diameter. In order to find  $u_{max}$ , we assumed a periodic function of the velocity (i.e., a sine wave); therefore, from the known value of its root mean square  $u_{rms}$ , the amplitude is found according to the following relationship:

$$u_{max} = \sqrt{2} u_{rms} \quad (9)$$

Eqns (4) and (7) represent a system of two differential equations with two unknown functions  $\dot{m}'_q$  and  $p'_q$ . Fox [5] provided the solution of this system; assuming known values of the pressure and the mass flow rate at the inlet of the heat exchanger are known, the equivalent outlet values are found by means of the Laplace transform [4, 5, 8], hence:

$$\begin{bmatrix} P'_q(L_q, s) \\ \dot{M}'_q(L_q, s) \end{bmatrix} = \begin{bmatrix} \cosh(\Gamma L_q) & -Z_{ct} \sinh(\Gamma L_q) \\ -Z_{ct}^{-1} \sinh(\Gamma L_q) & \cosh(\Gamma L_q) \end{bmatrix} \begin{bmatrix} P'_q(0, s) \\ \dot{M}'_q(0, s) \end{bmatrix} \quad (10)$$

With  $\Gamma = \sqrt{Z_t Y_t}$ , and  $Z_{ct} = \sqrt{Z_t/Y_t}$ ,  $Z_t = R_t + sL_t$  and  $Y_t = sC_t$ . Introducing the series expansion of the hyperbolic functions in eqn (10), the equivalent electrical equations are obtained. The equivalent electrical circuit shown Figure 2 is used to model the heat exchangers. The resistance ( $m^{-2}s^{-1}$ ), the capacitance ( $s^{-2}$ ) and the inductance ( $m^{-2}$ ) are given respectively by:

$$R_t = \frac{\kappa}{A_t}; C_t = \frac{A_t}{RT_q}; L_t = \frac{1}{A_t} \quad (11)$$

where  $T_q$  is the temperature of the heat exchanger.

### 2.3 Model for the regenerator

Along the length of the regenerator, which may be a porous media of metallic wires [16], the temperature cannot be considered constant; hence the energy equation must be solved simultaneously with the momentum and the continuity equation. As a result, a system of four equations should be treated, that is, the mass, momentum



and energy equation for the working gas and an additional energy equation for the regenerator matrix. Both gas and matrix temperatures have the form given by eqn (1). The state equation is introduced in eqn (3), which yields to:

$$\frac{1}{R\bar{T}_r} \frac{\partial p'_r(x, t)}{\partial t} + \frac{\bar{p}_r}{R\bar{T}_r^2} \frac{\partial T'_r(x, t)}{\partial t} - \frac{1}{A_r} \frac{\partial \dot{m}'_r(x, t)}{\partial y} = 0 \quad (12)$$

where  $A_r$  is the regenerator cross-section area. The momentum conservation equation is not affected by the introduction of the energy equations; however, the coefficient  $\kappa$  is not suitable for evaluating the viscous pressure drops in a porous media. The momentum equation for the gas in the regenerator is written as:

$$\frac{1}{A_r} \frac{\partial \dot{m}'_r(x, t)}{\partial t} + \frac{\partial p'_r(x, t)}{\partial x} - f \frac{\nu Re}{A_r dw} \dot{m}'_r(x, t) = 0 \quad (13)$$

where  $\nu$  is the kinematic viscosity ( $m^2 s^{-1}$ ) and  $f$  is the flow friction factor given by

$$f = C_{fd} + \frac{C_{sf}}{Re} \quad (14)$$

The values of  $C_{fd}$  and  $C_{sf}$  have been experimentally evaluated [19]. Their values are listed in Table 1, according to the measurements collected from five different works.

Table 1: Correlations for the flow friction factor in the regenerator.

| $C_{fd}$ | $C_{sf}$ | Reference                 |
|----------|----------|---------------------------|
| 0.5274   | 68.556   | Gedeon and Wood [6]       |
| 0.3243   | 44.71    | Tong and London [20]      |
| 0.4892   | 47.245   | Blass [1]                 |
| 0.337    | 33.603   | Miyabe <i>et al.</i> [12] |
| 0.5315   | 40.7413  | Tanaka <i>et al.</i> [18] |

We recall that the last term in eqn (13) should be divided by a factor 4, because the Darcy Weisbach's formula is four times the Fanning friction factor. For this reason, the coefficients listed in Table 1 have already been divided by 4. Moreover, the Reynolds number,  $Re$ , is calculated as:

$$Re = \frac{uw}{\nu} \quad (15)$$

where  $w$  is a characteristic length function of the matrix wire diameter  $d$  (m) and porosity of the regenerator  $\varepsilon$  estimated by [19]

$$w = d \left( \frac{1}{\sqrt{\sqrt{0.25 + \frac{4(1-\varepsilon)}{\pi}} - 0.5}} - 1 \right) \quad (16)$$

The energy equation for the gas is then written as:

$$C_{fr} \frac{\partial p'_r(x, t)}{\partial t} + \frac{\partial \dot{m}'_r(x, t)}{\partial x} + \frac{C_{tr}}{\tau_{gr}} [T'_r(x, t) - T'_s(x, t)] = 0 \quad (17)$$

where  $T'_s$  is the matrix regenerator temperature. The multiplicative coefficient are defined as

$$C_{fr} = \frac{A_r}{RT_r}; C_{tr} = \frac{\bar{p}_r A_r}{RT_r^2}; \tau_{gr} = \frac{\bar{p}_r V_r \varepsilon (c_p - R)}{RT_r h A_{HT}} \quad (18)$$

where  $c_p$  is the helium heat capacity ( $J kg^{-1} K^{-1}$ ),  $V_r$  is the regenerator volume ( $m^3$ ),  $h$  is the convective heat transfer coefficient ( $W m^{-2} K^{-1}$ ) and  $A_{HT}$  is the surface over which convection occurs. In a similar fashion, we derive the energy equation for the metallic matrix:

$$\tau_{sr} \frac{\partial T'_s(x, t)}{\partial t} - [T'_r(x, t) - T'_s(x, t)] = 0 \quad (19)$$

with  $\tau_{sr} = \frac{\rho_s c_s V_r (1-\varepsilon)}{h A_{HT}}$  and  $\rho_s$  and  $c_s$  are the density ( $kg m^{-3}$ ) and the heat capacity of the matrix, respectively.

Eqns (12), (13), (17) and (19) form a system of differential equations that can be solved in the complex domain. By introducing the expressions for  $T'_r(x, s)$  and  $T'_s(x, s)$ , which are the Laplace transforms of  $T'_r(x, t)$  and  $T'_s(x, t)$  from eqns (17) and (19), into eqn (12), yields

$$\frac{\partial \dot{M}'_r(x, s)}{\partial x} + C_{fr} s P'_r(x, s) = 0 \quad (20)$$

Nevertheless, the momentum equation remains unchanged (see eqn (13)):

$$\frac{\partial P'_r(x, s)}{\partial x} + (R_r + s L_r) = 0 \quad (21)$$

where  $R_r = f \frac{\nu Re}{A_r d w}$  ( $m^{-2} s^{-1}$ ) and  $L_r = 1/A_r$  ( $m^{-2}$ ). Eqns (20) and (21), which are equivalent to eqn (10), are linear. Therefore, both pressure and mass flow rate variations are modelled by the equivalent electrical circuit shown in Figure 2.



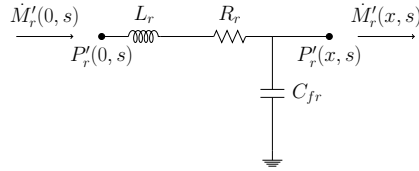


Figure 2: Equivalent circuit for the heater/cooler and regenerator.

### 3 Preliminary validation of the model

The model described above cannot be completely validated because it does not take into account all dynamic effects occurring in Stirling engines (i.e., power piston and displacer). Therefore, this section is limited to test its capability to predict the pressure losses in the heat exchangers. For this purpose, based on Figure 2, it is assumed that the pressure gradient  $\frac{dp}{dx}$  in these components corresponds to the steady-state AC voltages due to changes in current intensities (i.e., the mass flow rate  $\dot{m}$ ). Hence, we can express:

$$\frac{dp}{dx} = R\dot{m} \quad (22)$$

The integration of this equation gives  $\Delta p = R\dot{m}H_{reg}$ , with  $H_{reg}$  the regenerator length. The use of this equation is justified because the inductance behaves as a short circuit and the capacitance behaves as an open one in steady state conditions. In the following subsections, eqn (22) is used to estimate the pressure drops in the heat exchangers and in the regenerator of a conventional Stirling engine.

#### 3.1 Description of the RE1000 Stirling engine

The selected pressure drop data were collected at the NASA Lewis Research Centre [16] from their RE1000 system. Figure 3 shows a simplified schematic of this engine.

Helium is used as working gas. This gas flows back and forward along the expansion chamber throughout the heater, the regenerator, the cooler and the compression chamber. As shown in the figure, the pressure variations of the working gas in the compression chamber move a power piston. The total power produced by this piston is dissipated by friction in the dashpot shown in the figure. The displacer assures the gas flow from the expansion to the compression chamber (and vice versa) to occur. Its oscillatory motion movements are maintained by the spring effect of the working gas and of the gas spring.

#### 3.2 Pressure drops in the heat exchangers

The predictions of the model presented in the previous section have been compared with the experimental data collected by Schreiber *et al.* [16]. They measured the



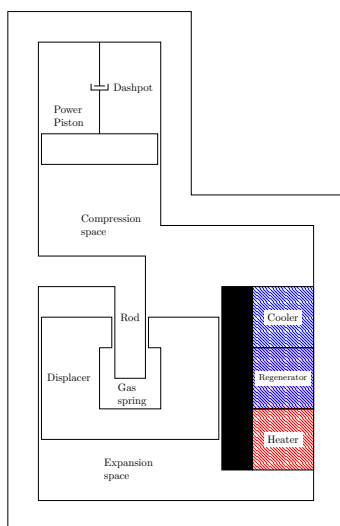


Figure 3: Sketch of the engine under analysis [16].

pressure drops for different values of mass flow rate in both heat exchangers as well as in the regenerator. Figures 4, 5 and 6 compare the pressure drops measurements for the RE1000 heat exchangers and the prediction of the electrical analogy model (eqn (22)).

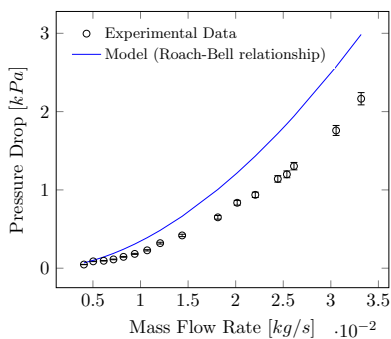


Figure 4: Pressure drops (cooler).

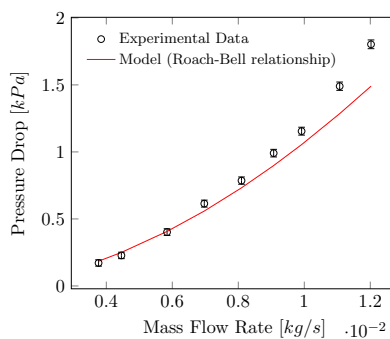


Figure 5: Pressure drops (heater).

For both the cooler and the heater, it can be observed that the predictions follow the experimental trends. Nevertheless, the model over estimates the cooler data, while it underrates the data for the heater. The mean relative error for the cooler is 33%, while for the heater the error is around 10%. These errors are partially due to the use of the experimental correlation proposed by Roach and Bell [14],

based on the value of the maximal velocity of the oscillatory flow. As discussed in Section 2.2, it has been calculated according to eqn (9) (i.e., the root mean square velocity is calculated from the mass flow rate data). It is observed that further studies should be performed to improve the model proposed for the heat exchangers.

Since the regenerator is the most critical element for Stirling engines, where the highest pressure drops are observed, five correlations have been applied to evaluate the friction factor coefficient required by eqn (14). Table 1 provides the list of the correlations used in this study and the results are shown in Figure 6. It can be observed that the minimum relative error of 9% is obtained when the Gedeon and Wood's correlation is used [6]. We recall that in their work, Huang and Chuang [8] used a relationship based on Tanaka *et al.*'s results. However, as shown in Figure 6, this correlation would under estimate the RE1000 data.

In order to reduce the errors of Gedeon and Wood's experimental correlation, a further analysis has been performed and a relationship having the form of eqn (14) has been evaluated from the data collected for two different porosities; hence:

$$f_{\epsilon=0.759} = 1.4 + \frac{79.24}{Re} \quad (23)$$

This equation is used to predict the pressure drop shown in Figure 6; the calculated friction factor is presented in Figure 7. As it can be seen, the accordance between the data and the predictions is satisfactory (the maximal relative error is 3.3%). However, it must be pointed out that this correlation has been evaluated for a regenerator of porosity of 75.9% and it cannot be used for other values. For instance, Figure 7 also shows the friction factor for a regenerator porosity of 81.2% [16], given by the following correlation:

$$f_{\epsilon=0.812} = 0.68 + \frac{86.79}{Re} \quad (24)$$

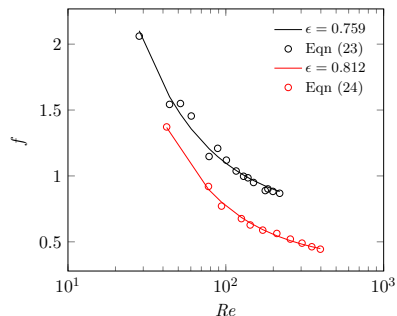
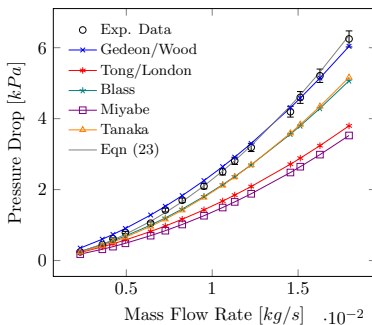


Figure 6: Regenerator pressure drops. Figure 7: Friction factor vs Reynolds number.

This means that the coefficients of the function given by eqn (14) cannot be considered constant, as proposed in the literature [1, 6, 12, 18, 20]. In fact, eqns

(23) and (24) show that there is a strong dependency of the coefficients with the porosity. Having access to a larger data bank could help to obtain a better correlation.

## 4 Conclusion

A model that describes the behaviour of the expansion and compression chambers, the heater, the cooler and the regenerator a conventional Stirling engine has been proposed. The model is based on the equivalent electrical circuits developed by means of an electrical analogy between pressure/voltage drop and mass flow rate/electric current. The results in Figures 4, 5 and 6 show that the model well predicts the pressure drops in the heater, while we acknowledge appreciable over estimation for the cooler. Since the pressure drops are estimated according to eqn (22), we assume that the evaluation of the coefficient  $\kappa$  is responsible of these errors. The use of eqn (9) should be revised and other experimental correlations are to be tested in order to attain a better agreement with the experimental data. The analysis of the regenerator pressure drop has led us to test five correlations proposed in the literature for estimating the friction factor. The comparison of the predictions of the pressure drop with the data collected on a regenerator shows a maximal relative error of 9% when the Gedeon and Woods correlation is used; however, a new relationship providing a relative error of 3.3% is proposed. Moreover, it has been observed that the regenerator friction factor not only depends on the mass flow rate, but also on the porosity. Additional work is still necessary to find a more reliable correlation.

It is worth to notice the usefulness of the model presented here: the thermodynamic analysis performed here could be coupled with a dynamic analysis; for instance, the Newton equation for the displacer and the power piston shown in Figure 2 can be introduced. In this way, it is possible to develop an equivalent network able to predict the behaviour of the Stirling engine, in the same way as Formosa *et al.* presented in their work [4].

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