An analytical study on the performance characteristics of a multi-stage thermoelectric cooling system

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Abstract

A thermoelectric module is a device, which can transfer heat from one surface to another surface when a current is applied. A cooling system employing the thermoelectric module is generally known as a thermoelectric cooling system. A thermoelectric cooling system has no moving parts and does not require any refrigerant, so they are environmentally friendly, inherently reliable, and virtually maintenance free. In this study, a numerical analysis for optimization of a thermoelectric cooler is presented. A single-stage system, cuboids type twostage system and pyramid type two-stage system are analyzed concerning maximum cooling capacity. For two-stage thermoelectric systems, which are composed of two thermoelectric modules, the optimum ratio of current for each stage is analyzed theoretically. The cooling load and heat sink design are also analyzed to optimize the cooling capacity. When the temperature of the hot side of the thermoelectric module was fixed for all systems, the pyramid type twostage system showed an approximately 10.48% higher cooling capacity than that of a single system. The method to improve the performance of the pyramid type two-stage system was suggested by reducing the wasted area of the module surface.

Keywords: thermoelectric module, multi-stage, refrigerator.

1 Introduction

As there are energy depletion issues all over the world, needs of new technologies to replace conventional compression-type refrigerator systems have arisen. One of the alternatives is the thermoelectric cooling system. A



thermoelectric module is a device, which can transfer heat from one surface to another when a current is applied to the module. The thermoelectric module has several advantages such as accuracy, no noise, a small size, and it is eco-friendly. For these reasons, the thermoelectric module has been widely used in the military industry, the medical field, and in many commercial products [1-4]. In this research, the theoretical design and analysis of a thermoelectric cooler is presented. In addition, a method to optimize a two-stage thermoelectric system is suggested.

2 Theoretical analysis

Figure 1 shows the thermoelectric cooler that is designed in this study. The thermoelectric module is attached on the inside wall of a refrigerator. On each side of the thermoelectric module, a heat sink composed of a fin and fan is placed so that it transfers heat from the inside of the refrigerator to the outside effectively. Fig. 2 shows the cases of thermoelectric arrangement considered in this research. Two types of thermoelectric modules with different sizes were used; 40 x 40 mm and 50 x 50 mm.







Figure 2: Arrangement of thermoelectric modules. (a) single-stage, (b) cuboid type two-stage, (c) pyramid type two-stage, (d) pyramid type two-stage with high conductance material.



2.1 Component analysis

Fig. 3 shows the thermal circuit for a thermoelectric cooling system. A basic experiment was conducted to find the thermal resistance of a cooling heat sink $(R_{hs,c})$ and the overall heat transfer coefficient (UA) of the refrigerator. For the basic experiment, a commercial 20 L refrigerator was remodelled to a thermoelectric cooling system. Fig. 4 shows the measured temperatures for each component. Table 1 shows the calculation results based on the measured temperatures.



Figure 3: Thermal circuit of the thermoelectric cooling system.





Table 1:	Calculation	results	based	on	the	experiment	
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Qc	28.10 W
Q _h	87.05 W
Heat transfer coefficient, UA	1.6600 W/°C
Thermal resistance, R _{hs,c}	0.1339°C/W



2.2 Analytical study overview

The flow chart for theoretical simulation analysis is shown in Fig. 5. Visual Basic 6.0 was used to conduct and calculate the flow chart. The calculation starts with assumption of T_c . Then, the cooling load of refrigerator and the cooling capacity of the thermoelectric module can be calculated individually. For steady state, the cooling capacity and cooling road must be the same. Iteration is conducted to satisfy energy balance until the cooling load of the refrigerator and the cooling capacity of the thermoelectric module become the same. In this way, Q_c can be obtained for specific current input. The change of Q_c with variable currents can also be calculated by repeating previous steps with different current inputs.

The input data for the analytical program is presented in Table 2. The information of thermoelectric modules was obtained from the manufacturer. Assume that the temperature on the hot side of the thermoelectric module is fixed at 40°C and the contact resistances are negligible.

Thermoelectric module	HMN 6040	HMN 1550	
Seebeck coefficient, α (VK ⁻¹)	0.051816	0.045974	
Electrical resistance, R (Ω)	1.956	0.882	
Thermal conductance, K (WK ⁻¹)	0.464	0.932	
M (Number of semiconductor)	254	254	
Size	40x40 mm	50x50 mm	
Ambient temperature	25°C		
T _h	40°C (fixed)		
Internal volume	20 L		

Table 2: Input data for the theoretical simulation program.

2.3 System modelling

 Q_c and Q_h of the thermoelectric modules are calculated from eqns (1) and (2).

$$Q_{c} = \alpha I T_{c} - \frac{1}{2} I^{2} R - K(T_{h} - T_{c})$$
(1)

$$Q_{h} = \alpha I T_{h} + \frac{1}{2} I^{2} R - K(T_{h} - T_{c})$$
⁽²⁾





Figure 5: Flow chart of the theoretical simulation program.

Each component of the right side represents heat from peltier effect, heat generation due to the electrical resistance, and heat transfer loss by conduction, respectively.

$$T_i = R_{hs,c}Q_c + T_c$$
(3)

$$Q_{\text{cooling load}} = UA(T_o - T_i)$$
(4)

In addition, $Q_{\text{cooling load}}$ can be obtained from eqns (3) and (4). By assuming T_c , not only Q_c can be calculated from eqn (1) but cooling load from eqn (4). For the two-stage system, eqns (5)–(8) are used to find optimum ratio of current for each thermoelectric module and maximum Q_c . Using $Q_{h,1}=Q_{c,2}=Q_m$, T_m is referred to eqn (9) from eqns (6)–(7). Eqn (9) is applied into eqn (5). Under the condition that dQ_c /dI=0, the maximum Q_c can be determined [5].

$$Q_{c,1} = \alpha_1 I_1 T_{c,1} - \frac{1}{2} I_1^2 R_1 - K_1 (T_m - T_{c,1})$$
(5)



$$Q_{h,1} = \alpha_1 I_1 T_{h,1} + \frac{1}{2} I_1^2 R_1 - K_1 (T_m - T_{c,1})$$
(6)

$$Q_{c,2} = \alpha_2 I_2 T_{c,2} - \frac{1}{2} I_2^2 R_2 - K_2 (T_{h,2} - T_m)$$
(7)

$$Q_{h,2} = \alpha_2 I_2 T_{h,2} + \frac{1}{2} I_2^2 R_2 - K_2 (T_{h,2} - T_m)$$
(8)

$$T_{\rm m} = \frac{\alpha_1 I_1 T_{\rm h,1} - \alpha_2 I_2 T_{\rm c,2} + \frac{1}{2} I_1^2 R_1 + \frac{1}{2} I_2^2 R_2 + K_1 T_{\rm c,1} + K_2 T_{\rm h,2}}{K_1 + K_2} \quad (9)$$

3 Results and discussion

Figure 6 presents the cooling capacity of the single-stage thermoelectric system for different input currents. As the current was increased, Q_c increased at first and decreased after the peak point. The peak point exists due to limitation of heat removal capacity of the heat sink on the hot side of the thermoelectric module. The maximum cooling capacity was 31.87 W at 6.4 A.

Figure 7 indicates Q_c of the two-stage systems against different input current ratios I_2/I_1 . In this figure, the cuboid type two-stage system showed the lowest Q_c since $Q_{c,2}$ was too small to cover $Q_{h,1}$ so T_m of the cuboid type two-stage system was much higher than that of the pyramid. In other words, the capability of the bottom thermoelectric module must be bigger than that of the top module to maximize Q_c . This phenomenon happens because Q_h is always bigger than Q_c for commonly occurring designs of thermoelectric modules, which explains why the pyramid type two-stage system shows better results.

The pyramid type with heat transfer enhancement material showed better results than those of the pyramid type without any material. Generally, the thermoelectric module is made of material with a low conductance to prevent heat loss from both surfaces. For this reason, $Q_{c,2}$ which is generated from exposed area of the bottom thermoelectric module cannot be transferred to the upper one effectively. However, if a material with high conductance is placed between two thermoelectric modules as shown in Fig. 2(d), most of the cooling capacity from the bottom thermoelectric module will be transferred to the top module. From these results, the maximum Q_c of the pyramid type two-stage system with high conductance material showed 20.75% higher than that of the pyramid-type without an enhancement material.

Table 3 shows the maximum cooling capacity of each thermoelectric cooler. The pyramid type two-stage system showed about 10.5% higher cooling capacity than that of the single system. Also, the cuboid type two-stage system showed the worse result than any other systems including the single-stage system.





Figure 6: Variation of Q_c with current for the single-stage system.



Figure 7: Variation of Q_c with current ratio for the two-stage systems.

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	0.1	Cuboid type	Pyramid type	Pyramid type
	Single-stage	two-stage	two-stage w/o material	two-stage w/ material
Max. Q _c (W)	31.87	27.95	29.16	35.21
Ratio of				
system to	-	87.70 %	91.50 %	110.48 %
single-stage				

Table 3: Maximum Q_c for each system.

4 Conclusion

In this research, the theoretical design and analysis of thermoelectric cooler is presented. Several thermoelectric cooling systems were analyzed respectively. From the analytical study, the pyramid type two-stage system showed about 10.5% higher cooling capacity than that of the single-stage system. The bigger bottom thermoelectric module covered more Q_h from the top thermoelectric module so it was more effective than any other system. In addition, it was helpful for inserting a high conductivity material between two thermoelectric modules to optimize the pyramid type two-stage thermoelectric system. The maximum cooling capacity of the pyramid-type material with high conductance material was shown to be 20.75% higher than that of an ordinary pyramid-type system. The high conductance material helped in transferring heat away from the contact area.

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