Transactions on Biomedicine and Health vol 7, © 2003 WIT Press, www.witpress.com, ISSN 1743-3525

Use of a mesoscopic dynamic assignment model for approaching the evolution of an urban transportation system in emergency conditions

M. Di Gangi[1], P. Velonà[2]
[1]*Department of Architecture, Planning and Transport Infrastructure, University of Basilicata, Potenza, Italy.*
[2]*Department of Computer Science, Mathematics, Electronics and Transportation, Mediterranea University of Reggio Calabria, Reggio Calabria, Italy.*

Abstract

In this paper a mesoscopic model for the simulation of traffic flow in urban areas of medium dimensions during evacuation conditions is applied. The simulation of traffic with a within-day dynamic approach is necessary, in order to represent the formation and dissipation of the vehicles queues, due to the presence of over saturation phenomena. The model, implemented inside a general assignment procedure, allows to obtain evacuation time and traffic parameters during the dynamic evolution of the evacuation phases.

1 Introduction

The evacuation of an area involves an amount of users (or vehicles) larger than the ones usually considered in the simulation of a steady state working conditions of a transportation system. A static approach does not allow a reliable representation of the dynamic evolution of demand profiles within a reference period; in this case an intra-period dynamic approach (within-day dynamics) is more suitable since it can give a satisfactory representation of both over-saturation and queues formation and scattering phenomena. The considered dynamic model is mesoscopic that is flow characteristics depend on link conditions defined within discrete time intervals; using this approach both evacuation time and temporal evolution of flow conditions can be evaluated. In the following, after a brief description of the properties of dynamic model in point 2, the proposed model will be described in point 3 and some indications for further developments are given in point 4.

2 Dynamic models

Assumptions concerning stationarity in time variations of demand in transportation systems made by static assignment models can be summarised as:

- Inter-period stationarity hypothesis; it is assumed that there is no variation in terms of demand among time intervals with similar characteristics;
- Intra-period stationarity hypothesis; it is assumed that there is no variation in terms of demand within a reference time interval whose length is sufficiently large to allow the system to reach a stationarity condition.

Many studies ([1], [2], [3]) highlighted the limits of such models, due to particular assumptions (i.e. the hypothesis of uniform spatial distribution of traffic), and underlined the importance of a dynamic analysis of traffic evolution to overtake those limits due to stationarity hypotheses.

As a matter of fact, static models do not allow to perform an analysis of phenomena connected to temporal variations in terms of both demand and supply, such as rising and scattering of queues due to temporary peaks of demand and/or capacity reductions of infrastructures.

Thus, in order to comply with these phenomena, it is necessary either to use static models in pseudo-dynamic assignment procedures ([4], [5], [6]), or to remove the intra-period stationarity hypothesis and choose within day dynamic assignment models (Dynamic Traffic Assignment models - DTA).

Passing from static models to dynamic ones is not straightforward; as a matter of fact it is necessary to consider not only time dimension, by introducing another variable, but, especially, to specify in a different way both supply and demand models.

Time dimension can be taken into consideration within the DTA problem in two different ways:

- by introducing a continuous variable τ defined within the reference time interval $[0, \Omega]$, thus simulating the system evolution continuously (continuous models);
- by dividing the time interval $[0, \Omega]$ in n sub-intervals whose length is θ, and introducing a discrete variable τ whose value represent the generic sub-interval (discrete models).

In general, a classification of DTA models can be made depending on the traffic variables representation (continuous or discrete) and on the aspect of the variables representing network performances (aggregate o disaggregate).

So called macroscopic models (or Flow-based Analytical Models: [7], [8], [9], [10], [11], [12], [13]), simulate network performances by means of aggregate variables (speed, density, flow) with explicit capacity, as in static models, and use a continuous representation of traffic; generally, in the formulation of macroscopic models, fluid-dynamic analogies of traffic are adopted.

A second type of models, named mesoscopic (or Packet-based models: [14], [15], [16], [17], [18], [19]), is similar to the previous one in the way of

simulating network performances (aggregate variables with explicit capacity are used), but it is different in terms of traffic representation; mesoscopic models peculiarity consists of a discrete flow representation for group of vehicles (users). A third type is made up by microscopic models, where individual trajectories of all the vehicles are simulated by using disaggregate variables with implicit capacity, and a discrete traffic representation.
The DTA model here presented is mesoscopic and it is based on the one proposed by Cascetta and Cantarella [18] and successively developed by one of the authors [19] e [20] [22] where users are assembled in packets that move on the network discretising demand for each origin-destination pair. The model here described consists of an evolution of the dynamic approach developed for evacuation purposes in [21].

2.1 Supply models

The specification of supply model is different from those commonly adopted in static approach for the explicit modelling of rising and/or scattering of queues at the terminal edge of each link.
In order to formulate a model, it seems necessary to introduce some concepts concerning within-day dynamic assignment:

- the flow of users that, in a certain interval τ is on a link varies from a section to another (it is not uniform), moreover the flow related to a certain section is not constant in time;
- time needed to cross a link can assume different values for different time intervals, depending on time variations in terms of supply and/or demand;
- in congested networks time needed to cross a link a in an interval τ generally depends not only on the characteristics of link a and, in case of not separable cost functions, on characteristics of other links, related to interval τ, but also on traffic characteristics of previous time intervals.

On the basis of these observations, the relation between link flows and path flows (path flow propagation model) can be described as:
Non congested networks

$$\mathbf{f}(\tau) = \Phi(\mathbf{F}(\tau'), \mathbf{t}(\tau') \ \forall\ \tau' \leq \tau)$$
$$\mathbf{T}(\tau) = \Gamma(\mathbf{t}(\tau') \ \forall\ \tau' \geq \tau)$$

Congested networks

$$\mathbf{f}(\tau) = \Phi(\mathbf{F}(\tau'), \mathbf{t}(\mathrm{f}(\tau'')\ \tau'' \leq \tau')\ \tau' \leq \tau)$$
$$\mathbf{T}(\tau) = \Gamma(\mathbf{t}(\mathrm{f}(\tau'')\ \tau'' \leq \tau')\ \ \tau' \geq \tau)$$

where:

Φ	represents the path flow propagation model;
Γ	represents the function giving path times starting from link times
τ, τ' e τ''	represent time intervals belonging to the reference period;
$\mathbf{f}(\tau)$	is the vector of exiting arc flows related to interval τ; the generic element is addressed as $f_a(\tau)$;

Transactions on Biomedicine and Health vol 7, © 2003 WIT Press, www.witpress.com, ISSN 1743-3525

$\mathbf{F}(\tau)$	is the path flow vector whose generic element $F_k(\tau)$ represents the flow of users following path k leaving during time interval τ;
$\mathbf{T}(\tau)$	is the travel time whose generic element $T_k(\tau)$represents the travel time of generic path *k* for an user leaving during interval τ; every travel time is evaluated taking into account the travel times obtained during intervals τ' when the various link making up path *k* are crossed;
$\mathbf{t}(\tau)$	is the link time vector whose generic element $t_a(\tau)$ defines the travel time of link *a* for the generic user entering link during interval τ.

2.2 Demand models

Demand model formulation is similar to the static case, so it is simpler to develop respect the supply model. Model specification slightly differs since it is necessary to define a demand profile, that is the evolution of demand in time, then to form the hypothesis that the choice of leaving interval depending on desired arrival time (it is also possible to define the model considering the desired leaving interval).

In the considered case of rigid demand, path flows are given by:

$$F_k(\tau) = d_{rs}(\tau)\ p_{rs,k}(\tau)(\mathbf{V}_{rs}(\tau))$$

$$V_k(\tau) = -T_k(\tau) - g_k(\tau)$$

where:

$p_{rs,k}(\tau)$	is the path choice probability of path *k* given leaving interval τ and o/d pair r/s;
$\mathbf{V}_{rs}(\tau)$	is the è vector of systematic utilities $V_k(\tau)$ of paths related to pair r/s;
$g_k(\tau)$	takes into account other components of generalised cost, different from travel time, associated to path *k* and perceived by generic user starting its travel during interval τ;
$d_{rs}(\tau)$	is travel demand from origin r to destination s, leaving during interval τ.

2.3 Interaction between demand and supply

The model of interaction between demand and supply comes from the integration of demand model with supply one.

Using the above introduced notations, it can be written as:

Non congested networks

$$\mathbf{f}(\tau) = \Phi(\mathbf{F}(\tau'), \mathbf{t}(\tau')\ \forall\ \tau' \leq \tau)$$

$$F_k(\tau) = d_{rs}(\tau)\ p_{rs,k}(\tau)(\mathbf{V}_{rs}(\tau))$$

Congested networks

$$\mathbf{f}(\tau) = \Phi(\mathbf{F}(\tau'), \mathbf{t}(\mathbf{f}(\tau'')\ \tau'' \leq \tau')\ \tau' \leq \tau)$$
$$F_k(\tau) = d_{rs}(\tau)\ p_{rs,k}(\tau)(\mathbf{V}_{rs}(\tau))$$

3 The proposed procedure

The proposed procedure takes advantage of the above described consolidated methodologies for the analysis of transportation systems. The proposed approach can be used both for the definition (design) and for the checking of evacuation plans: in the first case searching for strategies finalised to optimise the evacuation process; in the second one simulating pre-defined scenarios of the transportation system.
Within the proposed procedure it is possible to define the following three activities:

- definition of escape path;
- network loading;
- evaluation of evacuation times and flow characteristics.

3.1 Path and demand definition

Once defined origin-destination pairs, that is associating a safe destination to each origin zone, paths connecting the la m-th pair are generated by evaluating the k_m ($k_m \geq 1$) paths, in a way that depends on the requirements of the analysis (k-shortest paths, in terms of distance or time, not overlapping paths, etc.), and associating to each one a choice fraction $\pi(k)$.
Let n_{rs} be the number of origin-destination pairs (in the following r/s); let d_m be the number of users moving on the m-th pair r/s; let the reference period Ω be divided in n_τ intervals having the same time length θ and let users start at first n_j intervals; the path flow F_{jk}, that is the number of users leaving during interval j and following path k, in the hypothesis that starts are uniformly distributed, can be defined as $F_{jk} = (\pi(k)\cdot d_m)/n_j$ and its vectorial representation is $\mathbf{F}$.

3.2 Network loading

Assignment problem in the proposed procedure is approached by means of a dynamic network loading model using packets, initially defined in Cascetta and Cantarella [18] and successively developed by one of the authors [19], [22]. In the following a description of the proposed approach is briefly pointed out.

<u>*Demand model*</u>
Let $\mathbf{d}(j, u)$ the demand vector made up by the set of users belonging to class u travelling between the pairs r/s leaving during interval j, and let $K(m, u)$ the set of paths, connecting the m-th pair r/s, used by the users belonging to class u. A path choice probability $\pi(k)$ is associated to each path $k \in K(m, u)$.

Every starting interval j (whose length is θ) is divided into n sub-intervals that, for the sake of simplicity, are supposed to have the same length $\lambda = \theta / n$, and let η be one of these sub-intervals.

The generic packet P(η, k, u) will be made up by an integer number of users $x(\eta, k, u)$, belonging to class u leaving during sub-interval η and following path k, whose value is defined making the hypothesis that the distribution of starts being uniform within interval j.

The dimension of users' distribution among starting sub-intervals underlies path choice, so, at first the number of users choosing the path is computed for each path $k \in \mathrm{K}(m, u)$, then these users are distributed among the starting sub-intervals. The following cases can arise:

- the number of starting users $d_m(j, u)$ is less than the number of paths within the set $\mathrm{K}(m, u)$: in this case only the first $d_m(j, u)$ paths will be loaded, where paths are ranked in descending order respect to the choice probabilities $\pi(k)$;
- the number of users $d_k = d_m(j, u)\pi(k)$, $k \in \mathrm{K}(m, u)$, choosing path k is less than the number n of sub-intervals: in this case users following path k will start only in the first d_k sub-intervals.

A packet is made up by an integer number of users $x(\eta, k, u)$, so it must be verified that:

$$x(\eta, k, u) \in N$$

$$\Sigma_{\eta=1, n}\ \Sigma_{k\in \mathrm{K}(m,u)}\ x(\eta, k, u) = d_m(j,u)$$

where $d_m(j, u)$ represents the number of users belonging to m-th pair r/s of vector $\mathbf{d}(j, u)$.

The leaving instant of each packet P(η, k, u) coincides with the initial instant of sub-interval η to which it refers.

<u>Supply model</u>

The network is made up by nodes and links. In particular two kinds of link are considered [23], [19]: *running* in which packets move and *queuing* where packets are queued; the functionalities of these two link types can also be represented by using another kind of model that uses an unique link [21]. Points representing packets move on the *running* part depending on speed and cross *queuing* part depending on capacity. In the following, for the sake of simplicity, how model works will be described using the two link model.

<u>Point movement</u>

Point movement can be synthesized as follows: a point **p** located at time ω of interval τ at the abscissa x of link a, owns a temporal residual resource $\Delta\omega^* = \theta - \omega$ to be used to move before the end of the interval.

Let $w_a(\tau)$ the entity that allows the movement of the point on link a during interval τ and $z_{a\tau}(\omega)$ the impedance that point **p** must overcome to reach the end of link a starting from its current position. The resource $\Delta\omega$ necessary to point **p** to reach the end of the link can be expressed as:

$$\Delta\omega = z_{a\tau}(\omega)/w_a(\tau)$$

If the residual resource of the point is sufficient to entirely cross the link, that is $\Delta\omega^* \geq \Delta\omega$, point **p** leaves the link a during the interval τ and its residual resource

is updated by subtracting $\Delta\omega$ to the initial value and can be used to cross, during interval τ, the link following link a on path k.
If otherwise $\Delta\omega^* < \Delta\omega$, point **p** cannot leave link a during interval τ and the impedance that point **p** must overcome to reach the end of link a in the next interval is updated by subtracting $\Delta z_{a\tau} = \Delta\omega^* \cdot w_a(\tau)$ to the initial value.
The physical meaning of quantities $\Delta\omega$, $z_{a\tau}(\omega)$, $w_a(\tau)$ and the initialization of the value of $z_{a\tau}(\psi)$ when the point enters link a at time ψ depend on the type of the link, as described in the following.

Running link
Let L_a the length of link a and $v_a(\tau)$ the crossing speed of the link during interval τ. Point **p** located at abscissa $x_{a\tau}(\omega)$ at instant ω must cover a portion of link whose length is $\Delta x_{a\tau}(\omega) = L_a - x_{a\tau}(\omega)$ and the time needed to cover it is given by $\Delta\omega = \Delta x_{a\tau}(\omega) / v_a(\tau)$.
If residual time is sufficient to entirely cover the link, that is $(\theta - \omega) \geq \Delta\omega$, point **p** leaves the link and its residual time is updated by subtracting $\Delta\omega$ to the initial value; otherwise the point covers on the link a distance $(\theta - \omega)v_a(\tau)$ and stops, at the end of interval, at abscissa $x_{a\tau}(\tau) = x_{a\tau}(\omega) + (\theta - \omega)v_a(\tau) .= x_a(\tau)$
The value of $x_{a\tau}(\psi)$, when point enters the link at instant ψ, is initialized to 0.

Queuing link
Let $C_a(\tau)$ the capacity of the link, that is the maximum flow that can cross link a, and let $q_{a\tau}(\omega)$ be the number of users making up the queue that is in front of point **p** at time ω of interval τ. The time needed to cross the link is given by $\Delta\omega = q_{at}(\omega)/ C_a(\tau)$.
If residual time is sufficient to get over the queue in front of point **p**, that is $(\theta - \omega) \geq \Delta\omega$, point **p** leaves the link and its residual time is updated by subtracting $\Delta\omega$ to the initial value; otherwise the point remain on the link and a number of users $(\theta - \omega)C_a(\tau)$ located before of point **p** leave the queue. The value of the queue in front of point **p** is updated and the point is located at the end of interval at abscissa $q_{a\tau}(\tau) = q_{a\tau}(\omega) -(\theta - \omega) C_a(\tau) = q_a(\tau)$.
The value of the queue in front point **p**, when point enters the link, is initialized to:

$$q_{a\tau}(\psi) = \mathrm{MAX}\ \{ 0, q_a(\tau\text{-}1) + \varphi_{a\tau}(\psi) - C_a(\tau)\ \psi\}$$

where $\varphi_{a\tau}(\psi)$ is the number of users entering the link until the instant ψ when point **p** enters the link.

3.3 Evaluation of evacuation times and flow characteristics

The several link characteristics (flows, densities, queues) related to interval τ are computed, by means of the described model, tracking the movement of each point, representing a packet of users, along the followed path during the considered interval.

Every time one of these points representing the packets of users reaches its fixed destination, the time at which safe place is reached is evaluated and, depending on starting time, travel time are computed.
For every origin-destination pair it is possible to know the time needed to all the users belonging to the pair to clear the network. Evacuation time is thus defined as the difference (whatever is the origin-destination pair) between the starting time of the first user and the time the last users leave the network to reach the safe place.

4. Some remarks

To manage and control those events that modify in a drastic way the assessment of a transportation system in an urban area the use of tools and procedures able to acquire information through the simulation of several scenarios, representing the feasible alternatives to face up to the problem, is needed.
The proposed mesoscopic approach seems to a flexible tool able to simulate a complex event such as the evacuation of an urban area taking into account the connected flow phenomena like the building up and getting over of queues.
The procedure not only allows to simulate scenarios exogenously defined, but also to define an optimal distribution of starting times to reduce the time needed to evacuate an area.

References

[1] Ben Akiva, M., Dynamic network equilibrium research. *Transportation Research*. Pergamon Press, 1985.
[2] Friesz, T. L., Transportation network equilibrium, design and aggregation: key development and research opportunities. *Transportation Research*. Pergamon Press, 1985.
[3] Cascetta, E., *Transportation systems engineering: theory and methods*, Kluwer, 2001.
[4] Russo, F. & Vitetta, A., Urban road transportation analysis in emergency conditions: models and algorithms. *Proceedings of Urban Transport 2000*, Wessex Institute of Technology. Sucharov e Brebbia ed., WIT Press Boston, pp. 533-542, 2000.
[5] Russo, F. & Vitetta, A., Urban transportation system analysis in emergency conditions. *In this book*, 2003.
[6] Velonà, P. & Vitetta, A., Evolution of an urban transportation system in emergency conditions: analysis through a pseudo-dynamic assignment model. *In this book*, 2003.
[7] Merchant, D.K. & Nemhauser, G.L., A model and an algorithm for the dynamic traffic assignment problems. *Trans. Sci.*, 12(3), 183-207, 1978/a.
[8] Merchant, D.K. & Nemhauser, G.L., Optimaly conditions for a dynamic traffic assignment model. *Trans. Sci.*, 1978/b.

[9] Ben Akiva, M., Cyna, M. & De Palma, A., Dynamic Model of Peak Period Congestion. *Transportation Researche*, 1984.

[10]Ben Akiva, M., De Palma, A. & Kanaraglou P., Dynamic Model of Peak Period Traffic Congestion with Elastic Arrival Rates. *Trans. Sci.* 20 (2), 164-181, 1986.

[11]Janson, B. N., Dynamic traffic assignment for urban road networks. *Transportation Research*, 1991.

[12]Carey, M., Nonconvexity of the dynamic assignment problem. *Operations Research*, 35 No. 5, 58-69, 1992.

[13]Papageorgiou, M. & Korsialos, A., Short term traffic forecasting with metanet. *Daccord Short Term Forecasting Workshop* TU Delft., 1998.

[14]Smith, M. J. & Van Vuren, T., Traffic equilibrium with responsive traffic control. *Trans. Sci.*, 1993.

[15]Ben Akiva, M., Cascetta, E., Gunn, H., Smulders, S. & Whittaker, J., DYNA: a real-time monitoring and prediction system for inter-urban motorways. *First World Congress on Applications of Transport Telematica and Intelligent Vehicle-Higway Systems*. Paris, 1994.

[16]Ben Akiva, M., Bierlaire, M., Bottom, J., Koutsopoulos, H. N. & Mishalani, R., Development of a route guidance generation system for a real-time application. *8th IFAC/IFIP/IFORS symposium on transportation systems*. Chania, Greece, 1997.

[17]Ben Akiva, M., Bierlaire, M., Koutsopoulos, H. N. & Mishalani, R., DynaMIT: a simulation-based system for traffic prediction and guidance generation. *Daccord Short Term Forecasting Workshop* TU Delft, 1998.

[18]Cascetta, E. & Cantarella, G.E., A Day-to-day and Within-day Dynamic Stochastic Assignment Model. Transpn. Res. 25a (5), 277-291, 1991.

[19]Di Gangi, M., Modelling Dynamic Network Loading on Transportation Networks through a Continuous Packet Approach. *Mathematical Modelling of Systems*, vol. 2, issue 3, pp.175-196, 1996.

[20]Di Gangi, M., An algorithm for continuos flow Dynamic network loading. *VI World Conference on Transportation Research*. Lione, 1992.

[21]Di Gangi, M., Luongo, A. & Polidoro, R., Una procedura di carico dinamico per la valutazione dei piani di evacuazione. In *Metodi e Tecnologie dell'Ingegneria dei Trasporti, Seminario 1999* a cura di G.E. Cantarella e F. Russo pp. 137-150. Franco Angeli. ISBN 88-464-2643-6, 2001.

[22]Di Gangi, M., Modelli a pacchetti per il carico dinamico delle reti di trasporto. *V Convegno annuale su Metodi e Tecnologie dell'Ingegneria dei Trasporti*, 11-13 dicembre 2002, Reggio Calabria.

[23]Cascetta, E., Cantarella, G. E. & Di Gangi, M., Evaluation of Control Strategies Trough a Doubly Dynamic Assignment Model. *Transportation Research Record* No 1306, pp. 1-13 TRB National Research Council, Washington D.C., 1991.