Mathematical modelling applied to ecosystems: the Gödel's theorem

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Abstract

In the framework of evolutionary physics, we must deal with goal functions instead of state functions: ecodynamic models must be based on relations evolving in time; far-from-equilibrium thermodynamics (Prigogine) is the foundation for a new description of nature. *But if energy and mass are intrinsically conservative and entropy is intrinsically evolutionary, how can entropy be calculated on the basis of energy and mass quantities (entropy paradox)*? This question is still unanswered and all we can do is note that the ecodynamic viewpoint is different from that of classical physics and classical ecology. This paper is an attempt to deal with these concepts.

1 Introduction

Recently some studies in mathematical logic have examined the possibility of getting computers to understand the concept of the passage of time. Indeed, the study of real-time systems, in other words systems in which temporal evolution plays a primary role, has made interesting advances. Specifically, the properties to describe in these systems are not only qualitative, properties which classical temporal logic can express, but also quantitative.

It would be interesting to develop logics that express "eternal" constraints, such as the three dimensions, on one hand, and that tackle the real meaning of evolution, and hence the importance of *events* and their *successions*, on the other.

Nature is evolutionary in character. The more one seeks to comprehend her, in the etymological sense of *enclosing*, *imprisoning*, in our mental schemes, the more she creates relations and complexity, memories and creative possibilities. It

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is the passing of time that prevents us from capturing the fleeting moment of global knowledge.

It is also important to underline that:

- *Space, by its structure, is reversible;*
- *Time, by its structure, is irreversible.*

In order to achieve an ecodynamic description we need to shift our attention from state functions to goal functions and to configurations of processes.

Obviously the mathematical machine *par excellence*, the computer, cannot understand the concept of evolution, the arrow of time. As with all machines, it is indifferent to the irreversibility of time, incapable of understanding the real meaning of time.

We may also underline the following two statements by Jørgensen and Svirezhev [1]:

> The presence of irreducible systems is consistent with Gödel's theorem, according to which it will never be possible to give a detailed, comprehensive, complete and comprehensible description of the world. Most natural systems are irreducible, which places profound restrictions on the inherent reductionism of science.
>
> Many ordered systems have emergent properties defined as properties that a system possesses in addition to the sum of properties of the components: the system is more than the sum of its components. Wolfram [2] calls these *irreducible systems* because their properties cannot be revealed by a reduction to some observations of the behaviour of the components.

2 Discussion

2.1 Gödel theorem

In 1931, the young Viennese Kurt Gödel published a brief memoir on "formally undecidable propositions of *Principia mathematica* and similar systems" which concerned the incompleteness of a large class of formal theories, including arithmetic, as well as the impossibility of proving their coherence from within the theories themselves. Gödel's theorem [3] is often summarized as: "there is at least one formula of arithmetic that cannot be demonstrated" and with the following formula:

$$(\exists\, y)(x) : \; Dim(x,y) \tag{1}$$

Interpreted in meta-mathematical language, the formula says "there is at least one formula of arithmetic for which no sequence of formulae constitutes a demonstration".

Jørgensen and Svirezhev [1] and Wolfram [2] underline that Gödel's theorem requires that mathematical and logical systems (i.e. purely epistemic, as opposed to ontic) cannot be shown to be self-consistent within their own frameworks but only from outside. A logical system cannot itself (from inside) decide on whether it is false or true. This requires an observer from outside the system, and this means that even epistemic systems must be open.

The impossibility of completely knowing the world is linked to the principle of Pascal, according to which the whole is more than the sum of its parts. This deals a heavy blow to reductionism.

The mutual irreducibility of space and time makes it impossible to completely know living evolving systems.

2.2 Thermodynamic uncertainty

"At the instant when position is determined, the electron undergoes a discontinuous change in momentum. This change is the greater the smaller the wavelength of the light employed – that is, the more exact the determination of the position.

Thus, the more precisely the position is determined, the less precisely the momentum is known, and conversely" (Heisenberg [4]).

According to the laws governing the Compton effect, p_1 and q_1 are related by:

$$p_1 q_1 \approx h \tag{2}$$

$$E_1 t_1 \approx h \tag{3}$$

Equation (3) is equivalent to eqn (2) and shows that precise determination of energy can only be had at the cost of a corresponding uncertainty in time.

Another relation can be derived from the uncertainty between position and momentum. Let v and E be the velocity and energy corresponding to momentum p_x, then

$$v \Delta p_x \frac{\Delta x}{v} \geq h \tag{4}$$

$$\Delta E \Delta t \geq h \tag{5}$$

where ΔE is the uncertainty of energy corresponding to the uncertainty of momentum Δp_x, and Δt is the uncertainty in time within which the particle (or the wave packet) passes over a fixed point on the x-axis [5]. *Irreversibility of time is not considered, since in the quantum mechanical paradigm, time is assumed to be reversible.*

It is possible to link these concepts with the generalized uncertainty associated with the presence in the Universe of both conservative (space, mass) and evolutionary quantities (time, life span).

In dealing with evolutionary (living) systems, we may introduce a third concept: that of *Thermodynamic Uncertainty* related to the intrinsic irreversible character of time. Let us say that a thermodynamic uncertainty arises from the experimental existence of the arrow of time and from the experimental evidence that, during the measurements, time goes by.

Since time flows during the interval of an experiment (measurement), conservative quantities (energy and/or position) may also change leading to further uncertainty. Astrophysicists have recently discovered that the mass of a star is related to the star's life span; the greater the mass, the shorter the life span. This too may be related to the uncertainty principle. It seems that there is a sort of uncertainty relation between space and time, space being related to mass and energy, which are conservative quantities.

2.3 The role of entropy

Entropy breaks the symmetry of time and can change irrespective of changes in energy, energy being a conservative and reversible quantity, whereas entropy is evolutionary and irreversible *per se*. The flow of a non-conservative quantity, negentropy, makes life flow and the occurrence of a negentropy production term is the difference with respect to analysis based on exclusively conservative terms (energy and matter).

The situation is explained in Figure 1 "The death of the deer": at the moment of death, mass and energy do not change, whereas entropy does. There is an entropic *watershed* between far-from-equilibrium (living) systems and classical systems (the dead deer or any inorganic non living system).

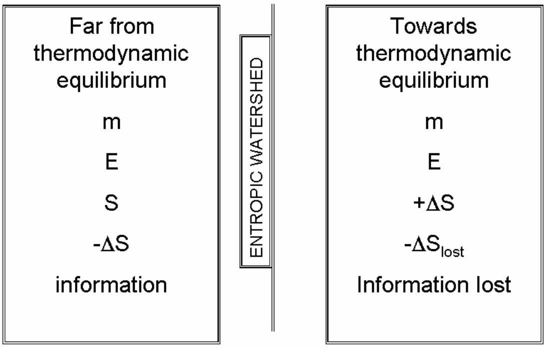

Figure 1: The death of the deer.

We may conclude that in systems far from thermodynamic equilibrium (biological and ecological), *entropy is not a state function, since it has intrinsic evolutionary properties*, strikingly at variance with classical thermodynamics.

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