One-dimensional finite volume simulation of real debris flow events

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Abstract

A numerical model for the simulation of mud flow and debris flow is presented. It is based on an alternative formulation of conservative balance equations, in which source terms are mathematically reorganized in order to guarantee an improved computational stability over complex geometry channels. For numerical implementation, the first order Godunov scheme with Roe's approximation is used. Source terms are computed with Euler's method and added by splitting. Such a simple basic scheme has been chosen to underline that the improved numerical stability depends on the proposed mathematical formulation, and not on a sophisticated numerical scheme. The correct wet-dry front velocity and propagation mechanism have been verified with standard dam-break test cases, and particular attention has been directed to the celerity computation inside the Roe's scheme when dealing with irregularly shaped cross-sections. The numerical model has already been verified with analytical tests and laboratory experiments. In this work, the model is applied to two real events that occurred in North-Eastern Italy. The first is a debris flow that took place in the Upper Boite Valley, in the proximity of Cortina d'Ampezzo, in 1998, the second is a mud flow event located in the Stava Creek Valley in 1985. These events have been chosen thanks to the wide documentation and significant amount of field data available, which include topographical surveys, flow velocity measures and flow depth estimations.
Keywords: mud flow, debris flow, wave propagation, source terms.

1 Introduction

The aim of the present work is to check a numerical model that is suitable for the simulation of mud flows and debris flows in channels of complex geometry. To

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fulfill this purpose, the model should have specific features, such as the treatment of wet-dry fronts, the handling of complex geometries and high bed slopes and the possibility of changing the model application field from Newtonian to non-Newtonian fluids, simply by changing the resistance law.

These features have previously been tested applying the model to different test cases that have been properly chosen [1]. The classic frictionless dam-break test has been used to verify the correctness of waves speed propagation and the capability of treating wet-dry fronts. A non-cylindrical frictionless ideal channel has been used to evaluate the model response to abrupt changes in cross-section wideness and bed elevation, then the effect of friction terms introduction has been checked using a mud flow dam-break. The first phase of the model verification ended with the simulation of laboratory experiments on a mud flow dam-break over a sloping plane.

In the present phase, the model is applied to two real events that occurred in North-Eastern Italy. The first is a debris flow that took place in the Upper Boite Valley, in the proximity of Cortina d'Ampezzo, in 1998, the second is a mud flow event located in the Stava Creek Valley in 1985.

The proposed model is based on an alternative formulation of conservative balance equations, which includes a particular mathematical expression of source terms ideated for natural channels, and which has already demonstrated important stability features under the numerical point of view [2, 3]. The numerical implementation is performed using the Godunov finite volumes scheme. This kind of numerical schemes are largely diffused in mud flow or debris flow treatment [4–6], together with the Roe's approximation for the solution of the Riemann problem. The presented model uses the same approach, but paying careful attention in conserving the general formulation suitable for complex geometry channels, in particular for what concerns the expression of the wave propagation celerity. This term is usually expressed as a function of water depth and cross-section width, but these hydraulic quantities often need to be corrected or mediated to be representative of irregular cross-sections. As an alternative, cross-section shape can be parameterized to be numerically handled [7]. In this work, celerity is determined referring to cross section wetted area and static moment, in order to ensure the formulation generality.

Source terms are handled using the splitting technique [8] and evaluated with the Euler's method. The pressure source terms, induced by the channel irregular geometry have been treated as in [2, 3], mathematically transforming the derivative of the static moment in order to eliminate the explicit dependence on the channel bed slope. This operation keeps its validity also in case of highly sloping channels, condition which often occurs in mud flow or debris flow phenomena. Friction source terms depends on the evaluation of friction slope, and therefore on the adopted resistance law. Like most of numerical models [5], the proposed model set up permits to easily change the resistance law and therefore to use the best fitting rheological model for each test case. It is worth noting that source terms numerical implementation has been kept as simple as possible, to put in evidence the stability features coming from the basis mathematical model.

2 Mathematical model

The mathematical model is based on an alternative formulation of shallow water equations for one-dimensional (1-D) flows in natural channels of complex geometry [2]. The continuity equation and the momentum balance equation are written in terms of state variables A and Q, considering no lateral inflows.

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0 \tag{1}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}+gI_1\right)=g\left.\frac{\partial I_1}{\partial x}\right|_{z_w}-gAS_f \tag{2}$$

where t is time, x is distance along the channel, A the wetted cross-sectional area, Q the discharge, g the gravitational acceleration, I_1 the static moment of the wetted area, defined as:

$$I_1=\cos\vartheta\int_0^{h(x)}b(x,z)\big(h(x)-z\big)\mathrm{d}z \tag{3}$$

I_2 is the variation of the static moment I_1 along the x-direction, $S_o=\sin\vartheta$, where ϑ is the angle between channel bottom and the horizontal, b is the cross-section width, h is flow depth.

The system closure equation for the evaluation of the friction term S_f will be described in detail for each examined test case, but the generally considered formulation is

$$S_f=\frac{\tau}{\rho gR} \tag{4}$$

in which S_f is the slope friction, R is the hydraulic radius, ρ is the mixture or the fluid density, and the shear stress τ depends on the adopted rheological model.

2.1 The source term

Differently from the commonly used formulation of shallow water equations, the proposed model does not include in the momentum balance equation source term a direct dependence on bed slope. Details on the mathematical treatment which led to eqn. (2) can be found in [3].

The classic momentum equation is

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}+gI_1\right)=gA\left(S_0-S_f\right)+gI_2 \tag{5}$$

Focusing on the source term, the pressure term I_2 has the following expression:

$$I_2=\left.\frac{\partial I_1}{\partial x}\right|_h=\cos\vartheta\int_0^{h(x)}\big(h(x)-z\big)\frac{\partial b(x,z)}{\partial x}\mathrm{d}z \tag{6}$$

Briefly, the pressure term I_2 can be expressed as the sum of two terms, one of which is the variation of static moment I_1 along x considering the water surface elevation z_w as a constant, while the other exactly balances gravitational forces in the momentum equation, unless the presence of the term $\cos\vartheta$ which arises in case of high slopes, and cannot be neglected when considering mud-flow or debris-flow phenomena.

$$I_2 = \left.\frac{\partial I_1}{\partial x}\right|_{z_w} - S_0 A \cos\vartheta \tag{7}$$

The substitution into (6) produces:

$$gA\left(S_0 - S_f\right) + gI_2 = gAS_0\left(1-\cos\vartheta\right) - gAS_f + g\left.\frac{\partial I_1}{\partial x}\right|_{z_w} \tag{8}$$

In this case, the term AS_0 does not disappear as illustrated in [2, 3], but it remains and it is multiplied by the factor (1-cosϑ). However, numerical proofs have demonstrated that this term is little if compared to friction terms, and can therefore be neglected. Eqn. (2) is therefore valid also for high sloping channel and debris flow simulation.

3 Numerical model

Shallow water equations have been numerically implemented using the first-order finite volumes Godunov scheme. Numerical fluxes are computed with Roe's method and source terms are evaluated with Euler's approach and taken into account adopting the splitting technique. Details on the different components of the numerical model can be found in Toro [8]. The resultant scheme is explicit, first-order accurate, and has a very uncomplicated structure, since it is built choosing the simplest solution technique for every element of the partial differential equations system. This approach has the intention to illustrate the intrinsic stability features of the mathematical model, which could otherwise be hidden by sophisticated numerical schemes.

Referring to shallow water equations in the vector form (eqn. (9)) the splitting approach for source terms treating, consists in separately solving the homogeneous partial differential equations system (eqn. (10)) and the ordinary differential equation (eqn. (11)). In detail, the solution obtained from eqn. (10) is used as initial condition for eqn. (11).

$$\mathbf{U}_t + \mathbf{F}\left(\mathbf{U}\right)_x = \mathbf{S}\left(\mathbf{U}\right) \tag{9}$$

$$\mathbf{U}_t + \mathbf{F}\left(\mathbf{U}\right)_x = 0 \quad \Rightarrow \quad \bar{\mathbf{U}} \tag{10}$$

$$\mathbf{U}_t = \mathbf{S}\left(\bar{\mathbf{U}}\right) \quad \Rightarrow \quad \mathbf{U}_{t+dt} \tag{11}$$

The Roe's scheme, used to solve eqn. (6), requires the definition of the Jacobian matrix

$$\mathbf{J} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}} = \begin{pmatrix} 0 & 1 \\ g\dfrac{\partial I_1}{\partial A} - \dfrac{Q^2}{A^2} & 2\dfrac{Q}{A} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ c^2 - u^2 & 2u \end{pmatrix} \tag{12}$$

Most of models proposed in the literature about the resolution of shallow water equations for debris flow or natural channels, based on approximate Riemann solvers (see for example [4, 5, 9]), adopt the same simplification in the evaluation of the term $\partial I_1/\partial A$, assuming

$$\frac{\partial I_1}{\partial A} \simeq \frac{A}{B} \simeq h \quad \Rightarrow \quad c = \sqrt{g\frac{A}{b}} \quad \text{or} \quad c = \sqrt{gh} \tag{13}$$

In the present model, in order to keep the formulation generality and to ensure the applicability to natural and complex channel geometries, the static moment derivative is explicitly computed as the variation of I_1 relative to the variation of A in the water depth variation range $h \pm \Delta h$

$$\frac{\partial I_1}{\partial A} = \frac{I_1(h+\Delta h) - I_1(h-\Delta h)}{A(h+\Delta h) - A(h-\Delta h)} \tag{14}$$

The celerity c is therefore defined as

$$c = \sqrt{g\frac{\partial I_1}{\partial A}} \tag{15}$$

Another important aspect of the Godunov finite volume method application to natural geometries is the quantification of cell water volume V and the definition of the relation between the state variable A and V. For every computational cell, A is defined as

$$A_i = \frac{1}{\Delta x}\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} A(x,t)\,\mathrm{d}x = \frac{V_i}{\Delta x} \tag{16}$$

V_i is computed as the volume of a pyramid which bases are irregular polygons, since the water profile is assumed to be parallel to channel bed.

$$V_i = \frac{\left(A_{i-\frac{1}{2}} + A_{i+\frac{1}{2}} + \sqrt{A_{i-\frac{1}{2}}A_{i+\frac{1}{2}}}\right)\Delta x}{3} \tag{17}$$

3.1 Source terms numerical treatment

Source terms are numerically included in computations by splitting, and they are simply computed by Euler's method

$$\mathbf{U}_{t+dt} = \mathbf{U}_t + \Delta t \cdot \mathbf{S}\left(t, \bar{\mathbf{U}}\right) \tag{18}$$

in which

$$\mathbf{S} = \begin{pmatrix} 0 \\ g\left.\frac{\partial I_1}{\partial x}\right|_{z_w} - gAS_f \end{pmatrix} \tag{19}$$

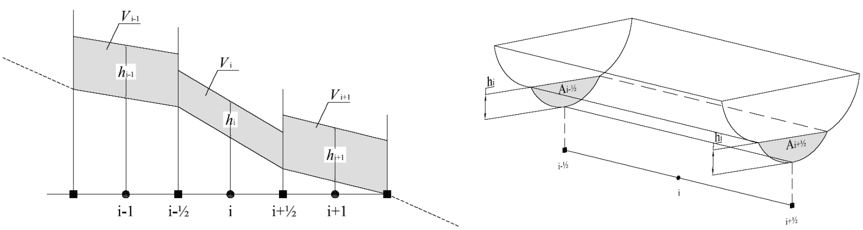

Figure 1: Computational scheme for V_i.

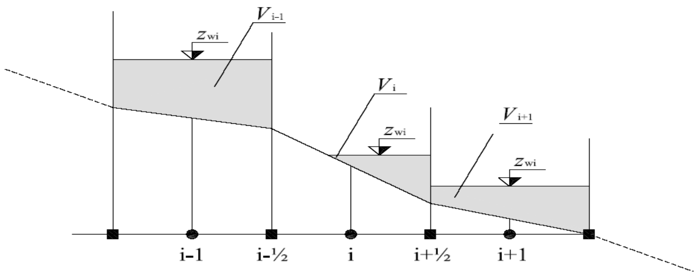

Figure 2: Computational scheme for momentum balance pressure source term.

Considering no lateral inflows, source terms are present only in the momentum balance equation. This term can be divided into two parts, that is the friction term and the pressure term, represented by the static moment variation along channel, taking the water surface elevation as a constant.

The computational scheme for the pressure term quantification in represented in Figure 2, and the variation of I_1 is computed as:

$$\left.\frac{\partial I_1}{\partial x}\right|_{z_w} \simeq \left.\frac{\Delta I_1}{\Delta x}\right|_{z_w} = \frac{I_1\left(h_{i+\frac{1}{2}}\right) - I_1\left(h_{i-\frac{1}{2}}\right)}{\Delta x} \tag{20}$$

4 Numerical tests

In this work the model has been applied to two real events. The first is a natural debris flow event, due to intense rainfall, surveyed at the Acquabona site in Northern Italy. It is of particular interest thanks to the large amount of available field data. The second is the Stava mud flow, a tragic episode occurred in a little town of Italian Alps. This event was caused by the collapse of two tailing dams, which released a huge quantity of water into the Stava Creek channel, causing the formation of a mud flow wave with an enormous destructive power.

4.1 Acquabona debris flow

The Acquabona debris flow has been widely surveyed and documented in the context of the “Debris Flow Risk” Project, funded by the EU. In particular, the UPD (resp. Prof. Rinaldo Genevois) has carried out a research on some debris flow prone watersheds in the Upper Boite Valley (Eastern Dolomites, Southern Alps) and surroundings, included in the municipality of Cortina d’Ampezzo [10]. A large quantity of field data is therefore available since an automatic, remotely controlled monitoring system has been installed at Acquabona on June 1997. The Acquabona site in characterized by one or more debris flow every year, which usually occur in summer and in early autumn and are associated to intense, spatially limited rainfall events. The monitoring system installed at Acquabona was fully automatic and remotely controlled. It consisted of three on-site monitoring stations and an off-site master collection station. Every station was equipped with a geophone, while at Station 3 also a superficial pressure transducer and an ultrasonic sensor were present.

In this work we refer to the event of the August 17th in 1998. The event was originated by a very intense rainstorm: 25.4 mm of rain were measured during 30 min by the rain-gage at Station 1. The volume of the deposits available for debris flow generation has been estimated to be around 8000-9000 m^3. The overall duration of the event was of approximately 38 min and more than 20 different surges have been surveyed at Station 3.

The geometry of the channel is available thanks to 19 surveyed transversal cross-sections, for a global channel length of 1120 m and a difference in height of 245 m. The longitudinal slope ranges from 10% to 55%. For model application a constant spatial step of 1 m has been adopted. Numerical simulations were performed adopting the rainfall hydrograph reconstructed by Orlandini and Lamberti [11], which has an extension of about 2.5 hours and a peak discharge of 2.3 m^3/s. An open boundary type condition is imposed at the downstream end. For the debris flow the bulk concentration is assumed to be 0.6 and mixture density 1850 kg/m^3, according to [7].

The rheological model adopted in the simulations is the Herschel-Bulkley model, which, for simple shear conditions may be written as:

$$\tau = \tau_c + K\gamma^{\eta} \tag{21}$$

in which K and η are rheological parameters. Referring to the simulations carried out by Fraccarollo and Papa [12] on the same event, K is assumed to be 150 Pa·$s^{1/3}$, τ_c is equal to 925 N/m^2, and η has been empirically set equal to 1/3.

In Figure 3 computed flow height is compared with the measured data collected by the ultrasonic sensor at Station 3. The model satisfactorily captures wave height and shape, but it underestimates their duration, overestimating as a consequence their number. Results arte however encouraging and comparable to those obtained by Fraccarollo and Papa [12] and Zanuttigh and Lamberti [7].

The average velocity of the different flow surges has been estimated through geophone log recordings. Available data refer to two 100 m channel reaches

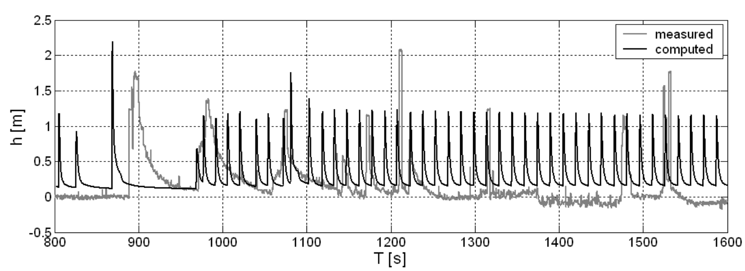

Figure 3: Comparison between the flow depth measured and calculated at Station 3.

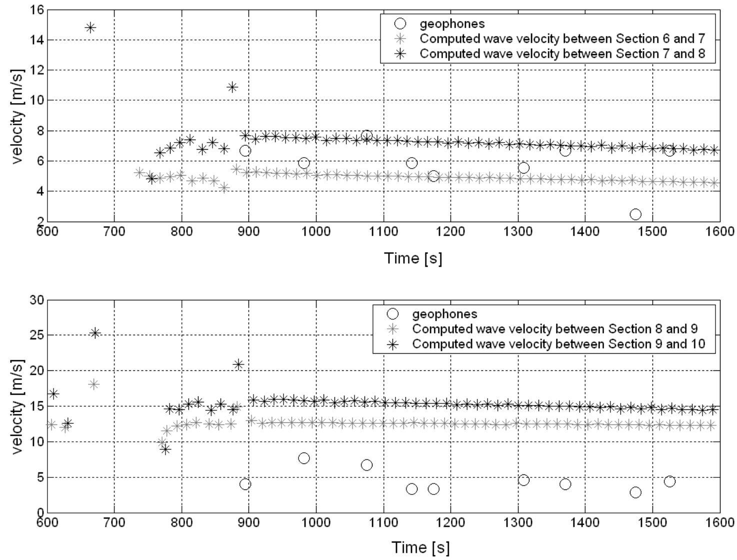

Figure 4: Comparison between measured and computed wave speeds upstream and downstream from Station 3.

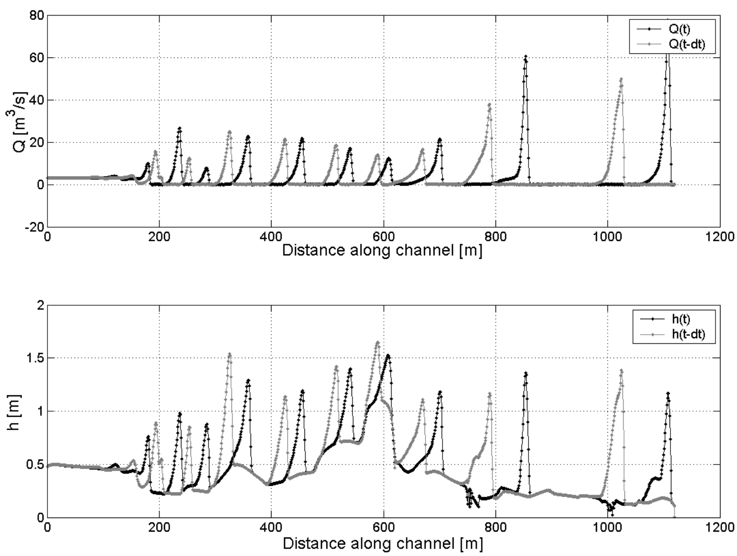

Figure 5: Longitudinal discharge distribution and flow depth profile.

located in the lower part of the channel before and after Station 3, which corresponds to the surveyed cross-section 8. Comparison is showed in Figure 4. In the upstream reach computed velocities compare well with field data, while in the downstream reach they are generally overestimated.

It is interesting noting that the flow regime is mainly characterized by the formation of roll waves, as it is evident observing the longitudinal distribution of discharges and flow depths at two subsequent time steps. Nevertheless, numerical solution is not affected by relevant numerical instabilities.

4.2 Stava mud flow

In July 19th 1985, two tailing dams suddenly collapsed in Tesero, a little town in the Italian Alps. The stored water, together with the dam body material flowed down to the Stava River as a big mud flow, claiming 268 human lives and destroying 47 houses. As reported by Takahashi [13], the Stava River before the disaster flowed with an approximately uniform slope of 5°. Although the mud flow had such an intensive destructive power, as well as fluidity, the Stava River channel itself had not suffered much erosion or deposition, and it can therefore be simulated as a fixed bed stream. In his report Takahashi gives important references also about mud flow solids concentration which was as high as 0.5, while the particle size was so fine that the relative depth, R/d, had a value of the order of 10^5. In this condition the resistance to flow is similar to that of a plain water flow and the Manning's equation can be applied. Takahashi obtained a Manning's roughness coefficient in each section by reverse calculation from the data on velocity computed with the Lenau's formula applied to measured flow superelevations at bends.

The channel description is also taken from Takahashi [13]. It includes 24 surveyed cross-sections, their planimetric location and the longitudinal profile. In this case bed slope ranges from 5% to 12%. The simulated reach is 3500 m long and a constant spatial step of 1.25 m has been used.

In Figure 6, discharge and depth computed hydrographs are compared with Takahashi numerical results obtained with the kinematic wave theory [13]. Referring to cross-section 10, located about 3000 m downstream the dams, there is a good accordance between the computed peak discharge and the value estimated by Takahashi (3500 m^3/s) as a result of product between the wetted cross-section area measured in situ (about 500 m^2) and the maximum velocity derived by the flow superelevation at the nearest bend (7 m/s)

The initial water profile condition reproduces the same hypothesis adopted by Takahashi, which is a uniform slide of the mud mass until Section 4, from which the mud flow is assumed to develop.

Figure 8 shows the comparison between computed and measured front arrival times at different locations. The measured values are estimated on the basis of a seismograph located in Cavalese, a nearby town. The computed times are in good agreement with the estimated ones along the entire channel.

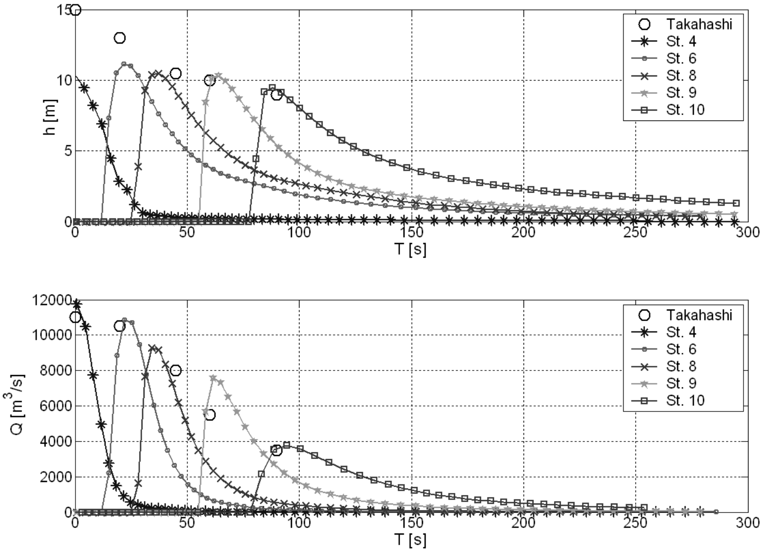

Figure 6: Depth and discharge hydrograph at different cross sections.

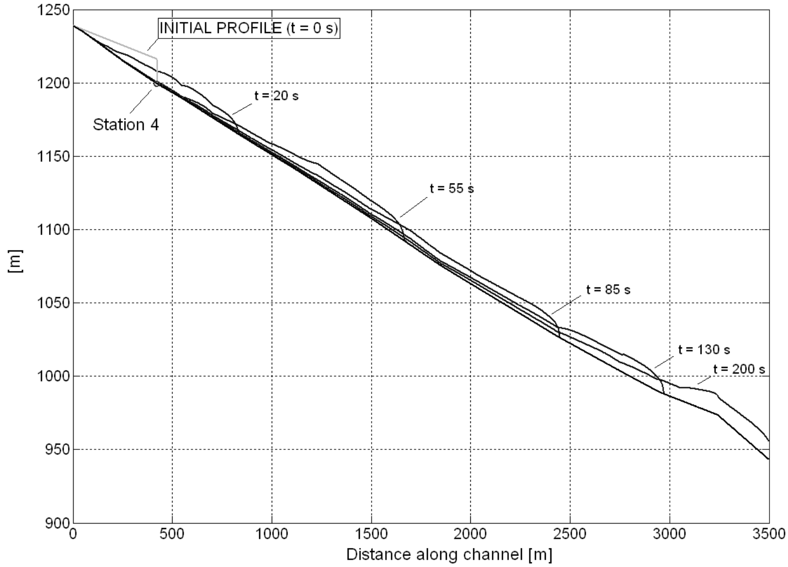

Figure 7: Initial conditions and flow profiles along the channel during the simulation.

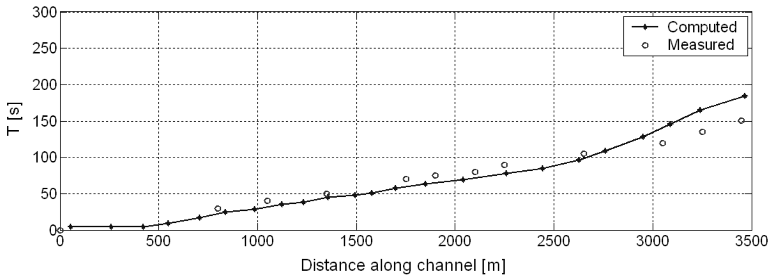

Figure 8: Comparison between computed and measured front arrival times at different locations.

5 Conclusions

A numerical model for the simulation of mud flow and debris flow natural events is presented. It is based on a mathematical model which main features are concerned with the propagation of the wet-dry fronts, the treatment of irregular and variable cross sections shape, and the applicability to highly sloping channels. Two real events have been chosen to test the model. The first is a natural debris flow event at Acquabona site. In this case a large quantity of field data was available and model results compared well with wave peak height and propagation velocities. The second test case refers to the Stava mud flow tragic event, originated by the collapse of two tailing dams. Also in this case good accordance between observed data and mud front propagation speed has been obtained. Simulation results have also been compared with the Takahashi analysis of the same event, showing good accordance for what concerns peak discharge estimation at different cross sections.

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