Optimizing model of a railroad yard's operations plan based on production scheduling theory

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Abstract

A railroad yard is the basic production unit of rail freight transportation, located at the junction of railway lines, mainly responding to the operations of arriving, sorting, assembling, marshaling, departing; its operation efficiency directly affects the economic benefits of the railroad industry. However, the problem of railroad yards' operations planning not only has a complex model and intractable algorithm, but also a lot of uncertainty factors exist. So searching for an efficient, effective, accurate planning method has always been a difficulty in this field. Different countries, different regions and different production environments vary their description methods, in addition to optimization objectives and constraint conditions. Because of this, it becomes very difficult for researchers from related fields to learn from each other, and the experimental results cannot be compared. So there is an urgent need for building a method of unified modeling based on theory. We consider the fact that operation planning problems of a railroad yard are related to particular production scheduling problems. Then, we use production scheduling theory to describe the rail yard's problem, which can strengthen the theoretical depth and facilitate the research of experts on related fields of scheduling, math, computer and control theory. Combined with the operation feature and overseas and domestic research status, this paper has unified the description method of the operation procedure, variable naming, problem description, objective function and constraint condition. A standard construction method of one switch engine, one hump model was given.
Keywords: railroad yards, operations plan, production scheduling, optimal model.

1 Introduction

A railroad yard, also known as a marshalling station, shunting yard or railway yard, is the core of the railway terminal, which is the distribution place for allocating wagon-flows coming from arrival trains to form the new trains. According to statistics, in China, the annual daily number of sorted wagons of a railway network railroad yards is 6000 and the efficiency and quality directly affect the work efficiency and economic benefit of railway transportation [1]. But the planning issue of a railroad yard's operations plan not only has complex model building, involves creating a difficult algorithm for solving it but also has uncertainty factors. Therefore, searching for an efficient real-time planning method is the difficult point in the field. Different countries, station types and operation equipment vary in their description, optimization objectives and constraint conditions making it hard to learn from each other from relevant research results and to compare the experimental results. The development of this field is restricted and there is an urgent need to build up a set of theoretical support for a unified modeling method. The planning issue of a railroad yard's operations plan is a large-scale production scheduling problem, a kind of classical problem of combination and optimization. From the point of view of computational complexity, this kind of problem has NP properties and it is hard to find the optimal solution. The approximate solution is usually obtained by using the approximate algorithm or the heuristic algorithm.

In this paper, combined with the modeling method of production scheduling theory, we build a wagon-flow allocation model of single directional, single pushing and single humping. This model is established on the optimal goal for the shortest dwell time of all cars during a stage plan time and with some constraint conditions such as car flows joining relation, destinations, full axis, rail cars in arrival or departure trains, hump operation type, sorting sequence, assembling sequence etc.

2 Problem statement

2.1 Production scheduling problem

The production scheduling problem [2] is one of the representative problems of combinatorial optimal problems in operations theory. It means making the reasonable production decision under the known condition of production tasks. According to the expended goals, it should determine how to most efficiently allocate the limited human and material resources to different tasks in time order. If one only considers the processing sequence of the tasks, it is a sequencing problem in mathematics, which is an urgent problem to be solved in the process of manufacturing production management [3]. In 1954, after the first article which described a permutation flow shop scheduling problem of two machines was published [4], in the following 60 years, scheduling problems have been the concern of researchers in the fields of mathematics, computers, industrial engineering and economic management.

According to the literature [5–9], the research can be divided into the following 5 stages: for the first stage (1950s–1960s), its technical characteristics are a combination of analysis; for the second stage (1960s–1970s), the main technical features are branch and bound, and dynamic programming; for the third stage (1970s–1980s), the main technical feature is computational complexity theory; for the fourth stage (1980s–1990s), the main technical feature is the approximation algorithm combined with heuristic strategies; for the fifth stage (1990s to date), the main technical features are theories of real time and uncertainty modern scheduling. The research of production scheduling has been accompanied by the development of mathematics and computer science. For different production processes, the mathematical models of the different production scheduling problems vary. As an easy description, in 1967, Conway *et al*. [10] proposed a method of four parameters representing a scheduling problem. Based on that, in 1979, Graham *et al*. [11] proposed a three-parameter representation method. Then a large number of studies followed using the description of three parameters $\alpha|\beta|\gamma$, and the definitions of Pinedo in [12] can be associated with our question as follows.

2.1.1 The possible machine environments specified in the α field

Identical machines in parallel (*Pm*): There are m identical machines in parallel.

Flow shop (*Fm*): There are m machines in series. Each job has to be processed on each one of the m machines. All jobs have to follow the same route, i.e., they have to be processed first on machine 1, then on machine 2, and so on.

Flexible flow shop (*FFc*): A flexible flow shop is a generalization of the flow shop and the parallel machine environments. Instead of m machines in series there are c stages in series with at each stage a number of identical machines in parallel. Each job has to be processed first at stage 1, then at stage 2, and so on. A stage functions as a bank of parallel machines; at each stage job j requires processing on only one machine and any machine can do. The queues between the various stages may or may not operate according to the *First Come First Served (FCFS)* discipline. (Flexible flow shops have in the literature at times also been referred to as hybrid flow shops and as multi-processor flow shops.)

2.1.2 The processing restrictions and constraints specified in the β field

Release dates (r_j): If this symbol appears in the β field, then job j cannot start its processing before its release date r_j. If r_j does not appear in the β field, the processing of job j may start at any time.

Preemptions (*prmp*): Preemptions imply that it is not necessary to keep a job on a machine, once started, until its completion. When preemptions are allowed *prmp* is included in the β field; when *prmp* is not included, preemptions are not allowed.

Permutation (*prmu*): A constraint that may appear in the flow shop environment is that the queues in front of each machine operate according to the *First In First Out (FIFO)* discipline.

> **Blocking** (*block*): Blocking is a phenomenon that may occur in flow shops. If a flow shop has a limited buffer in between two successive machines, then it may happen that when the buffer is full the upstream machine is not allowed to release a completed job.

2.1.3 Possible objective functions to be minimized in the γ field

Total weighted completion time $(\sum w_j C_j)$: The sum of the weighted completion times of the n jobs gives an indication of the total holding or inventory costs incurred by the schedule. The sum of the completion times is in the literature often referred to as the flow time. The total weighted completion time is then referred to as the weighted flow time.

2.2 Technical operations of railroad yards

Each railroad yards needs to set up a number of yards according to its own transport production demand; it can complete multi-destination technical operation such as train arrival and departure operations, car sorting and assembly operations. The function partition and flexible use of each yard are determined by the rational layout of the station, the capacity of the technical equipment, and the cars meeting a balance between inbound and outbound trains. Based on the practical operational process, the yards can be divided into functions of a receiving yard, a classification yard and a departure yard. In addition, there can be a hump in the linking area between the receiving yard and the classification yard, and also the pulling out tracks in the linking area between the classification yard and the departure yard. The hump has a certain degree of slope, so that it can help the wagon flow under potential energy to slip into the classification yard. Therefore it is always used for sorting. The pulling out tracks are the flat tracks used for marshalling or picking up, and also for no hump car sorting. Figure 1 gives a station chart of a bidirectional three-grade six-yard transversal type railroad yard which is commonly used for larger operations.
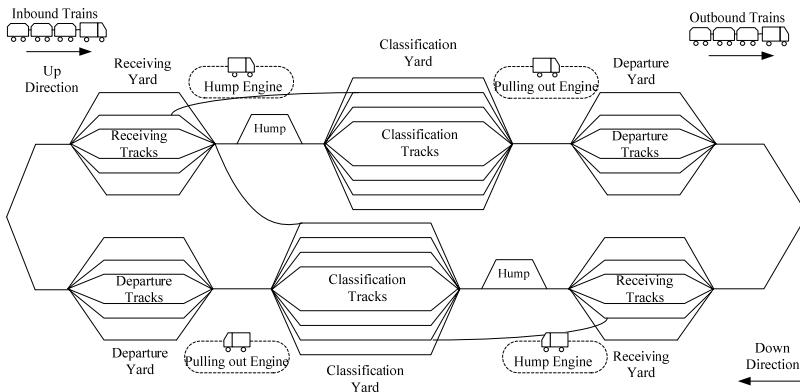

Figure 1:   Station chart of a bidirectional three-grade six-yard transversal type railroad yard.

Railroad yards have perfect transportation equipment in that they can carry out all of the work flow in the station such as arriving, sorting, accumulating, marshalling and departing.

There are many kinds of definitions about the functional assignment in the classification yard and receiving yard (see [13–17]). In this paper, we review one of the situations: the arriving technical operation refers to the task of meeting the arrival train on the receiving track and of its being inspected by an inspector group etc.; the sorting technical operation refers to disassembling the inbound train flow or train set onto the special track of the classification yard via the hump or pull out track according to the cars' destinations; the accumulating technical operation refers to the car halting process which starts from the time of humping into the classification yard and ends at the time of pulling out to the departure yard; the marshalling technical operation refers to the shunting operation which needs to select accumulated cars by the pulling out engine to make up wagon flow according to the rules of the freight train formation plan, the train diagram, and the regulations governing the railway technical operation on the pulling out tracks; the departure technical operation refers to the setup operation for the departing train in the departure yard.

2.3 The relationship between production scheduling and railway yard operation scheduling

The railway yard operations problem is the need to organize a variety of productive resources to complete the work of meeting arriving trains and making sure trains depart day and night according to production demand; it is a large-scale complex production scheduling problem. In China, the schedulers organize the production scheduling plan in accordance with a daily shift working plan which is made firstly by railway administration. According to the specific circumstances of the current station, taking 3–4 hours as a stage, they adjust the deviation in the daily shift plan and gradually make detailed operation plans to achieve the production indexes. The operation scheduling can be divided into the daily shift working plan, the stage plan and the shunting operation plan. The relationships between the different levels of the plans are shown in Figure 2.
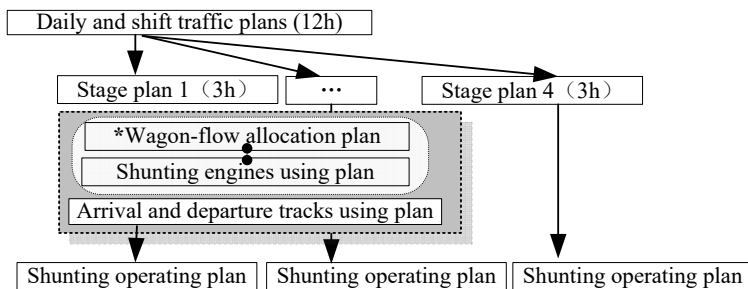
Figure 2:   The different levels of the railroad yard scheduling operation plans.

The stage plan is the core of the whole plan, which is like a connecting link between the up level plan and the down level plan. Establishment of a stage plan mainly includes car assignment, the humping sequence, the marshalling sequence, hump engine scheduling, pulling out engine scheduling and arrival and departure track occupation, through setting a wagon-flow allocation plan, a shunting engine usage plan and an arrival and departure track usage plan. The focus of the stage plan is the wagon-flow allocation plan which forms the basis for the other two plans. The wagon-flow allocation plan not only needs to consider the requirements of the departure trains in the current stage, but also the factors of humps, engines, and tracks comprehensively. It mainly determines the departure train: cars coming from which arrival trains, the detailed destinations, and the number of cars in one destination. The destination and all car numbers must obey the regulations of the formation plan. It is not only the embodiment of coordinated operation in sorting and assembling, but also an optimal problem of reasonable selection and collocation. The process of wagon-flow allocation can be divided into dynamic wagon-flow allocation and static wagon-flow allocation [18]. Dynamic wagon-flow allocation mainly solves how to set the order of sorting based on the demand of the outbound trains. Static wagon-flow allocation studies focus on how to make train allocation under the condition of determined sorting order. Therefore, the key to finding the optimum wagon-flow allocation plan is to choose a rational sorting order. The wagon-flow allocation is discussed in the following paragraphs.

The research into the railway yard operation problem involves many technical terms, which makes it difficult to understand for experts in the field of other scheduling problems from other majors. Therefore, we hope that this problem can be changed from a specific scheduling problem of railway yards to a more general production scheduling model. The forms of the production scheduling problem are various, and it is difficult to unify. In fact, many technical terms originate from manufacturing scheduling problems which first attracted attention, such as machine, job, operation and order. In order to unify the terms, the specific scheduling problems can be abstracted and converted into a standard form of three parameters.

Based on the description of the deterministic production scheduling problem, the model distributes receiving tracks, inspectors, humping engine, hump, classification tracks, pulling out engine and the number of departure tracks to different buffers, and makes an inbound train correspond to a job and technical operations correspond to different machines. So, the description of a wagon-flow allocation problem in a specific railway yard is as follows.

The total number of inbound trains is n, where the trains have different priorities. The yards consist of 18 receiving tracks, 2 humping engines, 1 piece of hump equipment, 30 classification tracks, 1 pulling out engine, 14 departure tracks, and 2 departure technical operation groups. Each inbound train needs four operating procedures in order: they are arriving technical operation, sorting technical operation, marshalling technical operation and departing technical operation. Each procedure can only be processed on one machine with no preempt. When the buffer is without enough spare space, the upstream machine can't release the job. Each procedure needn't meet the first in first out rule, that is to say the

model has no permutation constraint. In addition, the wagon-flow allocation plan should be constrained by the requirements of the formation plan and the train diagram; this problem also belongs to the scheduling problem and has a due date. As the stage plan is only a part of the daily shift working plan and the jobs have different arrival times, the total completion time is not suitable to this problem as it can't recognise the efficiency of inbound trains. We adopt the total wagon-flow dwell time (C_j') on the station during a stage plan time as the optimal function. Based on the above analysis, the problem can be described as a flexible flow shop scheduling problem with blocking. The problem described by the three parameters is: $FF_4|r_j, block| \sum w_j C_j'$. Figure 3 illustrates the job flow of the problem.
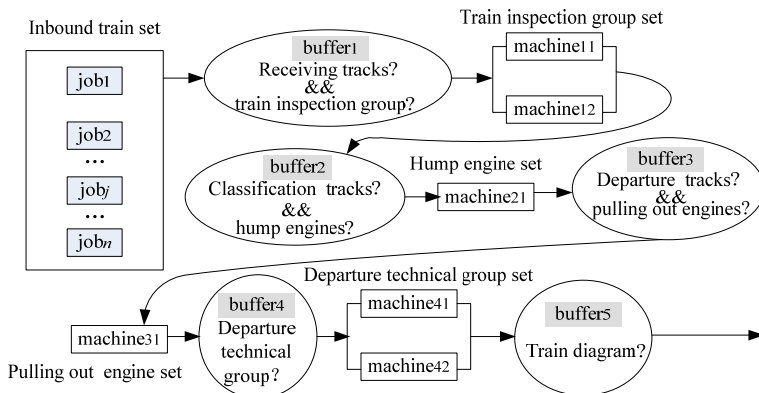

Figure 3:   The job flow of the wagon-flow allocation problem.

3   The optimal model of the scheduling problem of railroad yards

The model of single pushing and single humping is a fundamental scheduling formation in a wagon-flow allocation problem. Based on this model, many problems of other models can be transformed or expanded. Other modeling problems refer to double pushing and double humping, the yards that receive exchange cars, empty car distribution and multi-objective optimization. Compared with others, sorting is the core operation. We assume that under the condition of sufficient operation resources, which include the group of inspectors, the tracks of all kinds of yards, and the marshalling sequence being equal to the departure sequence according to the train diagram, the problem is simplified to a sorting sequence optimal problem. When we assume the case of one humping engine and one pulling out engine, the problem can be described as $F_2|r_j, block| \sum w_j C_j'$ in the format of production scheduling theory.

For the characteristics of the wagon-flow problem, the conventions are given as follows: the total number of trains including which are halting in the classification yard, having arrived or being expected to arrive in the receiving yard is $n+1$, we assume that all of the cars are in the classification yard at the beginning

of the stage plan at the 0th inbound train; the total number of outbound trains in the time of a stage plan is $m+1$, where we assume that the $(m+1)^{th}$ outbound train refers to all of the cars not assembled to any outbound train at the end of the stage plan; the maximum number of sorting intervals is defined as K, where sorting interval refers to the time window between the beginning and the ending of a whole sorting operating process, assuming T_0 to be the starting time of a stage plan, and letting the stage plan operating time be 3–4 hours; outbound trains must obey the train diagram to operate, but the outbound train is allowed to depart whilst not meeting the constraint of full axis so as not to waste the operation curves of the train diagram. The definition of the symbols used in the model are shown in Table 1.

Table 1:   Symbol definition.

Symbol	Definition	Vague range
D	inbound train	symbolic constant
C	outbound train	symbolic constant
CF	departure technical operation	symbolic constant
BZ	marshalling technical operation	symbolic constant
JT	sorting technical operation	symbolic constant
DD	receiving technical operation	symbolic constant
JTQJ	sorting interval	symbolic constant
BZQJ	marshalling interval	symbolic constant
j	inbound train numbering	$j=0,1,\cdots,n$
i	outbound train numbering	$i=1,\cdots,m,m+1$
f	destination	$f=\text{‘1’, ‘2’, }\cdots,\text{ ‘N’}$
k	sorting interval numbering	$k=1,2,\cdots,K$
h	marshalling interval numbering	$h=1,2,\cdots,m$
F	the set of all destinations on the railroad yards	$F=\{\text{‘1’},\cdots,\text{ ‘}f\text{’},\cdots,\text{ ‘N’}\}$
D_j	the jth inbound train	$D_j=D_0,D_1,\cdots,D_n$
C_i	the ith outbound train	$C_i=C_1,C_2,\cdots,C_{m+1}$
w_j	the weight of D_j' priority	
$F_{D,j}$	the set including all of fs in D_j	$F_{D,j}\subseteq F$
$F_{C,i}$	the set including all of fs conforming to C_i's marshalling demand	$F_{C,i}\subseteq F$
$D_{j,f}$	the set of cars in D_j on the destination of f	$D_{j,f}\subseteq D_j$
$|X|$	the number of set X	nonnegative integer
$C^f_{i,j}$	the set of wagons coming from $D_{j,f}$ and allocated to C_i on the destination of f	$C^f_{i,j}\subseteq D_{j,f}$
$x^f_{i,j}$	the number of $C^f_{i,j}$	nonnegative integer
$T_{C,i}$	departure time of C_i	nonnegative integer
$T_{D,j}$	arrival time of D_j	nonnegative integer
$t_{CF,i}$	departure operating time of C_i	nonnegative integer
$t_{BZ,i}$	marshalling operating time of C_i	nonnegative integer
$t_{JT,j}$	sorting operating time of D_j	nonnegative integer
$t_{DD,j}$	arriving operating time of D_j	nonnegative integer
r_i	the number of C_i's full axis	positive integer
$D_{JTQJ,k}$	D_j assigned to the kth soring interval	$D_{JTQJ,k}\in\{D_1,\cdots,D_n\}$
$T_{JTKS,k}$	the start time of the kth soring interval	nonnegative integer
$C_{BZQJ,h}$	C_i assigned to the hth marshalling interval	$C_{BZQJ,h}\in\{C_1,C_2,\cdots,C_m\}$
$T_{BZKS,h}$	the start time of the hth marshalling interval	nonnegative integer

Table 1:   Continued.

Symbol	Definition	Vague range
$Y^T_{i,j}$	0-1 variables represent the cars matching relation between D_j and C_i according time	$Y^T_{i,j}=0,1$
$Y^f_{i,j}$	0-1 variables represent whether the direction of f conforms to C_i' formulation rules according to the marshalling plan	$Y^f_{i,j}=0,1$
$Y^j_{JTQJ,k}$	0-1 variables represent whether D_j can be disassembled on the kth soring interval	$Y^j_{JTQJ,k}=0,1$
$Y^i_{BZQJ,h}$	0-1 variables represent whether C_i can be assembled on the hth marshalling interval	$Y^i_{BZQJ,h}=0,1$

The production environment, the processing restrictions and constraints, and the optimal functions of the model can be described as follows.

3.1 The production environment

1 humping engine and 1 pulling out engine, so it belongs to a 2 machine flow shop problem.

3.2 The processing restrictions and constraints

(1) Wagon-flow joining relation constraints:

$$Y^T_{i,j}=\begin{cases}0 & T_{C,i}-t_{CF,i}-t_{BZ,i}-t_{JT,j}-t_{DD,j}-T_{D,j}<0 \\ 1 & T_{C,i}-t_{CF,i}-t_{BZ,i}-t_{JT,j}-t_{DD,j}-T_{D,j}\geq 0\end{cases}, \quad 1\leq i\leq m,\ 0\leq j\leq n\ ; \quad (1)$$

$$T_{C,i},\ t_{CF,i},\ t_{BZ,i},\ t_{JT,j},\ t_{DD,j},\ T_{D,j}\geq 0,\quad 1\leq i\leq m,\ 0\leq j\leq n; \quad (2)$$

$$x^f_{i,j}\leq Y^T_{i,j}*|D_{j,f}|,\ 1\leq i\leq m,\ 0\leq j\leq n,\ 1\leq f\leq N; \quad (3)$$

Equation (1) is considered for the wagon-flow joining relation between D_j and C_i. If it is satisfied, $Y^T_{i,j}=1$; otherwise $Y^T_{i,j}=0$. Equation (2) shows that all the values of the time variables can't be negative. Equation (3) means the cars of D_j can be assembled to C_i if and only if $Y^T_{i,j}=1$.

(2) Destination constraints:

$$Y^f_{i,j}=\begin{cases}0 & \left((f\in F-F_{D,j}\cap F_{C,i})\wedge(F_{D,j}\cap F_{C,i}\neq\varnothing)\vee(F_{D,j}\cap F_{C,i}=\varnothing)\right) \\ 1 & \left((f\in F_{D,j}\cap F_{C,i})\wedge(F_{D,j}\cap F_{C,i}\neq\varnothing)\right)\end{cases},$$

$$1\leq i\leq m,\ 0\leq j\leq n,\ 1\leq f\leq N; \quad (4)$$

$$x^f_{i,j}\leq Y^f_{i,j}*|D_{j,f}|,\ 1\leq i\leq m,\ 0\leq j\leq n,\ 1\leq f\leq N\ ; \quad (5)$$

Equation (4) shows that $Y^f_{i,j}=1$ if and only if the destination f of D_js cars included in the C_i destination set; otherwise $Y^f_{i,j}=0$. Equation (5) means that the cars of D_j can be assembled to C_i if and only if $Y^f_{i,j}=1$.

(3) The car number assembling to the outbound train constraint:

$$\sum_{j=0}^{n}\sum_{f=1}^{d}x^f_{i,j}\leq r_i,\ 1\leq i\leq m\ ; \quad (6)$$

(4) The car number constraints between the inbound train and outbound train:

$$\sum_{i=1}^{m}\sum_{f\in F_{D,j}}x^f_{i,j}\leq\sum_{f\in F_{D,j}}|D_{j,f}|,\ 0\leq j\leq n\ ; \quad (7)$$

$$\sum_{j=0}^{n} \sum_{f \in F_{C,i}} x^f_{i,j} \leq \sum_{j=0}^{n} \sum_{f \in F_{C,i}} |D_{j,f}|, \ 1 \leq i \leq m ; \tag{8}$$

Equation. (7) shows that the number of cars in D_j must be greater than or equal to all the number of cars in C_i which comes from D_j; Equation (8) means that the number of cars in C_i must be less than or equal to the number of cars in D_j which satisfies the destination requirement of C_i.

(5) Hump and humping engine constraints (single pushing and single humping): if D_j can be arranged to sort on the kth sorting interval, it must meet the following constraints:

$$Y^j_{JTQJ,k} = \begin{cases} 0 & D_{JTQJ,k} \neq D_j \\ 1 & D_{JTQJ,k} = D_j \end{cases}, \ 1 \leq j \leq n, \ 1 \leq k \leq K; \tag{9}$$

$$\sum_{j=1}^{n} Y^j_{JTQJ,k} = 1, \ 1 \leq k \leq K; \tag{10}$$

$$\sum_{k=1}^{K} Y^j_{JTQJ,k} = 1, \ 1 \leq j \leq n ; \tag{11}$$

$$T_{JTKS,k} + \sum_{j=1}^{n} Y^j_{JTQJ,k} * t_{JT,j} \leq T_{JTKS,k+1}, 1 \leq k \leq K-1; \tag{12}$$

$$\begin{cases} T_{JTKS,1} \geq \max\left(T_0, \sum_{j=1}^{n} Y^j_{JTQJ,1} * (T_{D,j} + t_{DD,j})\right), k = 1 \\ \\ T_{JTKS,k} \geq \max\begin{pmatrix} T_{JTKS,k-1} + \sum_{j=1}^{n} Y^j_{JTQJ,k-1} * t_{JT,j}, \\ \sum_{j=1}^{n} Y^j_{JTQJ,k} * (T_{D,j} + t_{DD,j}) \end{pmatrix}, 1 < k \leq K \end{cases} ; \tag{13}$$

$$\sum_{j=1}^{n} \sum_{k=1}^{K} Y^j_{JTQJ,k} * t_{JT,j} \leq t ; \tag{14}$$

$$x^f_{i,j} = Y^j_{JTQJ,k} * x^f_{i,j}, \ 1 \leq j \leq n, \ 1 \leq i \leq m ; \tag{15}$$

Equation (9) shows that if D_j has been arranged to sort on the kth sorting interval, then the value of $Y^j_{JTQJ,k}$ is equal to 1; otherwise the value of $Y^j_{JTQJ,k}$ is equal to 0. Equation (10) shows that one sorting interval only can arrange one inbound train operation. Equation (11) shows that one inbound train only can occupy one sorting interval. Equation (12) shows that only when the upstream inbound train has completed its sorting operation can the current inbound train begin its operation by the same humping engine. Equation (13) means that only when D_j's earliest sorting time is less than or equal to $T_{JTKS,k}$ can D_j be arranged to the kth sorting interval. Equation (14) means that the total inbound train sorting operating time must be less than or equal to the stage plan time T. Equation (15) shows that the number of cars coming from the unassembled trains can't be the same as the number of $x^f_{i,j}$.

(6) Pulling out engine constraints:

$$Y^i_{BZQJ,h} = \begin{cases} 0 & C_{BZQJ,h} \neq C_i \\ 1 & C_{BZQJ,h} = C_i \end{cases} \ 1 \leq h, \ i \leq m ; \tag{16}$$

$$\sum_{i=1}^{m} Y^i{}_{\text{BZQJ},h} =1, \ 1 \leq h \leq m \ ; \tag{17}$$

$$\sum_{h=1}^{m} Y^i{}_{\text{BZQJ},h} =1, \ 1 \leq i \leq m; \tag{18}$$

$$T_{\text{BZKS},h} + t_{\text{BZ},i} \leq T_{\text{BZKS},h+1} , 1 \leq i \leq m, \ 1 \leq h \leq m-1 \ ; \tag{19}$$

$$T_{\text{BZKS},h} \leq T_{\text{C},i} - t_{\text{CF},i} - t_{\text{BZ},i}, \ 1 \leq h, \ i \leq m \ ; \tag{20}$$

$$\sum_{i=1}^{m} \sum_{h=1}^{m} Y^i{}_{\text{BZQJ},h} * t_{\text{BZ},i} \leq t \ ; \tag{21}$$

$$x^f{}_{i,j} = Y^i{}_{\text{BZQJ},h} * \ x^f{}_{i,j}, 1 \leq j \leq n, \ 1 \leq i \leq m \ ; \tag{22}$$

Equation (16) shows that if C_i has been arranged to assemble on the hth marshalling interval, then the value of $Y^i{}_{\text{BZQJ},h}$ is equal to 1; otherwise the value of $Y^i{}_{\text{BZQJ},h}$ is equal to 0. Equation (17) shows that one marshalling interval only can arrange one outbound train operation. Equation (18) shows that one outbound train only can occupy one marshalling interval. Equation (19) shows that only when the upstream outbound train has completed its marshalling operation can the current outbound train begin its operation by the same pulling out engine. Equation (20) means that only when C_i's latest marshalling time is greater than or equal to $T_{\text{BZKS},h}$ can C_i be arranged to the hth marshalling interval. Equation (21) means that the total outbound train marshalling operating time must be less than or equal to the stage plan time T. Equation (22) shows that if C_i hasn't accessed any marshalling interval, then the number of cars in the classification yard can't be the same as the number of $x^f{}_{i,j}$.

3.3 The optimal functions

Let $x^f{}_{i,j}$ be the decision variable, making the weighted total wagon-flow dwell time on the station during the stage plan time be the optimal function, as follows:

$$\min z(x^f{}_{i,j}) = \sum_{i=1}^{m+1} \sum_{j=0}^{n} \sum_{f \in F_{\text{C},i} \cap F_{\text{D},j}} w_j x^f{}_{i,j} * (T_{\text{C},i} - T_{\text{D},j}). \tag{23}$$

4 Conclusions

The railroad yard scheduling problem is more complicated than the same type of production scheduling problem. In any scheduling table, there is another NP problem which refers to the resource matching problem. Combined with different problems, using the historical data to generate the heuristic rules, the key point for further research is to build up a concise model and find an efficient algorithm.

Acknowledgements

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