The optimization of train timetable stop-skipping patterns in urban railway operations

A. Jamili, S. F. Ghannadpour & M. Ghorshinezhad
Department of Railway Transportation, MAPNA Group, Iran

Abstract

In this paper an operation mode, which is based on the determination of some patterns for train timetables, is analyzed. For this purpose, a new mathematical model is proposed to reach the optimized timetable patterns. In real-world applications, a small deviation from traffic analysis is acceptable, where passengers should stand more tightly inside the train. As a result, to reach a more practical and flexible solution, a fuzzy approach is utilized. At the end, a metro line is studied and the optimum patterns are presented and analyzed. It is found that in the stop-skipping approach, in comparison with the conventional all-stop approach, the number of stops reduces by 39%. By this new method, the number of trains that stop at stations differs according to the traffic that is usually measured by Passenger per Hour per Direction (PPHPD). Moreover, the commercial speed increases, and therefore, the number of required trains in operation reduces. However, this method results in a marked increase in the passengers' average waiting time at non-crowded stations.
Keywords: stop-skipping approach, train timetable patterns, fuzzy mathematical model.

1 Introduction

Urban railway lines are generally operated in two different classes: short loops and long loops. These modes of operations are defined based on the traffic analysis. Generally, short loop operation requires more facilities and a more complicated operation, but less repair and maintenance costs. In addition to this approach, skip-stopping approach, which increases the commercial speed and reduces the unwilling stops, can improve the system operation.

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The operation plan is one of the studies performed at the beginning of defining the project. In the operation plan study, based on the traffic studies as the input data, the suitable operation mode is defined for the system. In this paper, a new fuzzy mathematical model is proposed to specify a new operation mode, by defining different timetable patterns for trains.

Fig. 1 shows a simple sample of different operation modes. The squares are the symbol of stations. The colored ones indicate the stations that trains intend to stop. The hatched ones indicate those stations that are not visited by train and the blank ones shows the stations in which trains skips and do not stop for boarding and alighting purposes.

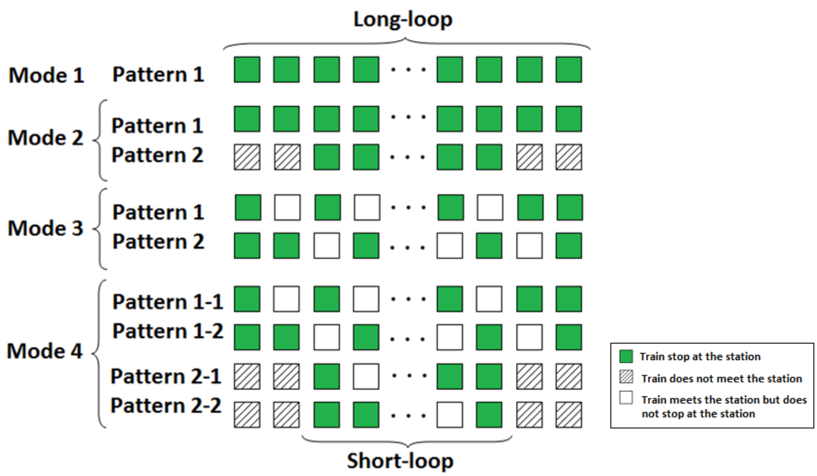

Figure 1: A sample of different operation modes.

It is necessary to mention that in the third, and fourth modes, different patterns are defined. The aim of this paper is to define the number of patterns as well as the proper blank squares for different patterns.

1.1 The effects of the stop-skipping approach

The effects of stop-skipping approach, which is studied in this paper, can be described in two different categories as summarized in the following:

1.1.1 The effects on the system

The following items are caused by considering the stop-skipping approach in urban railway systems:

1. The average number of stops at stations reduces.
2. The average traveling time reduces and consequently the average commercial speed increases.
3. The number of required trains in operation reduces.

(4) The operation and maintenance costs during operation life cycle time reduces.
(5) The signaling system and Passenger Information System (PIS) requires small modifications to the software.
(6) The number of parking areas and therefore, the land acquisition cost in project construction period reduces.

1.1.2 The effects on passengers

(1) In non-crowded stations, as some trains skip stopping, the average waiting times increases.
(2) As the commercial speed increases, the passengers' travelling time reduces.
(3) The passengers should pay attention to the PIS system located at the platform, before boarding the train, and should check the stations that the arriving train is going to stop.
(4) In some cases, the passengers have to change the train to reach the destination.

1.2 Previous works

During the past decade, the train scheduling problem has become one of the most interesting research topics. Zhou and Zhong [2] introduced a modified Branch and bound (BB) algorithm, which contains three methods to reduce the solution space in main line railway systems. A new multi objective mathematical model for train scheduling problems in main line railways introduced by Ghoseiri *et al.* [1]. Shafia *et al.* [3] proposed a robust timetabling model, and proposed a robustness measure to compute the required buffer intervals. Beside mainline train timetabling problem, in urban transport system, Albrecht [4] proposed a two level approach to generate demand-oriented timetable, where the optimal train frequency and the capacity of trains are first determined and then the schedule of trains are optimized. Wang *et al.* [5] proposed a detailed non-linear bi-level model of train movements with stop-skipping and the O-D dependent passenger demands, as well as a genetic alg.

In this paper, a new formulation is developed which addresses the problem of finding the optimum timetable patterns in urban railway systems. As the PPHPD is not a crisp value and a limited amount of violation are accepted, a fuzzy approach is used to reach a more flexible and practical solution. This method offers a more cost-saving and faster transport system.

The current paper is organized as follows: In section 2 the problem is defined, and a new mathematical model is proposed. In section 3, a fuzzy approach is utilized. As the resulted fuzzy model is not linear, the necessary variables as well as the required constraints are proposed. Section 4 deals with the validity of the proposed mathematical model through a metro line as the case study. Finally, the concluding remarks are given at the end to summarize the contribution of this paper.

2 Problem formulation

The problem is to find optimum patterns of train's timetables, so that (1) the number of train-stops at stations minimizes, (2) the required headway maximizes, (3) the time differences among consecutive patterns minimize. The PPHPD in each block section is computed based on the traffic study. The PPHPD is used to compute the required headway in each block section. In the next step, the required headway in each block section is used to define the patterns which determine the stations that trains should skip.

In this section, a new mathematical model is proposed. The employed notation is shown in Table 1.

Table 1: The employed notation.

Symbol	Definition
N	The set of trains, i.e. Maximum number of patterns
M	The set of block sections
S	The set of stations
hw_j	The crisp value of required headway for block-section j
$\widetilde{hw}_j$	The fuzzy value of required headway for block-section j
$\overline{hw_j}$	The upper bound of fuzzy number $\widetilde{hw}_j$
$\underline{hw_j}$	The lower bound of fuzzy number $\widetilde{hw}_j$
RT_J	The time difference between two scenarios (1) a train stops at station J for passenger boarding and alighting purposes, (2) a train just passes station J with no stop
MHW	Minimum possible headway can be reached by the signaling system
x_{iJ}	A binary variable, equals 1, If train i stops at station J, and 0, otherwise
HW	A variable indicates the headway between consecutive trains

The first objective is to minimize the trains' stops at stations.

It is necessary to mention that, as the headway reduces the number of required trains in operation increases. Therefore, the second objective is to maximize the headway.

A robust timetable is the one that the delay propagations are in the minimum level. Therefore, the third objective is to minimize the differences among consecutive patterns.

The proposed mathematical model is presented as follows:

$$Min\ Z_1 = \sum_{i\in N} \sum_{j\in M} x_{ij} \quad (1)$$

$$Max\ Z_2 = HW \quad (2)$$

$$Min\ Z_3 = \sum_{J\in S} \sum_{i\in N, i>1} \left(\sum_{K\le J} (1 - x_{i'K}) - \sum_{K\le J} (1 - x_{iK}) \right) \quad (3)$$

Eq. (1) specifies the first objective function which is to minimize the number of train-stops at stations. The second objective function, Eq. (2), maximizes the required headway. Eq. (3) is to minimize the differences among consecutive trains.

Subject to:

$$\frac{HW \times |N|}{\sum_{i \in N} x_{iJ}} < hw_j, \ \forall J \in S \tag{4}$$

Inequality (4) defines the stop-skipping patterns to reach the required headway for each block section. Note that in this inequality, block-section j is just located before station *J*.

$$HW - \left(\sum_{K \leq J} (1 - x_{i'K}) - \sum_{K \leq J} (1 - x_{iK}) \right) \times RT_J \geq MHW, \quad \forall J \in S, \forall i \in N - \{1\} \tag{5}$$

In Inequality (5), $\sum_{K \leq J} (x_{iK})$ specifies the number of train stops between stations $\{1, \ldots, J\}$.

3 Applying the fuzzy approach to the model

In real-world applications, the PPHPD in each block section is not an exact value. Therefore, it is more practical if the parameter hw_j is supposed to be a fuzzy number. It is necessary to mention that, in crisp linear programming, the violation of any constraint renders the solution infeasible, but in the mentioned case, the role of constraint (4) can be different, where a small but limited violation of constraints is accepted.

In this paper, the parameter hw_j of the proposed model, is supposed to be imprecise. This parameter affects the right-hand side of inequality (4). As the objective function assumed to be crisp, the Werner's approach is applicable.
The membership function is equal to Eq. (6).

$$\mu_j = 1 - \frac{\frac{HW \times |N|}{\sum_{i \in N} x_{iJ}} - hw_j}{\overline{hw_j}}, \ \forall J \in S \tag{6}$$

Note that in this inequality, block-section j is just located before station *J*.

Furthermore, the membership function of the objective function, f^i, is equal to Eq. (7).

$$\mu_G^i = \frac{c^{T,i} - f_1^i}{f_0^i - f_1^i}, \forall i = 1, 2, 3 \tag{7}$$

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where, $c^{T,i}$ equals the objective function value, and f_1^i is the optimum objective value of the proposed model in the crisp mode and f_0^i is the optimum objective value of the proposed crisp model when the parameter hw_j is replaced by $hw_j + \overline{hw_j}$.

Finally, by introducing one new variable, γ, the mathematical model with fuzzy constraint transforms to crisp model (8).

(Model (8): Fuzzy stop-skipping patterns generation mathematical model)

$$Max\ \gamma \tag{9}$$

Subject to:

$$\overline{hw_J} \times \gamma + \frac{HW \times |N|}{\sum_{i \in N} x_{iJ}} \leq \overline{hw_J} + hw_J,\ \forall J \in S \tag{10}$$

$$(f_1^1 - f_0^1) \times \gamma + \sum_{i \in N} \sum_{j \in M} x_{ij} \leq f_1^1 \tag{11}$$

$$(f_1^2 - f_0^2) \times \gamma - HW \leq f_1^2 \tag{12}$$

$$(f_1^3 - f_0^3) \times \gamma + \sum_{J \in S} \sum_{i \in N, i>1} \left(\sum_{K \leq J} (1 - x_{i'K}) - \sum_{K \leq J} (1 - x_{iK}) \right) \leq f_1^3 \tag{13}$$

$$(f_1^3 - f_0^3) \times \gamma + \sum_{J \in S} \left(\sum_{K \leq J} \left(1 - x_{|N|K}\right) - \sum_{K \leq J} (1 - x_{1K}) \right) \leq f_1^3 \tag{14}$$

Constraints 3 and 5.

By defining the variable $y_{iJ} = \gamma \times x_{iJ}$, one can replace the constraint (10) with the following new defined linear constraints.

$$y_{iJ} \leq M \times x_{iJ} \tag{15}$$

$$y_{iJ} \leq \gamma \tag{16}$$

$$y_{iJ} \geq \gamma - M \times \left(1 - x_{iJ}\right) \tag{17}$$

$$y_{iJ} \geq 0 \tag{18}$$

4 Case study: a metro line

The studied metro line contains 23 stations, 22 block sections and 2 shunting areas. Considering the estimated PPHPD in each individual block section of this line, the required headway in each of them is computed as shown in Table 2.

Table 3 shows the results in two scenarios: (1) Considering constraint (10) as the soft and (2) Considering constraint (10) as the hard one. The first row in each scenario shows the average number of trains' stops at stations. Obviously, the first objective seeks for minimum number of stops. The second row in each scenario indicates the fuzzy objective function value (OFV).

Table 2: The required headway in each block section.

Block sections		hw_J	$\overline{hw_J}$	Block sections		hw_J	$\overline{hw_J}$
B	C	68.7	9.0	O	P	6.4	0.8
C	D	40.2	5.3	P	Q	5.4	0.7
D	E	13.3	1.7	Q	R	7.3	1.0
E	F	10.3	1.4	R	S	10.0	1.3
F	G	7.8	1.0	S	T	13.6	1.8
G	H	6.7	0.9	T	U	18.4	2.4
H	I	6.4	0.8	U	V	25.1	3.3
I	J	6.3	0.8	V	W	34.7	4.6
J	k	6.3	0.8	W	X	48.1	6.3
k	M	7.1	0.9	X	Y	66.6	8.7
M	O	7.0	0.9	Y	Z	92.2	12.1

Table 3: Final solutions in different number of patterns.

N (i.e. number of patterns)		1	2	3	4	5	6	7
(1)	Average No. Stop/Train	22	16	14	12	11.8	11.5	11.4
	Fuzzy OFV	0	0.27	0.37	0.45	0.46	0.43	0.48
(2)	Average No. Stop/Train	22	16	15	13	12	12	11.7
	Fuzzy OFV	0	0.27	0.32	0.41	0.46	0.41	0.47

The results can be interpreted as follows:

1- As the number of patterns increases, the average number of train-stops at stations reduces.

2- Considering constraint (10) as a soft one improves the average final optimum solution by 3%.

3- As the number of patterns increases, passengers face with verity of different timetables, and may be more confused at stations.

Considering the last issue, and the achieved optimum solutions, the operators preferred the solution with four patterns. This solution does not provide some trips amongst some of the stations. In the case that, this type of solution is not valid for the decision makers, the authors proposed following constraints to be added into the Model (8):

$$\sum_{i \in N} \left(x_{iJ} \times x_{iJ+1} \right) = 1, \ \forall J \in S \tag{19}$$

Note that x_{ij} is a binary variable. One can replace $x_{iJ} \times x_{iJ+1}$ by a new binary variable z_{iJ}. Constraint 19 is non-linear and can be replaced with the following linear constraints.

$$z_{iJ} \leq x_{iJ} \tag{20}$$

$$z_{iJ} \leq x_{iJ+1} \tag{21}$$

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$$z_{iJ} \geq x_{iJ} - \left(1 - x_{iJ+1}\right) \tag{22}$$

$$z_{iJ} \geq 0 \tag{23}$$

Adding the above constrains into the model, results the following optimum solution (Fig. 2).

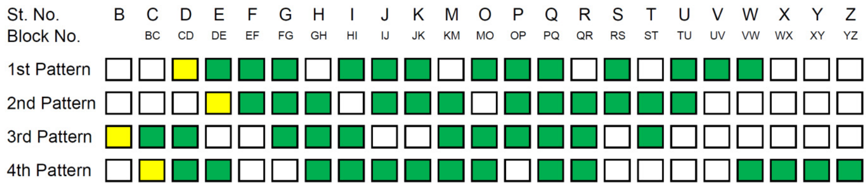

Figure 2: The optimum solution with four patterns.

Note that, the average number of trains' stops at stations increases from 12 to 12.5 in the latter case.

5 Conclusion

In this paper, an operation mode, which is based on the determination of some patterns for train timetables, also known as stop skipping approach, was analyzed. To that end, a new mathematical model presented to reach the optimized timetable patterns. The traffic analysis is considered as the most important input parameter of the model. As a result to reach a better solution, a fuzzy approach was proposed. Finally, a real-world case study was studied and the optimum patterns were presented and analyzed.

Acknowledgement

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