The building and implementation of a track unit selection model for a comprehensive track maintenance plan

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Abstract

To make the track comprehensive maintenance plan reasonably is an important means to ensure the safety of train operation. The track maintenance plan model analysis shows that making the annual maintenance plan is a process of finding the global optimal solution in a high dimension space. Reducing the model dimension is the key to simplify the complexity of a model under the premise of keeping the integrity of the model, so the maintenance plan model is divided into two parts, the track unit selection model and MTT job assignment model. In this paper, the first model will be introduced in detail. In this model, three unit selection constraints are considered, the optimized object is each lot of units and the objective function is to get the maximized sum of the maintenance, improving quantity of all the lots that are included in the selected unit. These selected units being optimized will be used as the input parameters of the MTT job assignment model, and this can achieve the purpose of reducing the dimension of the latter model. In order to solve and verify the first model, a genetic algorithm is introduced, constraint conditions in the generation phase of the initial population are pre-processed. The results, based on data from Shanghai-Kunming railway line, show that the solving efficiency is several times higher than the enumeration method under the circumstance of ensuring the average error is less than 15%. This proves that the model is practical and the genetic algorithm is effective in efficiency and precision, so the model can be used as a rapid and efficient approach for making a track maintenance plan.
Keywords: railway track, track longitudinal irregularity, comprehensive maintenance plan, unit selection, genetic algorithm.

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1 Introduction

Tamping and leveling are the main maintenance means for ballast track by using MTT (Multiple Tie Tamper). With the development of high-speed and heavy-haul railway, operation conditions have been changed greatly. Railway line quality changes faster and the time of maintaining sky-light is decreased, therefore, it is of great significance to build an optimal model for the track maintenance plan, which helps to resolve the conspicuous contradiction between maintenance workload and time.

The track maintenance plan model mainly contains a track geometry deterioration model, a track geometry restoration model and a track comprehensive maintenance plan model [1–11]. Zhou and Xu [4] took a multi-dimensional linear system as the research object in a track maintenance plan model. Andrade and Teixeira [5] developed a bi-objective optimization model for the planning of maintenance and renewal actions related to track geometry in a railway network, the research object became a nonlinear complex system with multi-dimensional and multi-objective.

However, time complexity and space complexity of a maintenance plan model performed exponential growth as the increasing number of the dimension of decision variables and nonlinear constraints in models in existing literature, the solution space became so large that it led to the curse of dimensionality [12].

In order to reduce the possibility of the curse of dimensionality and conduct the maintenance plan quickly and accurately, the maintenance plan model is divided into two parts, track unit selection model and MTT job assignment model. In this paper, a track unit selection model will be introduced in detail.

2 Track comprehensive maintenance plan model

2.1 Mathematical model analysis

Normally, tamping and leveling are mainly related to track longitudinal irregularity, so maintenance plan models are built for the objective of eliminating longitudinal irregularity, the method is also suitable for other irregularities [13].

The track comprehensive maintenance plan model aims to arrange the optional time and place for MTT within the minimal mean of track longitudinal irregularity.

Define parameters: 10 days set k = {1, 2, 3, …, 36}; track maintenance section d = {1, 2, … , $D^{\max}$}; unit u = {1, 2, 3, … , $j_i^{\max}$} (track maintenance section j).

Decision variable: W_{uk} = 1 if MTT is conducted in unit u on 10 days k and W_{uk} = 0 otherwise.

Considering holidays and equipment maintenance times in China, there are 28 × 10 days in 1 year to arrange a maintenance plan. The model can be described as follows [14]:

$$\max\sum_{u}\sum_{x=6}^{12}(37-x)\Delta S_u^r W_{ux}+\sum_{u}\sum_{y=14}^{27}(37-y)\Delta S_u^r W_{uy}+\sum_{u}\sum_{z=29}^{35}(37-z)\Delta S_u^r W_{uz} \tag{1}$$

such that,

$$W_{uk}\le 1 \tag{2}$$

$$W_{u_1 k_1}=1 \tag{3}$$

$$\sum_{u} W_{uk}\le A_k \tag{4}$$

$$\sum_{u\in U_1}\sum_{k\in R_u} W_{uk}=0 \tag{5}$$

$$\sum_{k} W_{uk}\le 1 \tag{6}$$

$$B\cdot W_{uk}+\sum_{u\in U_2^u} W_{uk}\le B \tag{7}$$

$$\sum_{k=1}^{k_c^u-1} W_{uk}=1 \tag{8}$$

$$\sum_{k}^{q_u-1} W_{uk}=0 \tag{9}$$

As for the above equations, Equation (1) is the objective function, ΔS_u^r represents the track geometry restoration rate after maintenance in unit u. Constraints expressed in Equation (2) to Equation (9), Equation (2) is total amount constraint of MTT, several track maintenance sections use one MTT, it can be arranged to any unit in different 10 days, but the total amount of MTT is only one. Equation (3) expresses special constraint of MTT, an MTT must be arranged to the specified track maintenance unit in the specified time, such as renewal actions and other major repairs, u_1, k_1: the specified track maintenance unit, the specified time. Equation (4) shows the upper limit constraint of units and A_k is the upper limit of unit sections which should be maintained in each ten days according to the operation characteristics of MTT; Equation (5) explores time constraint, work temperature of continuously welded rail will be distinct in different ten days and the time should be restricted when passenger flow is large, U_1 is one unit section which can be maintained but with time limited, R_u is the time sets that cannot be maintained in unit u which belongs to the set U_1; Equation (6) imposes maintenance times constraint, the maximum number of maintenance in each unit is only once a year; range of movement constraint of MTT explained in Equation (7), B is the maximum of $\sum_{u\in U_2^u} W_{uk}$ and U_2^u is the set of units that cannot be maintained in the same ten days with unit u; Equation

(8) determines upper limit constraint of deterioration state, $u \in U_3$, K_c^u is the set of the latest time to maintain in unit u, U_3 is the set of units that the geometric irregularity to be predicted to the upper limit in next year; Equation (9) is time interval constraint for laying guard rail, laying guard rail takes a long time to prepare, maintenance cannot be arranged in this time interval, U_4 is the set of units that have guard rail, q_u is the earliest month for maintenance in unit u.

The optimized object of the model above is the whole of the maintenance plan, the solution is in the form of a 1000 × 28 matrix if the annual maintenance plan is a 100 km line. Theoretically, each element in the matrix can be one of the two integer variables (0 or 1) without considering other constraints, then the solution space size of the model is $\sum_{i=1}^{28000} C_{28000}^{i}$, the order of magnitude is more than 10^{10}. Such redundant, sparse and high-dimension data make it complex to analyse and difficult to get the strict global optimal solution. Therefore, dimensionality reduction becomes necessary.

2.2 Dimensionality reduction

In the face of high-dimension space, it is necessary to find effective methods to reduce the dimension without sacrificing integrity and systematics of the model so as to simplify the decision problem. Dimensionality reduction is supposed to reveal much less computational cost and obtain coincident results with those got from the original data set.

The objective function of the model is to minimize the average value of track irregularity standard deviation in the plan period. Generally, it is believed that the track irregularity standard deviation is linear with improving quantity. When all considered lots are high dimensional data, we can choose lots with distinct improving quantity after the maintenance as the most representative lots to build a new optimization object set, that is, we obtain a set of low dimensional data as the optimization object.

The track unit selection model proposed by this paper aims at finding qualified units to ensure that the total maintenance improving quantity of all lots in the selected units is maximum.

The number of units can be used as input parameters of the MTT job assignment model, and becomes the number of matrix rows of the solution of the track comprehensive maintenance plan model. It can greatly reduce the dimensionality of the model and the possibility of the curse of dimensionality

3 Track unit selection model

3.1 Model definitions

The model is defined as follows.

1. Divide the planned maintenance line into lots and units, as shown in Figure 1. A lot is a basic unit of track irregularity to calculate standard deviation in 100 m length and predict geometric irregularity. A unit is the unit of the maintenance plan which is composed of a set of

continuous N lots. The value of N is determined by actual operation situation, such as maintaining sky-light. Each unit may not necessarily be continuous, but the sum length of all lots in the units must be less than the total length getting access to maintenance in a day.

2. Assuming that with the absence of maintenance for lots, track condition deteriorates shows a linear change, and the speed of deterioration ΔS_i of lot i is constant when the line is stable and MTT is fixed.

If the geometric irregularity standard deviation of lot *i* in time *t* is $\sigma_i(t)$, the standard deviation $\sigma_i(t+\Delta t)$ in time $t + \Delta t$ is in Equation (10). Equation (11) shows the amount of improving quantity after maintenance in $t + \Delta t$, therefore, it is considered that $\Delta\sigma_i$ is a parameter that only changes with Δt numerically, and this can be used to calculate the largest improving quantity of lot during the next maintenance.

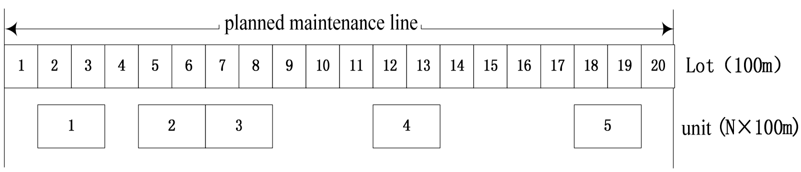

Figure 1: Division diagram for planned maintenance line.

$$\sigma_i(t+\Delta t) = \sigma_i(t) + \Delta S_i \Delta t \tag{10}$$

$$\Delta\sigma_i = \sigma_i(t) - \sigma_i(t+\Delta t) \tag{11}$$

where $\sigma_i(t)$: geometric irregularity standard deviation of lot *i* in time *t* (mm)

$\sigma_i(t+\Delta t)$: geometric irregularity standard deviation of lot *i* after time Δt without maintenance (mm).

ΔS_i: speed of deterioration of lot *i* (mm/d).

3.2 Proposed model

In the track unit selection model, the decision variable is integer, the decision object is the set of lots $L = \{1, 2, 3, \ldots, L_{max}\}$, the first one of the continuous lots is the decision variable if the unit includes more than one lot. There are three constraints and one objective function in the model [6].

3.2.1 Decision variable

$Vi = 1$ if *N* lots which start from lot *i* can compose a unit and $Vi = 0$ otherwise, $i \in L$.

3.2.2 Constraints

1. Theoretical constraint of unit selection:

We combine lots which are from lot $i + 1$ to lot $\min\{i + (N - 1), L_{\max}\}$ and cannot be the first one of a unit into the same unit when lot i has been selected as the first one of the unit, as shown in Equation (12).

$$\sum_{j=i}^{\min\{i+(N-1),L_{max}\}} V_j \leq 1 \quad (i \in \mathrm{L}) \tag{12}$$

As shown in Figure 1, take a line with a length of 2.0 km that consists of 20 lots as an example, when $N = 2$, the five selected units ($V_2 = 1$, $V_5 = 1$, $V_7 = 1$, $V_{12} = 1$, $V_{18} = 1$) are a case that imposed on the above constraint, all of the other decision variables are 0. It is not necessarily to select from the initial mileage or require the interval between two successive units. Equation (12) only needs that if lot i has been selected as the first one of a unit, the other lots in this unit cannot belong to other units anymore.

2. Constraint of specified lots:

The lots specified in the maintenance plan must be included in one unit.

$$\sum_{\mathrm{j=max\{i-(N-1),1\}}}^{\mathrm{i}} V_j = 1 (i \in \mathrm{L}_1 \subseteq \mathrm{L}) \tag{13}$$

L_1 represents specified lots, such as lots that track irregularity is more than comfort standard value but less than the safety limits.

As shown in Figure 1, when N = 2, and if lot 6 is specified, there are two available results: one unit composed of lot 5 and lot 6, the another unit composed of lot 6 and lot 7, and the former one is the selected unit 2.

3. Upper limit constraint of maintenance capacity:

Consider the upper limit of maintenance capacity as the max value to decide the sum of units according to maintaining sky-light and maintenance capacity of MTT.

$$\sum V_i \leq U_{max} \quad (\mathrm{i} \in \mathrm{L}) \tag{14}$$

Umax: the upper limit value of units that can be selected.

3.2.3 Objective function

The objective function is to maximize the sum of improving quantity $\Delta\sigma_j$ consists of all lots $N \times \sum V_i$ in all the selected units $\sum V_i$, explained in Equation (16). Si is the sum of the improving quantity of unit i that consists of N lots

$$S_i = \sum_{j=i}^{\min\{\mathrm{i+(N-1),L}_{max}\}} \Delta\sigma_j (\mathrm{i} \in \mathrm{L}) \tag{15}$$

$$\max Z = \sum_{i=1}^{L_{max}} S_i \times V_i \tag{16}$$

4 Implementation of the model

4.1 Algorithm selection

The model is a linear integer programming problem. The branch and bound method, cutting-plane method and implicit enumeration method can be used to solve integer programming generally, and these methods require the coefficient matrix of constraints to be indicated by a coefficient without a decision variable accurately, but the constraints shown in Equation (12) and Equation (13) are unable to meet the requirements. Consequently, this paper uses a genetic algorithm to optimize.

4.1.1 Chromosome code

The decision variables of the model is 0–1 integer variable; therefore, the coding form chooses binary. One row vector is a chromosome code, the number of the column vector indicates the number of lots. The starting position of a unit is Vi = 1. The code of the unit in Figure 1 is V = [0 1 0 0 1 0 1 0 0 0 0 1 0 0 0 0 0 1 0 0].

4.1.2 Initial population generation

Generally, the initial population is randomly generated in the solution space, but the quality of the initial population is not high. So it is better to estimate the range of the solution before the initial population is generated lest the initial population distribute far away from the code space of the global optimal solution, which may limit the searching range of genetic algorithm and the global optimal solution may not be achieved. It lightens the burden of time complexity as well.

Therefore, this model takes into account the constraints in the initial population generation. Firstly, generate a $m \times L_{max}$ all-zero matrix A, m is the individual number of the initial population, this paper takes 100; then generate $m \times U_{max}$ matrix B randomly, a random number for each row in matrix B is the position of Vi = 1 in initial population A. The model does not consider the cost of maintenance, thus simplifying the third constraint into an equality constraint, and making every individual in the initial population satisfy this constraint. Secondly, generate a $t \times N$ all-one matrix C, t is the lot number in L_1, N is the number of lots included in one unit, each row of matrix C is the number of the specified lot and N-1 lots before the specified lot. In addition, generate the N numbers of the row i of matrix C to the column i *of* matrix B randomly. Finally, find the number of U_{max} in each row of matrix A corresponding to the same row of matrix B, and assign "1", thus making the initial population satisfy the second constraint .

4.1.3 Fitness calculation

The selection of the fitness function affects convergence of the algorithm directly, the greater the fitness value of the individual, the more probable that the chromosome is inherited by the next generation. This model takes Equation (17) as the fitness function.

$$fitness = \sum_{i=1}^{L_{max}} S_i \times V_i \tag{17}$$

4.1.4 Selection, crossover and mutation

The roulette wheel selection method was applied to select outstanding individuals based on the proportion of individual fitness to fitness of all individuals in the population. Crossover and mutation of an individual use single point operation, crossover probability is 0.8, and mutation probability is 0.1. So it may avoid the premature phenomena of the genetic algorithm, increase the ability to explore new space and complete better convergence.

4.1.5 Algorithm termination condition

There are two commonly used termination conditions: one is setting the maximum number of evolution, which generally takes 100 to 1000 times; and the other one is setting a sufficiently small number ε. The algorithm will terminate when the difference between successive generations of the largest fitness is less than ε. The model in this paper uses the first one, and the number of evolution is 1000 times.

4.2 A case study

4.2.1 Data processing

Data of track longitudinal irregularity in the Shanghai-Kunming upline K226 + 000~K231 + 000 on January 4, 2009 and April 21, 2009 are used in this paper. There are 50 lots in the 5 km line when a lot is set to be 100 m, namely $L_{\max}$ is 50. Calculate the amount of improving quantity of all 50 lots according to the data of the two groups before and after April 21, the other parameters are $L_1 = [2, 12, 26, 42]$, $U_{\max} = 8$, $N = 3$. In addition, using data in the Shanghai-Kunming upline K267 + 900~K272 + 400 in July 10, 2009 and March 19, 2010, $L_{\max}$ is 45, the remaining parameters are the same as the above units. Maintenance improving quantity is shown in Table and Table 2.

4.2.2 Model calculation and analysis

The program code of the track unit selection model is written by MATLAB. It solves the optimal value of the objective function, determines the optimal unit selection plan by entering the amount of improving quantity of longitudinal irregularity and the parameters of genetic algorithm, and calculates the model with an enumeration method for comparison, the results are shown in Figures 2–5.

The black squares indicate the first lot of the unit composed of continuous N lots, namely the decision variables, Vi of which is 1, the decision variables Vi of white squares is 0.

In upline K226 + 000~K231 + 000, the value of the objective function calculated by the enumeration method is 4.46, but the time is more than 4 hours. The value of the objective function is 4.38 by genetic algorithm after iteration 1000 times, the error is 1.79%, and the average error is 14.57% when calculating 10 times randomly, the average computation time is less than 17 seconds. Thus it is evident that the efficiency of the genetic algorithm is relatively high, it can reduce maintenance planning time greatly. Figure 2 and Figure 3 show that two algorithms select 24 lots in eight units, and 20 lots of them are the same, accounting for 83.3% (the four different lots are lot 14, lot 47, lot 48, lot 49 chosen by genetic algorithm, and lot 1, lot 5, lot 9, lot 17 chosen by the enumeration method), which highlights the reliability of the genetic algorithm for this model for accuracy.

In upline K267 + 900~K272 + 400, 20 lots are the same in eight units selected by two algorithms, the probability is 83.3%, the four different lots are lot 7, lot 8, lot 9, lot 13 chosen by the genetic algorithm, and lot 10, lot 34, lot 41, lot 45 chosen by the enumeration method, as shown in Figure 4 and Figure 5. The maximum value of the objective function is 4.68 by the genetic algorithm and 4.65 by the enumeration method, the error is 0.64% for genetic algorithm and the average error is 10.48% when calculating 10 times randomly. Therefore, the error of the short maintenance plan length can be considered small under the premise of planning efficiency for the annual maintenance plan.

Table 1: Amount of improving quantity of longitudinal irregularity in upline K226 + 000~K231 + 000.

Start mileage	Improving quantity /mm	Start mileage	Improving quantity /mm	Start mileage	Improving quantity /mm
K226 + 000	0.22	K228 + 000	0.05	K230 + 000	0.39
K226 + 100	0.23	K228 + 100	0.13	K230 + 100	0.24
K226 + 200	0.36	K228 + 200	0.17	K230 + 200	0.20
K226 + 300	0.30	K228 + 300	0.27	K230 + 300	0.23
K226 + 400	0.24	K228 + 400	0.31	K230 + 400	0.21
K226 + 500	0.31	K228 + 500	0.25	K230 + 500	0.26
K226 + 600	0.25	K228 + 600	0.18	K230 + 600	0.23
K226 + 700	0.31	K228 + 700	0.14	K230 + 700	0.24
K226 + 800	0.30	K228 + 800	0.00	K230 + 800	0.19
K226 + 900	0.24	K228 + 900	0.04	K230 + 900	0.29
K227 + 000	0.18	K229 + 000	0.15		
K227 + 100	0.13	K229 + 100	0.06		
K227 + 200	0.09	K229 + 200	0.01		
K227 + 300	0.18	K229 + 300	0.15		
K227 + 400	0.23	K229 + 400	0.24		
K227 + 500	0.38	K229 + 500	0.25		
K227 + 600	0.19	K229 + 600	0.27		
K227 + 700	0.17	K229 + 700	0.37		
K227 + 800	0.29	K229 + 800	0.31		
K227 + 900	0.13	K229 + 900	0.37		

Table 2: Amount of improving quantity of longitudinal irregularity in upline K267 + 900~K 272 + 400.

Start mileage	Improving quantity /mm	Start mileage	Improving quantity /mm	Start mileage	Improving quantity /mm
K267 + 900	0.20	K269 + 400	0.10	K270 + 900	0.05
K268 + 000	0.14	K269 + 500	0.07	K271 + 000	0.14
K268 + 100	0.15	K269 + 600	0.04	K271 + 100	0.03
K268 + 200	0.30	K269 + 700	0.19	K271 + 200	0.14
K268 + 300	0.13	K269 + 800	0.06	K271 + 300	0.25
K268 + 400	0.14	K269 + 900	0.09	K271 + 400	0.23
K268 + 500	0.27	K270 + 000	0.03	K271 + 500	0.19
K268 + 600	0.08	K270 + 100	0.06	K271 + 600	0.2
K268 + 700	0.12	K270 + 200	0.09	K271 + 700	0.21
K268 + 800	0.13	K270 + 300	0.12	K271 + 800	0.23
K268 + 900	0.15	K270 + 400	0.13	K271 + 900	0.11
K269 + 000	0.24	K270 + 500	0.1	K272 + 000	0.23
K269 + 100	0.09	K270 + 600	0.31	K272 + 100	0.29
K269 + 200	0.00	K270 + 700	0.15	K272 + 200	0.27
K269 + 300	0.09	K270 + 800	0.07	K272 + 300	0.21

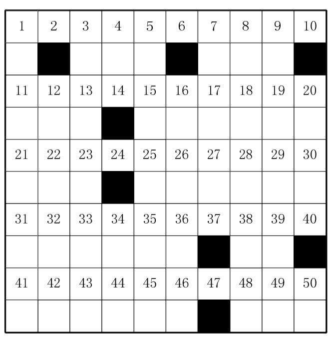

Figure 2: Track units in upline K226 + 000~K231 + 000 selected by the genetic algorithm.

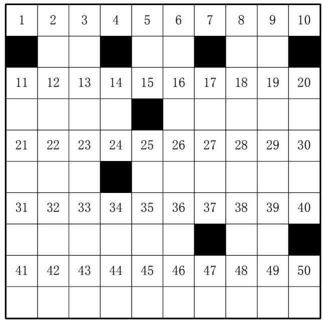

Figure 3: Track units in upline K226 + 000~K231 + 000 selected by the enumeration method.

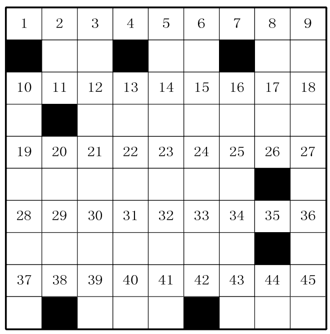

Figure 4: Track units in upline K267 + 900~K272 + 400 selected by the genetic algorithm.

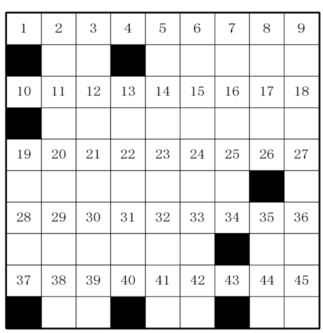

Figure 5: Track units in uplink K267 + 900~K272 + 400 selected by the enumeration method.

5 Conclusions

This paper builds a track unit selection model, solves the model based on a genetic algorithm, validates the effectiveness of the model and algorithm combining with field data of track irregularity. The conclusions are as follows:

1. Human resources, material resources and limited maintaining sky-light time can be used in track units that need maintenance mostly through the track unit selection model, which helps to keep track structure in a good irregularity state to conduct more effective and reasonable preventive maintenance.
2. The track unit selection model of this paper ensures that the sum of improving quantity of all units is maximal in the maintenance period, and eliminates the units in which the amount of improving quantity is not obvious so that they are no longer input parameters of the MTT job assignment model when using the track comprehensive maintenance plan to determine MTT job location, which reduces the possibility of the curse of dimensionality when programming the annual maintenance plan model.
3. A genetic algorithm is more effective than an enumeration method when solving models, and the lack of accuracy can be made up by pre-treating constraints in the stage of initial population generation.

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