A model to analyze railway delay to support the"performance regime" evaluation

P. L. Guida[1], A. Libardo[2] & G. Salerno[3]
[1]*Rete ferroviaria italiana, Italy*
[2]*Universitá IUAV di Venezia, Italy*
[3]*Universitá degli studi di Firenze, Italy*

Abstract

This research tries to improve an existing model of the authors concerning "performance regime": it showed an innovative methodology to check rail operations and to attribute delay responsibilities to every event and every train. This paper also considers other features of the infrastructure management dimension as: (a) railway company and external; (b) delay causes. A programming language has been used to set and define an algorithm that has been used in a key role in the "Perseo" project of the Italian railway manager (Rfi).
Keywords: performance regime, railway operation, delay responsibilities.

1 Introduction

Currently, one of the most important issues (features) of the management of railway infrastructure is *Performance regime*: it was traditionally called *Control train performance*. In the present state(ment) of liberalization, according with European directives, it has become necessary to be able to attribute correctly delay causes and responsibilities to the various rail transport actors (infrastructure manager included).

The estimation of a generic train delay and its effects on the circulation is a topic close but not the same as the one here presented as the objectives aimed at in the scientific literature mainly concern to the assessment of capacity (Line and station) and time reliability (Carey and Carville [1]).

Many researches investigate the performance improvement referring it to values derived from the statistical analysis of time delays and of their propagation

WIT Transactions on The Built Environment, Vol 127,
www.witpress.com, ISSN 1743-3509 (on-line)
doi:10.2495/CR120221

(Yuan [2]), but they do not solve completely the attribution and the validation of the causes of delay.

In many cases, the detection and quantification of the causes of delays are dealt separately distinguishing the conflicts generated in the station from the ones generated on the line. Yuan and Hansen [3] have tried to evaluate the impacts generated by the conflict between routes and by the propagation of delays in the station systems with a model that has taken into account random variations of occupancy time, of the necessary constraints to ensure safety and signalling system, determining the maximum frequency achievable once a delay that has been generated.

In terms of network effects, in 1994 Carey and Kwiecinski [4] have set a stochastic algorithm to determine the relationships between the deviation amount from the timetable and the next train. In 2007, Meester and Muns [5] formalized a model to quantify the running time propagation of delays which determines the secondary delay (caused) from the distribution of the primary delays (generators).

Recently, Daamen *et al.* [6, 8] and Goverde *et al.* [7] have developed a tool that can identify, basing on the conflicts recorded by the system when a train encounters a signal at danger, what is the triggering event (failure to the rolling stock or to infrastructure) and what would be the value of its own delay and of the one of the induced chain on the other trains in operation.

However, the issue is only partially resolved as shown by the first steps of the European Performance Regime (EPR) project, coordinated by the Rail Net Europe (RNE) and by the International Union of Railways (UIC) and the European Coordination Action projects such as the IMPRINT-NET [9].

Those projects aim to boost the rail system performance through the economic exploitation of the delays recorded at the end of journey and, as suggested by the Performance Regime, through the definition of common and measurable quality standards of the service (recognised in Italy in 2005 and applied in Network Informative Prospectus [10]) and subsequent monetization of the generated delays (with penalties for those who cause disturbances to the movement and compensation for those who suffered them).

This discipline, considering service increases due to demand elasticity (Libardo and Nocera [11]), should encourage the various operators, increased as consequence of the railway liberalization [12], to keep attention to individual performances [13], in order to achieve an overall service improvement, as shown in the case of transit quality by Nocera [14], as well as in the study experience by Giovine [15].

However, the definition of responsibility share of the different actors involved in the operations has proved to be a problematic process. England was one of the first countries in developing the system performance concept in the mid 90s. The model, well known as the Star Model, provides the monetary compensation between companies for delays longer than 3 minutes. Responsibilities have been defined in accordance to the types of service disruption: the infrastructure manager is responsible for everything related to the network, failures or vandalisms; while the operators are responsible for all the rolling stock. Nevertheless not existing so far, at date, a method of objective measurement that many disputes arise for the recognition of their own faults of delay.

The present research tackles and solves the issue of the attribution of delay causes. It represents a valid method to identify those individual services (or parts of infrastructure) that have penalized the global operation. To identify and quantify the overall responsibility of the different actors involved (network operator or a single railway company), will carry out the financial compensations in order to delete or minimize the current disputes.

The proposed method results in an operational tool for the concept that the delay incurred by a train is not automatically and fully attributable to the train itself. But that or total delay may be caused by one or more other trains that have influenced the movement or by other causes: due to the infrastructure manager or other external causes. With this in mind, it is necessary to split conceptually the delay of the train (accumulated in a given time and at a point on the line) into two components. The former endogenous, is the part that should be recognized as ownership of the train itself, and the latter exogenous, is due to the interaction and to the responsibilities of other trains or external events.

The methodology described in this paper allows:

1. to separate the endogenous and exogenous parts of variation of the delay which undergoes each train in an instant and at a point of its running;
2. to analyze at a point on the line (section or station) the interactions among all the trains that circulate in order to identify any indirect responsibilities (delays inflicted delaying one or more trains interposed between the two);
3. to combine the delay variations calculated locally along the line, in order to share correctly the total delay accumulated so far into two components (endogenous and exogenous); and further on to share out the exogenous delay allocating the responsibility of each train responsible, even if the interference has been verified in a earlier point of the line (in the space or in the time);
4. to explicitly assign those responsibilities to the subject among: the different railway companies, the infrastructure manager, the external causes, or more.

The proposed method is developed in several stages. It begins with the *reading* of the real movement along a single line such as it happened, and it merely considers the deviations from the theoretic timetable on the basis of a sort of interaction happened between one train and the one which precedes it.

Next, probable recording relating to anomalies which have been officially assigned specific responsibilities and timely are taken into account.

For example, from the recordings made by the staff of infrastructure manager, you may find that a train is actually departed late from a station, but that this delay (or part thereof) is due to a case linked to the exercise of the station and that without such a cause outside itself, on the train would not have an increase in its own delay.

Normally, the same recordings (not otherwise detectable by the automatic recording of the events of movement), are detected and incorporated into computer systems by executive managers and traffic managers.

In order to bring all the elements to a single, homogeneous model, these situations are handled by simulating the presence of a *ghost train*, conveniently positioned just before the train which suffered the delay.

The same ghost train, appropriately classified and coded, can be used in the reconstruction of the causes of delay and in identifying the subject responsible.

The model is immediately applicable in the network management practice and has been computationally implemented and included as one of the defining elements of the Perseus project by Rfi. The article presented here, even to contain it in a reasonable size, will be purely methodological, postponing the presentation and discussion of implementation issues and applications to a following article.

2 Conventions and notation

To facilitate the interpretation of symbols, some rules have been adopted:

- capital letters refer to time intervals and lowercase letters with the instants;
- arrival parameters are indicated by an apex ($'$), departure parameters are indicated by a double apex ($''$); for instance the delay in the arrival of a train in a station will be denoted by S', while the difference in the departure time will be denoted by S'';
- the use of the apex point ($^\bullet$) refers to a parameter that belongs to the *train which precedes* at the station the train under examination; the use of point with the subscript ($_\bullet$), refers to a parameter of the train under examination but referred to the *previous station*. For instance $p^\bullet$ denotes the instant in which the previous train has left (the station) and $p_\bullet$ denotes the departure of the train under examination in the previous station;
- the asterisk sign at the superscript ($*$) indicates the running of a train at the scheduled time. For instance a^* shows the scheduled arrival time in the station examined;
- parameters often have an index that indicates a train or a location. If the index is not reported, it means that the recipient is clear from the context or that it is applicable to all possible cases. This applies both to superscripts and subscripts as above;
- in space-time diagrams the actual train paths are drawn with a full line while the theoretical ones are dashed.

3 Criteria and rules

A train can earn delay (endogenous or exogenous both) in line and in station. For the calculation of this delay we will use the following operating rules:

1. any train that is on its time path may be charged of an endogenous or exogenous delay that has affected another train;
2. if a train causes a delay to another one because it is prevented by a third train to circulate regularly, it transfers part or the whole exogenous delay to the last train;

3. a train that has a station departure delay, must decrease or totally absorb it, using the elongation at the disposal;
4. every recovery requires an equivalent reduction in responsibility, first of all for the train itself, and if greater than the responsibility acquired till that time, also for other trains, starting from the first train that has been delayed at the earliest instant;
5. if a train does not make a recovery, although the possibility exist, this period will be called non-recovery time;
6. also the lack of recovery can be divided into endogenous and exogenous components, depending on the circumstance that train is queued or not, to the train which precedes it in line. In fact, in the case of a train, seeking to exploit its elongation, is queued to a delayed train that precedes on line, the latter will be held responsible for the non-recovery of the other train (possibly partially);
7. the non-recovery times do not modify the delays of trains, but result in a redistribution of the responsibilities;
8. the increase of delay that is induced on another train cannot exceed its own total delay;
9. block headway values at each station are assumed known, otherwise you give the minimum values in the normal specific timetable design;
10. the responsibility for an increasing delay is always assigned, if possible, to the train that immediately passed before at that line point (inbound or outbound from the station), the amount that is not possible to assign to this train (because it would exceed its total delay), will be "forwarded to the previous train".

4 Responsibility and contact matrices

4.1 The matrices of responsibility

The part of delay that a train accumulates and that has a responsibly cause (eventually the train itself) is named responsibility. If, during the period of analysis, the total number of trains that have circulated on the line (even if partially), is n, the responsibilities will be expressed through a *responsibility matrix* $\mathbf{R}$, of size $n \cdot n$, where each value R_{ij} represents the responsibility of the train j against train i in a specific point of the line.

The basic properties of the matrix $\mathbf{R}$ are:

1. the ordering of the trains of the matrix is arbitrary and must not satisfy any specific criterion, since each value represents a link one by one and is not influenced by its position in the matrix;
2. therefore individual values of the matrix express delays and must be non-negative:

$$R_{ij} \geq 0; \qquad (1)$$

3. the value R_{ii} on the main diagonal represents the train i endogenous deviation, that is the delay attributable to its own responsibilities;
4. the sum of row i must be equal to the total deviation of the train i:

$$\sum_j R_{ij} = S_i; \tag{2}$$

5. the sum of column j is the total responsibility of the train.

The responsibility matrix is related to special points of the line, that is to arrival and departure from each station. Those are also the detection points of train transits. The responsibility matrix in the next special point of the line will result by the summation of the responsibility matrix $\mathbf{R}$ in a special point and of a matrix of variations $\mathbf{V}$. In contrast to the matrix $\mathbf{R}$, the matrix $\mathbf{V}$ can be composed also by negative values (in case of recovery, either real or missed).

The primary purpose of the algorithm that is presented in this paper is represented by the construction of the variation matrixes, one for the standing time at the station and one for the journey time.

Therefore, the overall process, starts from the matrix $\mathbf{R}$ relative to the starting point of the line, calculates and adds the matrix $\mathbf{V}$, to get the second $\mathbf{R}$, and so on up to the end of the line. To manage this process in a modular way, it is established as a basic module the sequence consisting of the line journey time, the arrival in the next station, the standing at station and the departure from station.

The specific points under consideration are:

- the station entrance;
- the departure from the station.

The sequence will be formalized by the following recursive expressions:

$$\begin{aligned} \mathbf{R}'_k &= \mathbf{R}''_{k-1} + \mathbf{V}'_k \\ \mathbf{R}''_k &= \mathbf{R}'_k + \mathbf{V}''_k \end{aligned} \tag{3}$$

The first matrix of responsibilities to which you can assign values is the matrix $\mathbf{R}'_1$, i.e. the one that refers to arrival in station 1. This matrix will consist in the delays only due to arrival in the first station, considered endogenous and then placed on the main diagonal. Even for the trains that appear on the line for the first time at an intermediate station, we consider the delay which they occur with endogenous. Of course, if the records of the causes there will be a different responsibility, then we will take into account from explicitly the introduction of a ghost train, as will be explained later. If the stations are m, the last matrix $\mathbf{R}$ is the one in which the arrival happen:

$$\mathbf{R}'_m = \mathbf{R}''_{m-1} + \mathbf{V}''_m \tag{4}$$

Preliminary calculation of the matrix $\mathbf{V}$ is the analysis of elementary interactions between two trains passing at a specific point of the line, either arriving or departing from a station in succession. The following section explains how to make this analysis.

www.witpress.com, ISSN 1743-3509 (on-line)

4.2 Contacts matrix

La *contacts matrix* **C** is a matrix that report (in each specific line point) the most recent instant in which two trains have been in contact generating or not a delay to a train and for responsibility of the other. The matrix **C** has the same size and same ordering of the matrices **V** and **R**.

The matrix **C** is essential as a proper mechanism in decreasing the responsibilities. When a train recovers part of its delay, that is considered a “virtuous behavior” and the recovered amount of time will be removed from its responsibility. But if the train has not endogenous delays or has an amount smaller in respect than the recovery gained, it will be diminished the exogenous responsibility to other trains starting from the interference between the train observed and the one most distant in time. It is important to know the latest moment when a real contact between two trains has happened.

5 Elementary interactions between two trains

The different dynamics of interaction that take place in line and in station require a separate analysis of the two phenomena.

5.1 Interactions at the station

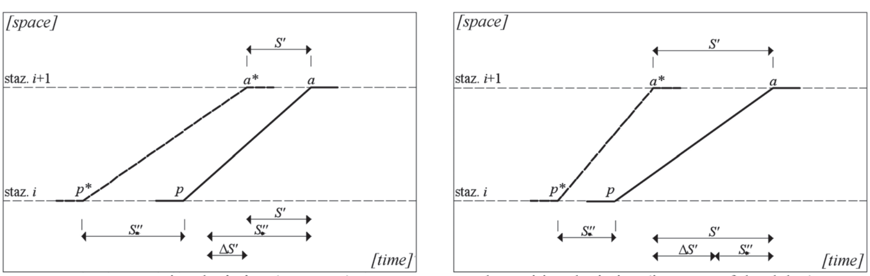
Train deviations from timetable are depicted in figure 1.

The variation of delay in departure $\Delta S''$ that the train produces at the station, is given, by definition, by the difference between the deviation in departure and on arrival. The deviation charge, if negative, corresponds to a recovery at the station.

The value $\Delta S''$ is calculated, as defined, by the following expression:

$$\Delta S'' = S'' - S' \tag{5}$$

starting from the train with the most distant in time interference with the one observed.



a. negative deviation (recovery) b. positive deviation (increase of the delay)

Figure 1: Deviations in arrival and departure from a station (in hatch the theory path).

While negative variations will be exclusively assigned to the train under examination, the positive ones will be separated in their two components, endogenous $\Delta I''$ and exogenous $\Delta E''$. This split will be achieved in steps, the first one, presented in this section, is the analysis of the elemental interaction between the train left immediately before the one to be analyzed.

We define free time in departure F'' the interval between the route release time of the previous train and the departure of the train under examination (it corresponds to the time interval between the two departures p and $p^{\bullet}$ minus the departure distancing D''). It is shown in figure 2, in which appears also the similar value F', arrival free time, which will explained later in the section on line interactions.

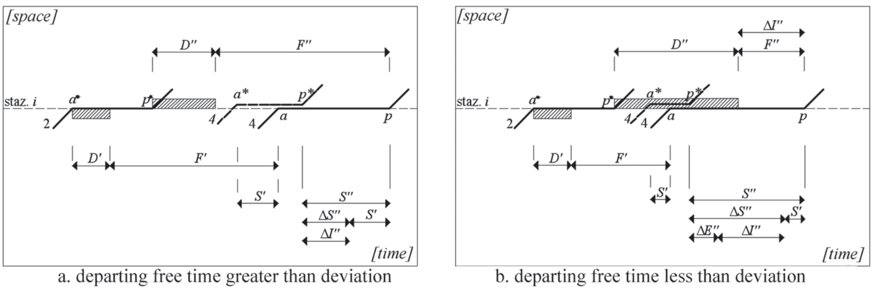


Figure 2: Endogenous and exogenous components related to an increase in delay at the station.

As you can see in the picture, the departing free time F'' is the interval between the instant when the train would have been able to leave respecting the rules of block headway, and the moment when it actually started. Therefore, it represents a possible increase of delay due to the train, or the maximum time that the train in case of necessity could recover.

Departing free time, therefore, is calculated by the following relationship:

$$F'' = p - p^{\bullet} - D'' \quad (6)$$

If there is an increase of delay at the station, the train will be responsible for up to the limits of the departing free time F'' . Therefore, the rate of change of endogenous delay attributable to the train, is the minimum value between the time free at the start and the variation of the difference recorded at the station:

$$\Delta I'' = \min(F'', \Delta S'') \quad (7)$$

relationship (applying)even in case of recovery, ie when the $\Delta S''$ is negative.

Using the exogenous variation definition, one can calculate the $\Delta E''$ by the difference between the total change in offset and the endogenous components:

$$\Delta E'' = \Delta S'' - \Delta I'' \tag{8}$$

valid applying even in case of recovery.

It should be noted that it is not guaranteed that the share of the exogenous value $\Delta E''$ calculated with (8), and ascribed to the train that has left just before the train under examination, respects the general rule according to which "increasing delay which is induced on another train can not exceed its overall delay". Therefore, the calculated responsibility is provisionally attributed. A next procedure will redistribute the excess exogenous component of other trains which left earlier and have affected behavior (responsibility propagation).

5.2 Line interactions

In the analysis of on line movement, in addition to the possibility of increase ($\Delta S > 0$) or recovery ($\Delta S < 0$) of a delay, it may happen that a feasible recovery, cannot be achieved or is only partially realized as already said. This form of irregularity can be endogenous or exogenous.

5.2.1 Variations of the line deviation

The procedure for calculating the variation in the arrival deviation $\Delta S'$ (ie developed along the route to before a station) and of the endogenous $\Delta I'$ and exogenous $\Delta E'$ components is similar to the one used for the calculation of similar values at the departure, and illustrated in Figure 3.

The variation of the line deviation is the difference between the deviation in arrival and in departing from the previous station (figure 4):

$$\Delta S' = S' - S''_{\bullet} \tag{9}$$

The endogenous part of the variation of the deviation is whichever is the lowest value between free time coming and the whole variation of the same deviation

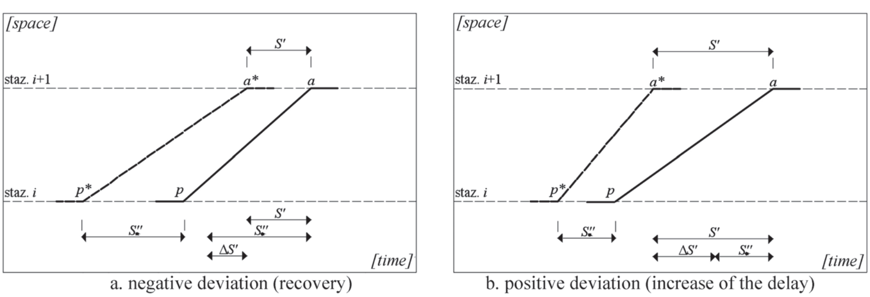
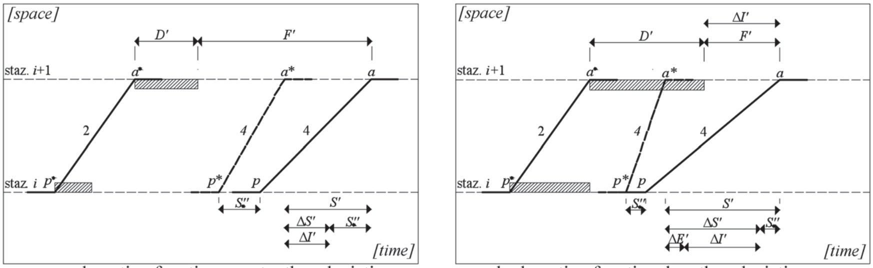


Figure 3: Variations of the line deviation (in hatch the theory path).



a. departing free time greater than deviation   b. departing free time less than deviation

Figure 4: Endogenous and exogenous components related to an increase of line delay.

(possibly negative):

$$\Delta I' = \min(F', \Delta S') \tag{10}$$

And finally, the exogenous part is derived from the definition itself:

$$\Delta E' = \Delta S' - \Delta I' \tag{11}$$

5.2.2 Failure to recover

In designing timetable for each train and each line can be defined an *elongation* L that represents the maximum possible reduction of the line travel time. It defines a margin of regularity used by that train on that route. Consequently, the elongation represents the maximum possible recovery for a delayed train, ie the recovery value that the infrastructure manager requires or imposes on the train and on the transport company in all possible cases. Note that the recovery can only be endogenous.

The recovery of a train is anyway possible only if another train does not circulate on the line immediately ahead making it impossible to take the maximum average speed permissible without violating the constraint of block headway (which is guaranteed by the signaling system).

The recovery cannot be higher than elongation L, but not even than the delay in departure from the previous station S'', since with such recovery the train would return on time, consequently, the possible recovery L^* will be calculated with the expression as follows:

$$L^* = \min(S''_{\bullet}, L) \tag{12}$$

In the case of a delayed train using only a part of the elongation provided without canceling the delay itself you can define the recovery unused L^{**} (figure 5a) by the following equation (it is assumed that trains will not have explicitly advance the race, whose possible exploitation shall not be interpreted as a cause of delay):

$$L^{**} = L^* + \min(\Delta S', 0) \tag{13}$$

In this way L^{**} is equal to L^*, if the train maintains unchanged or increases its delay, while it is lower and equal to $L^* + \Delta S'$ in case of recovery (taking into account the negative sign of recovery).

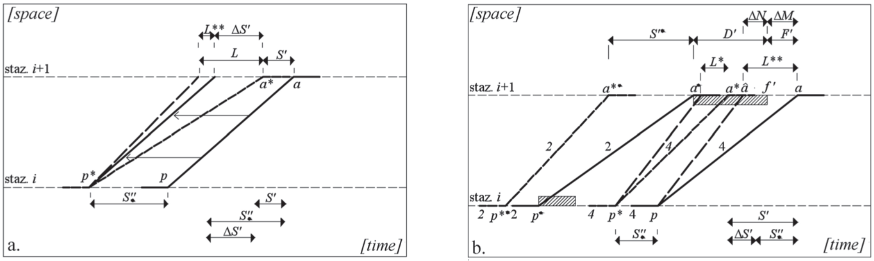


Figure 5: (a) Elongation and (b) endogenous and exogenous components related to a failure to recover.

Even the lack of recovery may be divided into part ΔM *(endogenous non-recovery)*, attributable to the train under analysis, and part ΔN *(exogenous non-recovery)* imposed on the train by the conditions of the line circulation (figure 5b). To this end, we define two auxiliary variables: *the optimal arrival* $\hat{a}$, representing the arrival of the train in the system, if the train had exploited all the possible recovery, and *the instant of line release* f' which represents the instant from which the train can enter the station (assuming the availability of a route and the clearing of a proper stopping or transit track):

$$\hat{a} = a - \Delta S' - L^* \tag{14}$$

$$f' = a^{\bullet} + D' \tag{15}$$

These measurements permit the calculation of the endogenous and exogenous failure to recover of a train line:

$$\Delta N = \min[\max(f' - \hat{a}, 0), \min(L', S'^{\bullet})] \tag{16}$$

Expressing (16) as a function of the primary values defined above, we get:

$$\Delta N = \min[\max(L^* + \Delta S' - F', 0), \min(L', S'^{\bullet})] \tag{17}$$

The endogenous non-recovery is calculated as the difference between the recovery unused L^{**} and the failure of exogenous recovery:

$$\Delta M = L^{**} - \Delta N \tag{18}$$

As for the delay variations, it should be noted that even in the case of the calculation of exogenous non-recovery the Train manager is detected and that the identification is temporary (of local nature), until the final stage of the analysis.

5.3 The local matrix of variations

The results of the elementary interactions just calculated can be collected in matrix form to compose the *local matrix of variations* **v**. If, in the entire time interval covered by the analysis, the total number of trains which have moved on that specific point of line is n_l, the local matrix of variations $\mathbf{v}$ has size $n_l \cdot n_l$. Each local matrix $\mathbf{v}$ reports initially for each train (lines) two values: the endogenous delay train i (on the main diagonal) and the exogenous delay generated by the train $i-1$ that preceded it (on the cell to the left of the main diagonal).

5.3.1 The local matrix at departure from the station

First, we analyze the local matrix of variations at departure from a station. The method and its essential properties, are the following:

1. trains are strictly ordered according to the actual transit time of that point (and not after the scheduled timetable);
2. on the main diagonal v_{ii} are placed the endogenous values $\Delta I''$, and in the cell left to its $v_{i,i-1}$ the exogenous values $\Delta E''$; values on the main diagonal could be both negative or positive, while the others outside it must be non-negative:

$$v_{ij} \geq 0 \qquad (\text{per } i \neq j) \tag{19}$$

3. the sum of the row i coincides with the total variation of the deviation of the train i compared to the previous point (in this, as in next formula, values are written without the quotes, which indicate the departure location, in order to have a valid writing also for arrival location):

$$\sum_j v_{ij} = \Delta S_i \tag{20}$$

4. the single value v_{ij} cannot be higher than the total delay of the train j S_j:

$$v_{ij} \leq S_j \tag{21}$$

However the last constrain (21)), is not automatically checked by the values that are derived from the analysis of elementary interactions, except for the endogenous increase, and if not met, you will have to impose it giving the later on, the difference $v_{ij} - S_j$ to another or to more trains. The procedure that accomplishes that is illustrated in the next paragraph and is called *propagation*.

5.3.2 The local matrix of arrival at the station

Also in the case of online route and of arrival in station, the matrix $\mathbf{v}$ must maintain the characteristics just mentioned, with respect to the constraints (19), (20) and (21).

The difference depends on the fact that we must consider also the failure to recover. To do that, the matrix $\mathbf{v}$ is constructed by adding the offset variations and the missed recoveries. Stands on the main diagonal $v_{ii} = \Delta I' + \Delta M$ and in the cell to its left $v_{i,i-1} = \Delta E' + \Delta N$.

Also this matrix has to respect the condition (20), which cannot happen for the presence of failure to recover. This is imposed by adding to the values on the main diagonal of $\mathbf{v}$ the sum of non-recovery, sign reversed. The value then v_{ii} will be calculated using the following expression:

$$v_{ii} = \Delta I' - (\Delta M + \Delta N) \tag{22}$$

The expression (22) is not a convenient analytical artifice to ensure that the condition (20) is respected, but it corresponds to the logic of the phenomenon. A recovery failure, in fact, is a possible negative variation decrease of deviation that the train did not realize anyway. The values ΔM and ΔN are calculated as a responsibility to be attributed to the train i or to the train $i-1$ respectively. But a similar quantity must be subtracted from the responsibility of some trains. In this phase of matrix construction **v**, this can be attributed to the train disclaimer i. Also in this case, the propagation procedure will have to define the reductions of train i and of other trains.

5.4 Infrastructure manager responsibility or other causes

Once calculated the elementary interferences and before evaluating their propagation, you have to take into account any records related to abnormal movements to which are assigned specific responsibilities (by the infrastructure manager), unless there are procedures of agreement and verification with the railway companies. If there is a recording that gives a time ΔG_i as variation of the delay of the train i due to a different cause, then this amount must be subtracted from that train and, if necessary, diminished the responsibility for the train that preceded it.

One way to account for this possibility is to simulate the presence of a *ghost train* between the train i and the train $i-1$, which becomes the train $i-2$. The figure 6 shows two cases, the former attributed the delay to the ghost train is lower than the endogenous component i of the train, while in the latter it is larger.

The values of the rows i and $i-1$ are determined as follows:

$$\begin{aligned} v_{ii} &= \max(0, \Delta I_i - \Delta G_i) \\ v_{i,i-1} &= \Delta S - v_{ii} \\ v_{i-1,i-1} &= \Delta G_i \\ v_{i-1,i-2} &= 0 \end{aligned} \tag{23}$$

and, for consistency, the values ΔS_{i-1} ed F_{i-1}, in order to ensure proper propagation process must comply with the following reports:

$$\begin{aligned} \Delta S_{i-1} &= \Delta G_i \\ F_{i-1} &= \max(0, F_i - G_i) \end{aligned} \tag{24}$$

This will simulate a train that has a endogenous delay equal to the recorded delay and that has transferred it to the train i. The value of F_{i-1} ensures the possibility of correctly transferring any surplus to the previous trains.

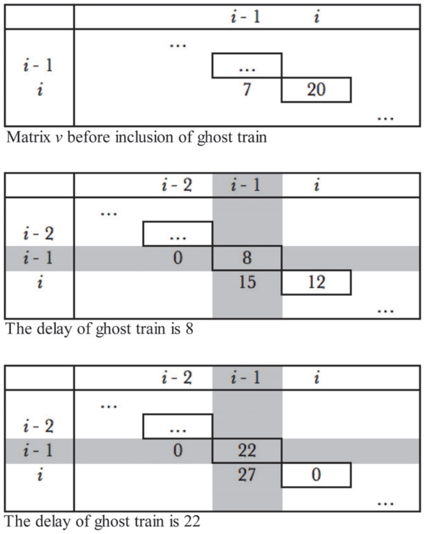


Figure 6: Two cases of inclusion of ghost trains (one with a delay $8 < 12$ and the other $22 > 12$). In the gray columns and rows are input the values related the ghost train. The train $i - 1$ becomes the train $i - 2$.

Each ghost train appears with its individuality in all matrices, just like all the other trains. It will be the task of computer procedure to assign each a "number train" or other marks, by which to date back to the recording and to collect, if necessary, in a single item all the responsibilities of the manager and of each one among the other responsible.

6 Interaction propagations

The process of the propagation of the interactions is the process that transforms the local variation matrix $\mathbf{v}$ into the matrix of variations $\mathbf{V}$. The process will modify the rows where it is required a redistribution of responsibilities, so there may be a transfer of responsibility from the train i to the train $i - 1$ or from these to other trains passed previously. In the end, in contrast to the matrix $\mathbf{v}$, the number of non-null values in a row may also be higher than 2.

The first formal difference between the two matrices is their size and their ordering. The first, $\mathbf{v}$, has size $n_l \cdot n_l$, where n_l represents the number of trains that have passed on that specific point of the line and ordered according to the actual time of transit; the second, $\mathbf{V}$, has size $n \cdot n$, where n is the total number of trains passed at any point of the line and has a completely arbitrary order (but of course always the same and equal to the one of the matrix $\mathbf{R}$ and $\mathbf{C}$).

The reasons why for you may require a redistribution of responsibilities in order to obtain a proper matrix $\mathbf{V}$, are three:

1. if the exogenous variation $v_{i,i-1}$ of a train i is higher than the total deviation of the previous train S_{i-1}, then the difference $v_{i,i-1} - S_{i-1}$ should be attributed to the train $i-2$, and, if still excessive, the surplus to the previous train, and so on;
2. if a train makes a recovery, ie when $v_{ii} < 0$, this value must be subtracted (algebraically added) from the endogenous delay stored so far; however is not excluded that $|v_{ii}| > R_{ii}$, ie the recovery, can be greater than the responsibility stored by the train itself; in this case the train in addition to cancel completely its own responsibility, will reduce the responsibility of some other train, as described later;
3. a train responsible for a non recovery (either endogenous or exogenous)will increases in its responsibility, even if it doesn't cause offset variations; that means that the same quantity must be taken away from one or more trains, as in the previous case.

The steps to be taken to correct the propagation depends on the sign of the variation:

1. if $v_{ij} > 0$, it means that train j (eventually i itself) has contributed to an increase in the delay of the train i (cases 1 and 3);
2. if $v_{ii} < 0$, it means that the train i has obtained a recovery (case 2); keep in mind that negative values in the matrix $\mathbf{v}$ can only be found on the main diagonal (in contrast to the matrix $\mathbf{V}$, as will be seen later).

6.1 Propagation of an increase of delay

In case that it is meet the condition (21) for the train $i-1$

$$\Delta E_i = v_{i,i-1} \leq S_j$$

no further operation is needed (that v_{ii} is $\leq S_i$,is automatically guaranteed by the calculation procedure ΔI).

In the event that the condition is not met, the surplus $\Delta E_i - S_j$ must be assigned to the previous train $i-2$, but always respecting the condition of not exceeding the total delay of the latter S_{i-2}.

Another constrain is to verify the conditions why there may be an interference with the train $i-1$, ie that the free time (in departure or in arrival) between the train $i-2$ and the train $i-1$ is zero:

$$F_{i-1} = 0 \tag{25}$$

If there is still a residual responsibility to be assigned to a previous train you continue moving up the chain of steps over time. This is represented by the following recursive expression:

$$v_{ik} = \min(S_k, \Delta E_i - \sum_{j=k+1}^{i-1} v_{ij}) \tag{26}$$

To start the recursion, you will use the index k (decreasing) equal to $i-1$ and the value $v_{i,i-1}$ equal to:

$$v_{i,i-1} = \min(S_{i-1}, \Delta E_i) \tag{27}$$

The recursion ends when one of the following three conditions is met:

1. index k is zero: all the trains circulating before the train in examination i have been treated;
2. $F_k > 0$: free-time non-zero means that there hasn't been any interference between the train k and the train $k-1$ and, as consequence, even between the train $k-1$ and the train i, even in the mediated form by other trains;
3. $\Delta E_i - \sum_{j=k+1}^{i-1} V_{ij} = 0$: it has been divided the total exogenous variation of the train offset i among the trains that have preceded it.

If any of the first two conditions occurs before the third, then you have not been able to distribute the total endogenous variation of the delay ΔE_i to the previous trains. It means that train i might have given the opportunity to leave or to pass in a moment antecedent, but it did not do. Then it is a legitimate presumption that all the responsibilities previews and not yet redistributed are to be charged to the same train i: from the analytical point of view it means that, at the end of recursion, to the endogenous componentΔI_i must be added any amount still to be assigned:

$$v_{ii} = \Delta I_i + \Delta E_i - \sum_{j=1}^{i-1} v_{ij} \tag{28}$$

When applying the propagation to a train, it must have already been applied to the previous trains, so that the procedure will apply sequentially from the second train to the last (for the first one the operation does not make sense).

Figure 7 shows the application of (26).

6.1.1 Linear transformation v to V

Once realized the propagation and obtained the matrix $\mathbf{v}$ with the correct linear transformation, you can obtain the matrix $\mathbf{V}$ equal to its established order (identical to that for $\mathbf{R}$ and $\mathbf{C}$).

6.1.2 Updating the matrix of contacts

At the end of the propagation process, you must register, on *the matrix of contacts* $\mathbf{C}$, the most recent instants (time clock) when you have had interference, direct or mediate between two trains: each value C_{pq} will indicate the most recent time in which the running train q has been disturbed by train p (have been used indices different from the usual i and j, to show that the matrix $\mathbf{C}$, as well as matrices $\mathbf{V}$ and $\mathbf{R}$, is structurally different from the matrix $\mathbf{v}$).

The matrix $\mathbf{C}$ is updated by inserting therein, in correspondence with the positive values of $\mathbf{V}$ the instant a or instant p, depending on whether you are dealing with the arrival or the departure from the station of the train indicated in

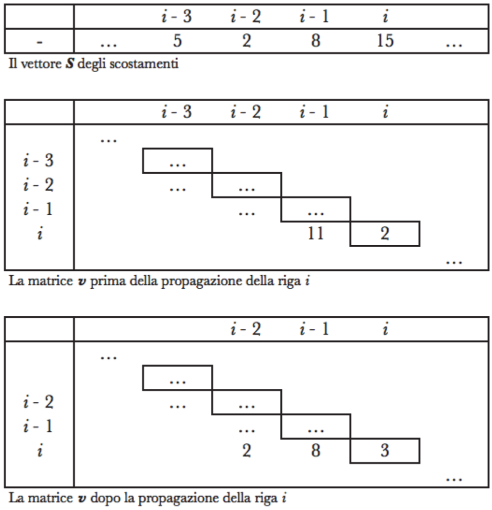


Figure 7: Responsibility propagation to previous trains: in the example it was assumed that $F_{i-3} > 0$, it is not possible to propagate on the train $i - 3$ the residual of 1 that is added to v_{ii}.

the column:

$$\begin{aligned} \text{Se} \quad V_{pq} > 0 \qquad C_{pq} &= a_q \qquad \text{if in arrival} \\ &= p_q \qquad \text{if on departure} \\ \text{Se} \quad V_{pq} \leq 0 \qquad C_{pq} &= C_{pq} \end{aligned} \tag{29}$$

The matrix $\mathbf{C}$, at the beginning of the algorithm is started with infinity value or, anyway, with an instant following the end of the period of analysis, so that in the search for the interference further on time, are not taken into account the interferences between two trains p and q that have not occurred.

6.2 Propagation of a recovery

Both in the matrix $\mathbf{v}$ and in the matrix $\mathbf{V}$, the negative values can only be found on the main diagonal. In fact a recovery can be performed by a train on its own behalf and no train can take credit for the recovery of another train. So, if $v_{ii} < 0$, this value will be assigned to corresponding to V_{pp}, so that added to $\mathbf{R}$ it will reduce the responsibility for the train i of the amount of the recovery made.

To ensure that the nonnegativity condition (1) is met, V_{pp} will be equal to:

$$V_{pp} = \max(v_{ii}, -R_{pp}) \tag{30}$$

and if $|v_{ii}| > R_{pp}$, you must still give the remain $w = v_{ii} + R_{pp}$ (negative value, taking into account the sign of the terms,) to some other train on matrix $\mathbf{V}$.

Index	…	*a*	…	*b*	…	*c*
Value	…	7	…	13	…	5

Row p of matrix R

Index	…	*a*	…	*b*	…	*c*
Value	∞	1000	∞	300	∞	700

Row p of matrix C

Index	1	2	3
Value	*b*	*c*	*a*

Vector K with contacts sorted

Recovery to assign w=17

Index	…	*a*	…	*b*	…	*c*
Value	…	0	…	-13	…	-4

Row p of matrix V

Figure 8: Recovery propagation. Firstly is canceled the total responsibility of the train b (equal to 13), with a residual of 4 that reduces the responsibility of the train c; the responsibility of the train a remains unchanged.

The sequence of trains that decrease the responsibility is building according to the times in which there was the last interference between the that train and the train i. The times are those in row i of the matrix $\mathbf{C}$; indices q of the only trains with $R_{pq} > 0$ are selected and are sorted (in ascending order) according to the values of C_{pq}, so you get "a vector" $\mathbf{K}$ with the sequence of indices of the trains to be taken into account.

The residue assignment to these trains is carried out according to the following recursive relationship:

$$V_{p,K_r} = \min[0, \max(-R_{p,K_r}, w - \sum_{s=1}^{r-1} V_{p,K_s})] \tag{31}$$

starting the recursion with $r = 1$, with $\sum_{s=1}^{0} V_{p,K_s} = 0$ and ending when $V_{p,K_r} = 0$.

Figure 8 shows the application of (31)

6.2.1 Updating the matrix of contacts

In the case of a recovery, it is not necessary to update matrix $\mathbf{C}$.

7 Outline and procedure of the algorithm

This section presents the operational flow of the algorithm.

Data reading –

The procedure for data reading obviously depends on the context and will provide for each train, in addition to the necessary technical data organization (such as elongation D' and D''), the data describing the performance of the circulation, ie the arrivals and departures from stations and the related delays:

$$a', \quad S', \quad p'', \quad S''$$

in addition to the delay increases ΔG attributed to other causes and recorded by the staff of the infrastructure manager;

Initialization – The initialization is to define the matrix of contacts $\mathbf{C}_0''$, and the matrix of responsibilities $\mathbf{R}_0''$ departing from the fictions station 0:

$$\begin{aligned} \mathbf{C}_0'' &= \infty \\ \mathbf{R}_0'' &= \mathbf{0} \end{aligned} \qquad (32)$$

For each module of the line – The procedure applies to the section station modules according to the progressive order along the line, maintaining the same criteria within the module, first on the section and then at the station.

1. Elementary interactions are calculated:
   - if the section: $\Delta I', \Delta E', \Delta M$ and ΔN (as reported in section 5.2);
   - if the station: $\Delta I''$ and $\Delta E''$ (section 5.1);
2. The local matrix of variations it is built $\mathbf{v}$ (section 5.3);
3. Any ghost trains are inserted (section 5.4);
4. Propagation of interaction is realized (section 6) and the matrix of variations $\mathbf{V}$ is obtained. Within this procedure is included the updating of the contacts matrix $\mathbf{C}$ (section 6.1.2 both for the increases of delay and 6.2.1 for the recoveries);
5. The matrix of responsibility $\mathbf{R}$ is updated, according to the formulas (3) of the section 4.1;

End of the process – Once all the modules have been processed the sequence of all matrices $\mathbf{R}'$ and $\mathbf{R}''$ will be built showing the responsibility of every train at every point and in every moment of its movement on the line, including those of other subjects besides Railway concerns as a ghost trains.

8 Conclusions

As mentioned in the introduction, the motivation behind the research consists in the need to have an operational tool that would properly ascribe the responsibility for traffic delays. This is a strong necessity in the current arrangements for the liberalization of the railways. The difficulty of the problem is in the fact that a train can have a negative impact on the running of other trains directly or also indirectly.

The developed procedure allows to treat carefully the many mechanisms of the phenomenon and to objectively determine the responsibility of each of the different actors of the transport to all the others, this is the essential basis for applying the mechanisms of financial compensation provided by the performance regime.

This article describes the model and its methodology, postponing to a later work the presentation of the informatic tool that implements it, and of the results of some application realized both on theoretical cases and on a real one. Data are elaborated on the basis of the database PIC (Piattaforma Integrata Circolazione), a computer system of Rfi, including the historical basis of the movement of trains and the causes of delay.

Possible and interesting developments that may take a cue from this work can concern:

- identifying the most appropriate methodology to define economic compensations between different railway operators using the matrices of responsibilities $\mathbf{R}$, by themselves or in combination with other aspects of transportation, such as the categories of the trains, the passengers carried, or otherwise;
- the determination of appropriate performance indicators regarding the carriers and the network operator;
- the statistical verification of the circulation during periods of comparable time;
- the analysis, also based on statistical considerations, of the reliability of the delivery carriers;
- the analysis of the timetable “robustness” with the identification of “weak paths”, those involving the high risk that any delay of the train that takes, them can cause a strong propagation to the overall circulation on the line (even if these trains terminate their own runnings exactly on time);
- probabilistic analysis of a new timetable using stochastic simulation.

References

[1] Carey, M. and Carville, S., Testing schedule performance and reliability for train stations. *Journal of the Operational Research Society*, **51(6)**, pp. 666–682, 2000.

[2] Yuan, J., *Railway Timetable and Traffic*, Eurail Press, chapter Statistical Analysis of Train Delay, pp. 170–181. 10, 2008.

[3] Yuan, J. and Hansen, J., Optimizing capacity utilization of stations by estimating knock-on train delays. *Transportation Research Part B*, **41**, pp. 202–217, 1997.

[4] Carey, M. and Kwiecinski, A., Stocastich approximation to the effects of headways on knock on delays of trains. *Transp*, **4**, pp. 251–267, 1994.

[5] Meester, L.E. and Muns, S., Stochastic delay propagation in railway networks and phase-type distributions. *Transportation Research Part B*, **41**, pp. 218–230, 2007.
[6] Daamen, W., Houben, T., Goverde, R.M.P., Hansen, I.A. and Weeda, V., Monitoring system for reliability of rail transport chains, 2006.
[7] Goverde, R.M.P., Daamen, W. and Hansen, I.A., Automatic identification of secondary delays based on train describer systems, 2007.
[8] Daamen, W., Goverde, R.M.P. and Hansen, I.A., Non-discriminatory automatic registration of knock-on train delays. *Network and Spatial Economics*, **9(1)**, pp. 47–61, 2008.
[9] Matthews B., N.J., Nash C., Implementing pricing reforms in transport - networking; pricing reforms in the rail sector. Technical report, European Commission, 2008.
[10] Rfi, Prospetto informativo della rete. Technical report, Rfi, 2009.
[11] Libardo, A. and Nocera, S., Transportation elasticity for the analysis of italian transportation demand on a regional scale. *Traffic Engineering and Control*, **49-5**, pp. 187–192, 2008.
[12] European Parliament, E.P., Directive 2001/14/ec on the allocation of railway infrastructure and the levying of charges for the use of railway infrastructure, 2001.
[13] UIC, Uic code 450-2: assessment of the performance of the network related to rail traffic operation for the purpose of quality analyses - delay coding and delay cause attribution process. Technical report, International Union of Railways (UIC), 2009.
[14] Nocera, S., The key role of quality assessment in public transport policy. *Traffic Engineering and Control*, **52-9**, pp. 394–398, 2011.
[15] Giovine, V., Il performance regime: il sistema delle valorizzazioni delle prestazioni ferroviarie. *Ingegneria Ferroviaria*, **12**, pp. 1031–1039, 2007.