

A novel research on the relation between the number of passengers and the braking distance of a metro

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Abstract

Due to heavy traffic at peak times, it is necessary to ensure that trains have an absolute safety braking distance. For this problem, this paper not only analyzes various factors that influence a metro's stopping time, but also analyzes a time model of a metro's stopping in the station. Secondly, based on the model and combined with the physical process of a metro's approach, this paper calculates the braking distance of oncoming trains. Finally, a novel relation between the number of passengers and the braking distance of an oncoming metro is established. Theoretical analysis and simulation experiments indicate that the braking distance of an oncoming metro can be effectively calculated according to the number of passengers on the platform.

Keywords: braking distance, oncoming metro, peak time.

1 Introduction

With the rapid development of the economy, the pressure of urban public transport is increasing, and the metro plays an more important role in the whole city traffic.

Stations are junctions of park and shift, but are also the bottleneck of passenger transport. The phenomenon that oncoming trains stop-start frequently often occurs. Meanwhile, passengers are not able to leave punctually because of



the delay of trains. Worse, the problem often arises that trains get into the station behind schedule. Finally, it is difficult to increase the efficiency of transport.

This paper aims to find the correspondence between the number of passengers and the braking distance of trains. Based on the factors that influence the braking distance of oncoming trains, this paper firstly points out the key factor, namely the number of passengers on the platform. Then, it analyzes a model of metro's stopping time. Based on the model and according to actual data, the paper establishes the braking distance of oncoming metro.

2 Decomposition of the entire research process

In order to meet the needs of practical problems, the whole process of research is divided into two parts, including analyzing a model of metro's stopping time in the station, and model making of the braking distance of trains which are drawing up at the station.

2.1 Distribution graph of metro trains

As shown in figure 1, there are metro trains stopping at the station, and passengers are getting on and off the trains, while the oncoming train calculates the braking distance according to the time spent by passengers getting on and off the trains as well as relevant influence coefficients. By means of the distance, the oncoming trains are guided to draw up at the station steadily, accurately, and safely. And the delaying time of arriving trains is also efficiently decreased, leading to a high frequency.

In Figure 1, the 1st train and 2nd train are respectively stopping on the 1st and 2nd platform while the 3rd train is ready to draw up at the station. What this paper aims to study is to calculate the braking distance of the 3rd train according to the number of passengers getting on and off the 1st and 2nd train which is stopping in the station.

2.2 Stopping time of trains that are in the station

In this paper, the model of stopping time derives from the model which is made by Zhuge Cheng-xiang and Gao Jian who are in Beijing Jiaotong University. The

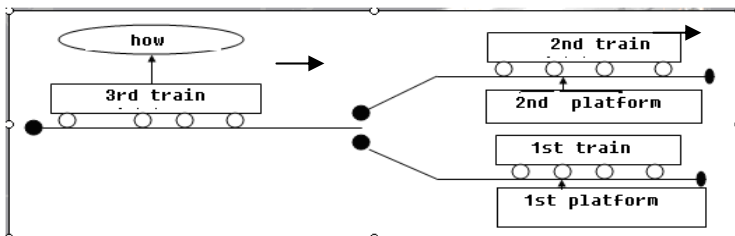


Figure 1: Distribution map.

stopping time of trains which are in the station are influenced by many factors, including the total time of passengers' getting on and off train, the time of opening doors(t_1), the time of closing doors(t_2), the delaying time of trains' stopping(t_0). Therefore, the stopping time [7] is expressed as follows:

$$t = t_0 + t_1 + t_2 + t' \quad (1)$$

In this formula:

t_1 and t_2 are primarily determined by the type of trains;

t_0 is set for the purpose of the trains' safety. Because of a certain distance between both adjacent trains, they must remain a certain distance.

t' is set at the maximum value of time spent by passengers in getting on and off trains. Because the number of passengers getting on and off every door is different, t' is expressed by:

$$t' = \max\{t'_1, t'_2, t'_3, \dots, t'_n\} \quad (2)$$

In the above formula, n is the number of carriages, and t'_i is the time spent by passengers in getting on and off the i th carriage of the train stopping in the station.

2.3 The amount of time to get on and off the train

The following part is to analyze a model of time spent by passengers getting on and off the i th door of the train. The following precondition is given:

1. The number of passengers getting on the i th carriage is N_i and the number of passengers getting off the i th carriage is M_i .
2. The amount of time for a passenger to get on the train is represented by the letters a , and the amount of time to get off the train is represented by the letter b .

Then the time for the i th door to keep open is expressed [7]:

$$t_i = N_i \times a + M_i \times b \quad (3)$$

The above formula holds only if the influence between passengers is not considered. However, the size of the crowdedness in the carriage and the availability of vacant seats actually have influences on the efficiency of getting on or off trains. We can define the crowdedness coefficient as K , and also define the crowding level of the i th carriage as K_i to quantify such influence. Table 1 [7] shows the concrete value of K_i :

Table 1: Degree of crowdedness inside a carriage.

Degree of crowdedness	A	B	C	D
influence coefficient	0.9	1	1.18	1.2

Note: A—vacant seats available; B—sufficient standing space; C—not much standing space; D—crowded with standing passengers.



Table 2: Values of influence coefficient.

Degree of influence	a	b	c	d	e
influence coefficient	1	1.05	1.1	1.15	1.25

Note: a—no influence; b—a little influence; c—general influence; d—severe influence; e—more severe influence

Table 3: Values of influence coefficient.

Degree of influence	A	B	C	D	E
influence coefficient	1.1	1.0	0.9	0.8	0.7

Note: A—more severe influence; B—severe influence; C—general influence; D—a little influence; E—no influence

The next part refers the influence of passengers being on and off the train. Such influence will increase the amount of time to get on or off train. An influence coefficient S is defined and S_i is the influence coefficient of the i th door. The table 2 lists the values of different influence coefficients and the concrete value of S .

Concerning the passengers' luggage, the size and amount of luggage will also affect the efficiency of getting on and off trains. We define a luggage influence coefficient as J and J_i is the influence coefficient of the i th door. The table 3 lists the values of different influence coefficients and the concrete value of J .

After the definition of coefficients K_i , S_i and J_i , the amount of time passengers to get on and off the i th is given by:

$$t'_i = S_i \times K_i \times J_i \times (N_i \times a + M_i \times b) \tag{4}$$

As for the whole train, the total amount of time to get on and off train is the maximum of t'_i , which is given by:

$$t' = \max \{ (S_1 \times K_1 \times J_1 \times (N_1 a + M_1 b)), (S_2 \times K_2 \times J_2 \times (N_2 a + M_2 b)), \dots, (S_n \times K_n \times J_n \times (N_n a + M_n b)) \} \tag{5}$$

2.4 The amount of actual time for trains to stop at the station

Theoretically the dwell time of the train could be retrieved from the mentioned formula. By use of linear regression methods, we can find the relationship between the theoretical result and the empirical one, which is expressed [7] as follows:

$$f(x) = \begin{cases} 11.497e^{0.0235x} & \text{Peak time} \\ 10.775e^{0.0252x} & \text{Usual time} \end{cases} \quad (6)$$

where x is the theoretical result and $f(x)$ is the actual dwell time.

2.5 The distribution of passengers in peak time

It is supposed that the average passengers' number of every train in each station is d . According to the relevant data, the crowdedness coefficient of carriage is usually 1, and the inference coefficient of passengers' getting on and off trains is usually 1.05, and the influencing coefficient of cargo is 1, and the velocity of passengers' getting on the train is 1.05, and the velocity of passengers' getting off the train is 1, and the time of opening each door is 2.4s, and the time of closing each door is 2.4s. In case of reasonable case, the delay time of metro is 0s. Except for the upper concrete parameters, the conditions of passengers' getting on and off trains are unknown. Due to great randomness of passengers' getting off the train, the number of passengers getting off the train is set to be b . The number of passengers getting on the train has certain regularity.

Because of great randomness of passengers' getting off trains, we suppose that the number of passengers getting off train is b . By contrast to the number of passengers getting off the train, the distribution of passengers getting on trains on the platform has certain regularity, which is mainly influenced by passengers' behavioural features. According to the relevant information, the distribution of passengers in peak time is shown in Figure 2.

It is shown in Figure 2 that the number of passengers is much more in the middle of platform while less at the extremes. The number of passengers in the i th carriage is expressed as follows [7]:

$$P_i = d \times \beta \quad (7)$$

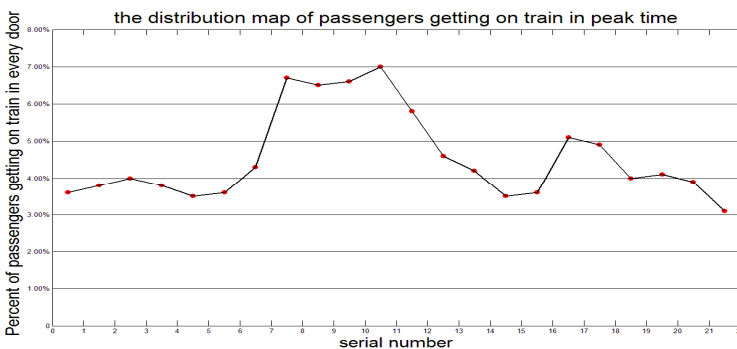


Figure 2: Distribution of passengers getting on the train in peak time.

In the upper formula, d is the passenger flow volume of the station while β is the percent of passengers getting on the i th door. It is shown in Figure 2 that the maximum of β is 7.0%. It is supposed that if the total number of passengers is n , the time of passengers' getting on train should be the time spending in getting on train through the door in front of which stand the most of passengers. Therefore, the time [7] of passengers' getting on the train in peak time should be:

$$t'' = n\beta_{\max} a \tag{8}$$

In the upper formula, the letter a is the time for every passenger to get on train, and the letter n is the total number of passengers getting on train, the letter β is the percent of the most passengers through each door.

2.6 The parameter value of a train's stopping time

The stopping time of trains is expressed as follows:

$$t = t' + t_0 + t_1 + t_2 \tag{9}$$

The parameter value of the upper formula is summed up as follows:

- t_0 is the delaying time which is set to be 0s;
- t_1 is the door opening time, which is set to be 2.4s;
- t_2 is the door closing time, which is set to be 2.4s;
- t' is the total time of passengers' getting on and off trains, which is expressed in formula (5);

S_{11} is the influencing coefficient of the eleventh carriage, which is set to be 1.05;

m is the number of passengers who are on the platform, and the letter m is a variable;

K_{11} is the crowdedness coefficient of the eleventh carriage, and is set to be 1.

J_{11} is the influencing coefficient of goods of the eleventh carriage, and is set to be 1.

a is the time of each passenger's getting on trains, and is set to be 1.05;

b is the time of each passenger's getting off trains, and is set to be 1;

In formula (9), t' , t_0 , t_1 , t_2 are respectively set to be concrete values as follows:

$$\begin{aligned} t' &= S_{11}K_{11}J_{11} \times (N_{11} * a + M_{11} * b) \\ &= 1.05 * 1 * 1 * (n * 7\% * 1.05 + n * 7\% * 1) \\ &= 1.05 * 0.1435 * n \\ &= 0.150675 * n \end{aligned} \tag{10}$$

According to the upper formulas, the actual stopping time of trains is calculated as follows:

$$t'' = 10.775e^{0.0252(t' + t_0 + t_1 + t_2)} \tag{11}$$

According to the upper formulas, t'' is calculated as follows:

$$t'' = 10.775e^{0.00379701 * n + 0.12096} \tag{12}$$



3 Model building of the metro's braking instance

3.1 The premises of the model of the metro's braking distance

- i The frequency of metro is high;
- ii In the premise of safety braking distance, the successive trains' distance should be short greatly.
- iii Once a train has stopped in the station, the sequential train decides its braking distance according to the stopping time of the train in the station.
- iv the velocity of the metro drawing up at the station is uniform.
- v the train is at a steady speed before drawing up at the station.
- vi the braking time of the sequential train is equal to the stopping time of the train in the station.

3.2 The braking distance of an oncoming train

Under the premise of ensuring the fore mentioned assumptions, the braking time and distance of the trains are related with acceleration and initial velocity of the metro train, and can be expressed as follows:

$$S = v_0 t + \frac{1}{2} a t^2 \quad (13)$$

The parameters of the formula are listed as follows:

- v_0 : initial velocity of the train that is drawing up at the station;
- t : time taken to brake;
- a : acceleration of the train that is drawing up at the station;

With regard to the equation (11), its precondition is in the process from start of braking to stopping of oncoming train, speed is uniformly reduced. Braking distance of train can be calculated by the following formula:

$$S = \bar{V} * t \quad (14)$$

\bar{V} is the average speed of the braking train. As the oncoming train's braking is uniformly decelerational and the terminational speed is 0m/s,

$$\bar{V} = \frac{(V_0 + 0)}{2} \quad (15)$$

From the equation (12) (13), we can see:

$$S = 0.5 * V_0 * t \quad (16)$$

When the initial braking velocity of the train is constant, the braking distance of the train merely has a relation with time. The braking time is connected with the stopping time of the fore train and the transmission time of the signal.

We can calculate the stopping time t'' of fore train by use of the formula (10). Suppose the transmission delay time of signal is t_5 , the braking time of the oncoming train meets the following relationship:

$$t = t'' + t_5 \quad (17)$$



From the formula (14) (15), we can see:

$$S = 0.5 * V_0 * (t'' + t_5) \tag{18}$$

From the equation (10) (16), we can see:

$$S = 0.5 * V_0 * (t_5 + 10.775e^{0.00379701*n+0.1296}) \tag{19}$$

Parameters' value of the above formula :

V_0 : initial brake velocity of the oncoming train, for instance, $V_0 = 40\text{m/s}$;

t_5 : transmission delay time of the signal, which is assumed to be 0.5s ;

By calculating, S is expressed as follows:

$$S = 10 + 215.5e^{(0.00379701*n+0.1296)} \tag{20}$$

Experiment simulation is shown in the following section.

4 Analysis of the simulation result

As can be shown in figure 3, when the train's delay time was respectively 0.1s, 0.2s, 0.3s, 0.4s, 0.5s, and the number of passengers getting on and off is certain, the braking distance of the oncoming trains is almost the same. This shows that if the delay time of trains is within a certain extent, it has little influence on the braking distance. The figure 4 indicates the relation between the braking distance and the delay time of trains.

In figure 4, various curves reflect that trains' braking distance is different in case of different number of passengers. Meanwhile, different delay time brings about different braking distances of the follow-up trains. However, within a certain time extent, the change of oncoming train's braking distance is not large.

As can be shown in figure 5, corresponding to different initial braking velocity V_0 , there are three curves, including $V_0=80\text{m/s}$, $V_0=60\text{m/s}$ and $V_0=40\text{m/s}$.

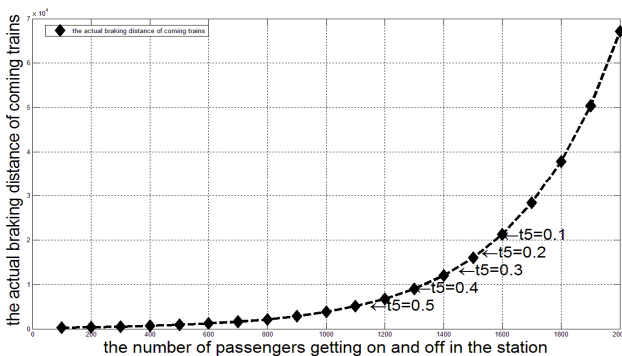


Figure 3: Curves of trains' braking distances in the case of different delay time.

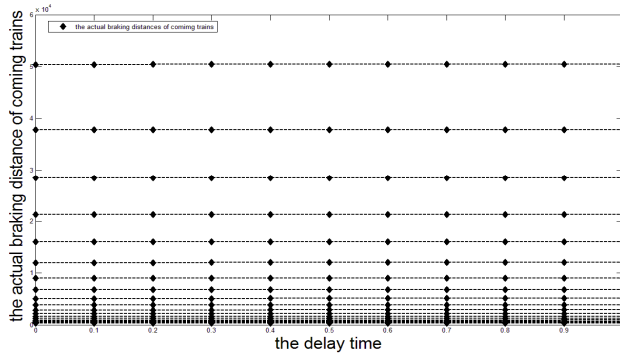


Figure 4: Curves of metro' braking distances in the case of a different number of passengers.

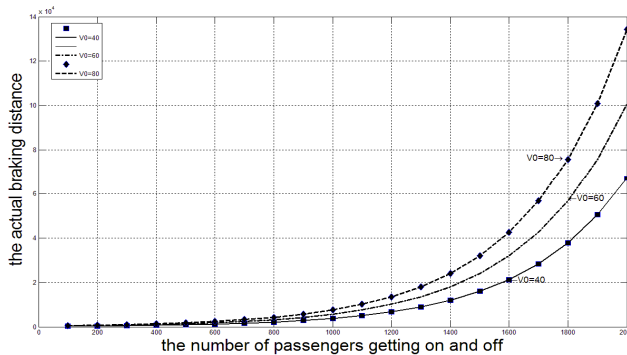


Figure 5: The curve about the braking distance in the case of different initial velocity.

Figure 5 shows that when the initial velocity of a train is certain, its braking distance increases when the number of passengers at the fore station increases; When the number of passengers on platform is certain, the braking distance of an oncoming trains increases with the increase of the initial velocity of trains that are drawing up at the station.

5 Conclusion

This paper does research on the relation between the number of passengers on the platform and the braking distance of an oncoming train. Firstly, this paper analyzes a model of metro's stopping time. Secondly, based on the established model and operation conditions, the braking distance of oncoming trains is calculated. In the next stage, it is important to make the expression of braking distance more accurate in order to reduce the error between the actual value and

the expected one. In addition, the braking distance of an oncoming train will be tested to access whether or not the braking distance is really reasonable.

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