

A new train GPS positioning algorithm in satellite incomplete condition based on optimization and the digital track map

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Abstract

The train positioning plays a key role in the train control system. The current train positioning is determined by the track circuit or balise, which cost a lot to build and maintain. GPS (Global Positioning System), one kind of GNSS (Global Navigation Satellite System) positioning technology, provides a cheap and real-time option. However, the inherent defect of GPS positioning is the so-called incomplete condition of GPS when less than four satellites are effective. This paper presents a new train GPS positioning algorithm based on the digital track map and optimization method for the incomplete condition of GPS. First, the track piece where the train is located is identified at the moment when the GPS satellite signals become incomplete. Then, a straight-line equation constrained by the pseudo-range equation is deduced. Finally, the estimated train position is obtained by minimizing the sum of the squared errors, which is solved by the gradient descent method and compared with the actual location in the digital track map. After the experiments were carried out in Sanjia dian Station, Beijing Railway Station, to get the field GPS positioning data, the performance of the proposed algorithm was evaluated and analyzed. The results demonstrated that the accuracy and stability of train positioning employing the proposed method were improved in GPS satellite incomplete condition (SIC).

Keywords: railway, GPS, incomplete condition, digital track map, optimization method.



1 Introduction

With the rapid growth of the Chinese economy, railway transportation plays a more and more important role in the national social and economical development. In the train control system, the train positioning is one of the key techniques. Obtaining accurate train position data is a prerequisite for the train safety and control. The present train positioning mainly depends on ground equipments, such as track circuits, balise and so on. However, ground equipments cost a lot to build and their security and the maintainability are not easy, which has greatly increased railroad worker's labor intensity. Furthermore, it is obviously advantageous to use GPS positioning for the train control system in reducing cost in infrastructure and maintenance, especially for low-density railways [1].

GPS is one kind of modern navigation technology whose applications are getting more and more widespread in transportation, surveys, geodetic and so on. However, GPS positioning also has its flaws: GPS receivers cannot work in satellite incomplete condition (SIC). When vehicles travel in some areas, such as urban tall building areas, tunnels and multi-level crossing bridges, some GPS satellite signals are often covered. In this case, the number of satellites is less than four or the geometric distribution received from satellite is non-uniform [2]. In particular, when GPS is applied in the train integrity inspection, the GPS antenna installed in the vehicle hook in the train rear part is easily occluded by the compartment [3].

Lin studied the problem of ground emitter positioning by a satellite cluster composed of three satellites and proposed an iterative algorithm based on a digital map for the urban traffic application [4]. Zou proposed a DR (Dead reckoning) positioning algorithm using Doppler and range data as the complementary information when the number of effective satellites was three. [5]. Liu proposed a positioning algorithm using a virtual satellite when the satellite number is three for train integrity inspection, checking with the GPS receivers in the head and tail of a train [6]. In a certain moment when the receiver in the tail of a train only receives three satellites signal because of carriage occlusion, the fourth constraint equation is added by making the height in the head and in the tail the same, which is called the virtual satellite assisted positioning method. Then, the three satellites with visual simultaneous equations can obtain the position solution of the rear. This method is easy to understand with few errors, but still needs three satellites and employs the four-star location model restrictions. Li proposed a GPS autonomous integrity detection method and a train-positioning algorithm assisted by a digital track map [7]. However, the algorithm was also in accordance with the four-star positioning mode. The linearized pseudo-range equation and track map data were used as the fourth constraint. However, these algorithms must be given an initial value, which has a deep influence on the results. The process assumed zero elevation changes, which cannot apply to larger areas of undulating terrain.

In this paper, we propose a solution for this GPS positioning incomplete condition, which combines the digital track map information and the



optimization method [8]. As the train is running on a fixed track, there are some strict regulations: Tracks approximately approach straight lines or curves with big curvature radius; any track has lots of nodes, such as turnouts, signals, insulation sections, kilometer marks and so on. All of this makes digital track maps easily described. In practice, a large number of low-cost GPS track data and a small number of high-precision nodes (turnout generally, signals and other properties points) can be used to describe the digital track map. Track straight lines can be fitted into the equivalent linear equations; curved tracks can be divided into several sections of line segments for approximate description and each track section of the endpoint nodes are high-precision [9].

In SIC, the results of position resolution equations due to the lack of conditions cannot be solved. However, we can use the digital track map as a constraint to achieve satellite positioning in the SIC according to the features of the digital track map. In this way, we not only use characteristic of the digital track map, which is not easily affected by outside influences and has high stability, but also use the optimization method to decrease the positioning error.

2 Positioning model in the satellite incomplete condition

When the number of available satellites is more than four, it is defined as the satellite complete condition (SCC), where the train's position is calculated by the traditional pseudo position method, as shown in Fig. 1.

The distance between satellites and a GPS receiver can be calculated by equation (1).

$$\rho = \Delta t \cdot c \quad (1)$$

c : The velocity of light;

Δt : The signal propagation time.

Then, the train's position can be calculated by equation (2):

$$\rho_j = \sqrt{(x^j - x_u)^2 + (y^j - y_u)^2 + (z^j - z_u)^2} - c(d_{tr} - d_{ts}^j) - d_{ion}^j - d_{trop}^j \quad (j = 1, 2, 3, 4 \dots) \quad (2)$$

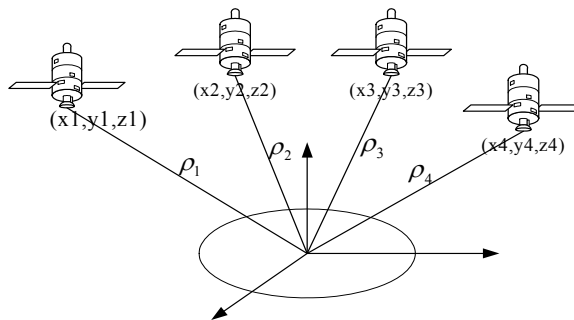


Figure 1: Schematic diagram of positioning theory.

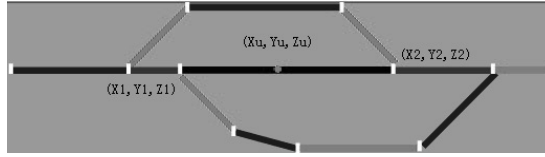


Figure 2: Schematic diagram of the track segment.

ρ_j : The geometric distance between the satellite position by satellite broadcast ephemeris and receiver;

x^j, y^j, z^j : The three-dimensional coordinates of satellite J;

d_{tr} : Receiver clock error;

d_{ts}^j : Clock error of satellite J;

x_u, y_u, z_u : The three-dimensional coordinates of the receiver;

d_{ion}^j : The distance deviation by the ionospheres' effects of satellite J;

d_{trop}^j : The distance deviation by the troposphere's effects of satellite J.

When the number of available satellites is less than four, which is defined as the SIC, the train's position is calculated with the help of the digital track map. First, the track segment that the train belonged to is judged, as illustrated in Fig. 2.

$$\frac{x_u - X_1}{X_1 - X_2} = \frac{y_u - Y_1}{Y_1 - Y_2} = \frac{z_u - Z_1}{Z_1 - Z_2} = k \quad (3)$$

$$k \in [0, 1]$$

From equation (2) and equation (3), we can get equation (4).

$$\begin{aligned} \rho_j = & \sqrt{(x^j - (k(X_2 - X_1) + X_1))^2 + (y^j - (k(Y_2 - Y_1) + Y_1))^2 + (z^j - (k(Z_2 - Z_1) + Z_1))^2} \\ & - c(d_{tr} - d_{ts}^j) - d_{ion}^j - d_{trop}^j - \rho_j \quad (4) \\ & (j=1,2,3) \end{aligned}$$

Then the train positioning can be achieved by using the gradient descent method, which calculates k by minimizing the E ($E = \sum_i (pf_i)^2$). Assuming that the initial value of k is 0.5, we get the updated value of k as follows:

$$k_{i+1} = k_i - \eta \frac{\partial E}{\partial k} \bigg|_{k=k_i} = k_i - 2 * \eta \sum_i pf_i \times \frac{\partial pf}{\partial k} \bigg|_{k=k_i} \quad (5)$$

where η is the learning rate, we set it as 0.1 in the beginning of the algorithm. A heuristic rule is applied to assure the stability of the optimization method. If the

E decreases two times continuously, we increase η at 10%. However, if E decreases and increases alternately twice, we decrease it also at 10%.

3 The proposed positioning algorithm

According to the model, the steps of the proposed positioning algorithm are as follows:

Step 1; Judge the number of satellites received. If the number is more than four, use the traditional pseudo positioning method; if the number is less than four, go to step 2.

Step 2; Judge the number of segments the train belongs to and then load the coordinates of the nodes of the segment.

Step 3; Using the node coordinates and the satellite information to calculate the position of the train by the descent gradient method.

The flow chart of the algorithm is shown in figure 3.

4 Results and analysis

The data collection and experimental validation was conducted in Sanjia dian Station, Beijing Railway Station. Experimental equipments were the Novatel (DL-4) GPS Receiver and the cart especially designed for track measurement, as shown in fig. 4.

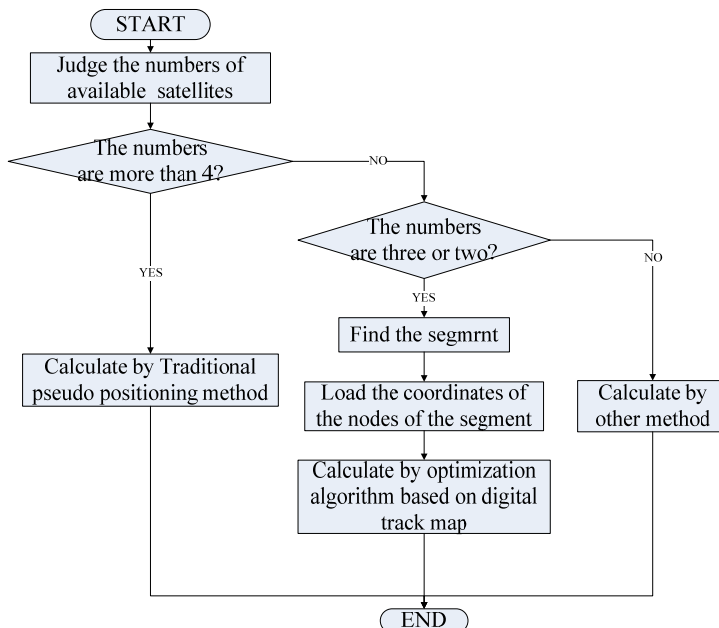


Figure 3: The flow chart of the algorithm.



Figure 4: Measurement cart and Novatel (DL-4) GPS receiver.

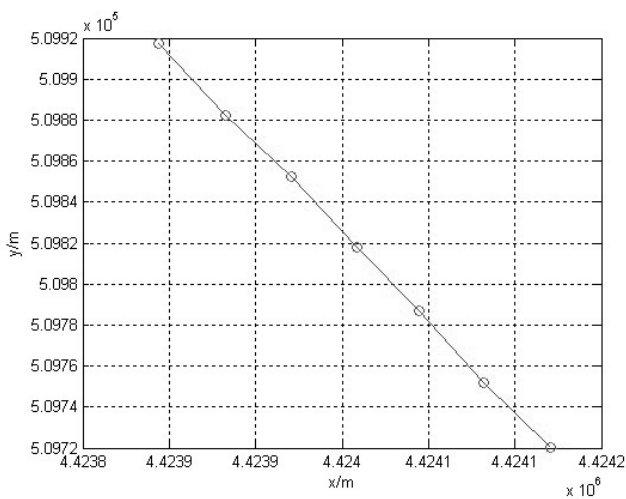


Figure 5: Digital track map model on the plane coordinate system.

The experimental steps were as follows:

Step 1: Select the track section and turnout section in the track map database as the experimental section;

Step 2: Collect GPS satellite ephemeris and pseudo-distance and other data using the Novatel (DL-4) GPS Receiver;

Step 3: Data extraction and processing;

Step 4: In the SCC, calculate the position using the algorithm. The satellite incomplete condition was created by deleting some satellite signals to make the number of satellites be two and three;

Step 5: Results comparison and analysis.

Track 16 of the station was chosen as the map model to test the proposed algorithm, then its coordinates were transferred from Gauss coordinates into plane coordinates, as shown in figure 5. In this model, "o" means the map nodes where signals or turnouts are placed. The line between two nodes indicates the

fragment. From the mathematical point of view, the head node and the corresponding tail node constructed a space straight-line equation. In SCC, the position can directly be calculated by equation 2. As in figure 6, the dots on the line in the figure are the true values; the star dots around the line are the value of point locations obtained by a GPS receiver. The positioning error is shown in figure 7, which means that the positioning error is less than 3.5m.

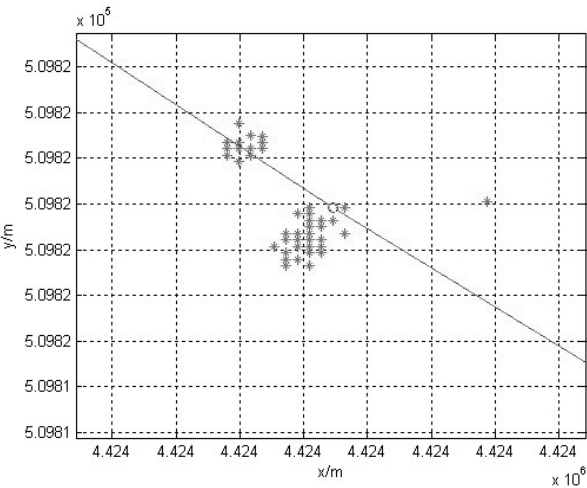


Figure 6: Result of positioning in SCC.

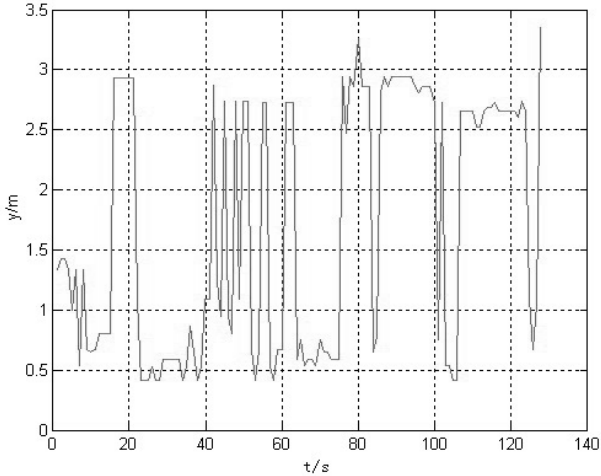


Figure 7: Positioning errors in SCC.



In the experiment, the satellite condition was very good and about 10 satellites signals were received. To construct the satellite incomplete condition, we deliberately chose two or three satellite signal data to validate the proposed algorithm. Of course, the positioning errors are different with the different satellite selection. There are many methods in satellite selection, which we do not want to overemphasize due to the page limits.

In SIC, the result of three satellites is shown in figure 8 and the positioning error is shown in figure 9, which means that the positioning error is less than 3 m, even less than the positioning error in SCC without using the digital track map.

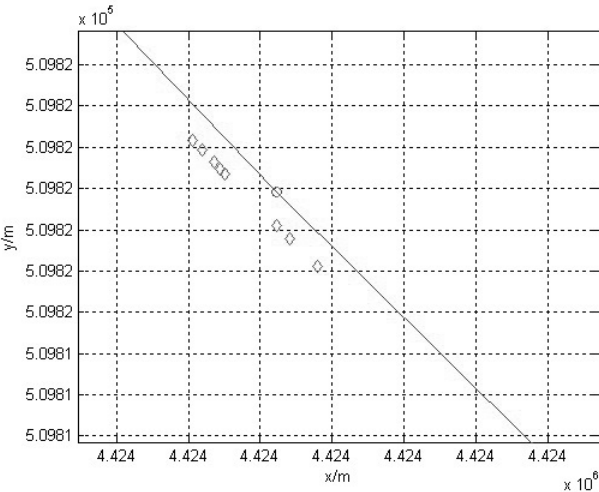


Figure 8: Positioning result for three satellites.

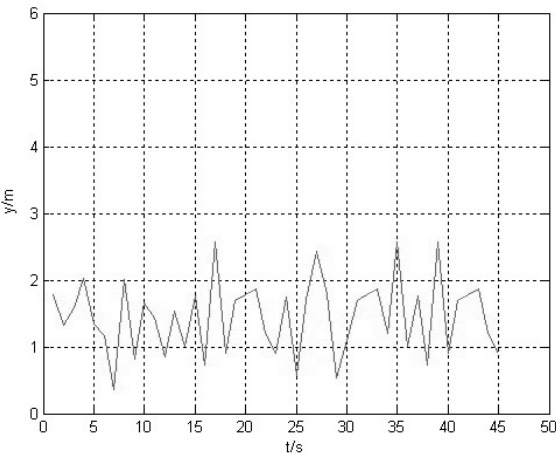


Figure 9: Positioning errors for three satellites.

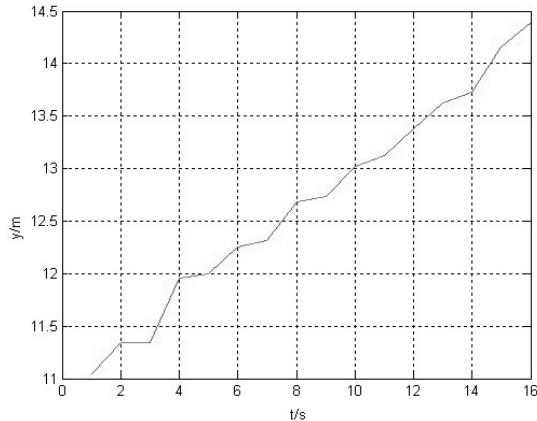


Figure 10: The positioning errors for two satellites.

In SIC, the positioning errors of two satellites are obviously influenced by the geometric distribution between the GPS receiver and satellites. The positioning errors are shown in figure 10, which means that the positioning error is greater than 11m. It can be found that the positioning error is much greater than that in the three satellites condition.

5 Conclusions

In this paper, we propose a positioning algorithm using the digital track map and optimization method in the SIC. The algorithm broke through the limit that GPS positioning must need four or more satellites. The experimental results show that the positioning accuracy obtained by the algorithm proposed in this paper can meet the positioning requirements if three satellites are available. In addition, the algorithm provides a valuable supplement and improvement for the application of GPS technology in the railway.

Due to the limited experimental conditions, we did not do large-scale experimental tests. Only some simulation experiments were carried out, but the results have some reference value. In addition, the positioning error of two satellites is still large, so how to improve the algorithm to make it work better for two satellites or even one satellite still needs further research.

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