A study on a mathematical model of the track maintenance scheduling problem

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Abstract

This paper presents a mathematical model of the long-term track tamping scheduling problem in the Korean high-speed railway system. The presented model encompasses various operational field constraints and moreover improves a state-of-the-art model in extending the feasible space. We show the model is sized up to intractable scale, then propose another approximation model that can be handled with the present computer system and commercial optimization package directly. The aggregated index, lot, is selected, considering the resolution of the planning horizon as well as the scheduling purpose. Lastly, this paper presents two test results for the approximation model. The results show the approximation model to be quite promising for deployment into an operational software program for the long-term track tamping scheduling problem.
Keywords: tamping scheduling, mathematical model, index aggregation.

1 Introduction

In the ballasted track system, the amount of gauge irregularity increases in proportion to the accumulated tonnage passed. When the amount of gauge irregularity is greater than a certain level, it results in not only the worse quality of traveling, but also a serious effect on traveling safety of trains. Therefore, the maintenance works of track systems are carried out based on the predetermined

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standards which is set to beforehand prevent from damaging the quality and safety of traveling [1, 2].

Tamping ballast is given the most importance among the maintenance works for correcting gauge irregularity. In Korea Railroad, tamping ballast is performed by hand or by multiple tie tamper (MTT), but MTT is used in most tracks except the some tracks not to be accessible.

Track tamping scheduling problem (TTSP) is to schedule MTT works for maintaining the regular amount of gauge irregularity below the criterion. To do so, target lines are divided by the 'lots' with the previously defined length (the basic length is 0.2 km) and the various parameters of gauge irregularity and actual data are maintained based on the lots. After all, TTSP is to find the schedule of MTT works performed on the lots in order to prevent the standard deviation of gauge irregularity of each lot from being below the criterion.

In this paper, we present mathematical models for TTSP. Many combinatorial problems are intractable if applied to the real field. But our models are developed to be solvable with the current computer system and commercial optimization packages.

This paper is organized as follows; Section 2 presents a mathematical model of TTSP. In Section 3, the mathematical model is scrutinized to devise an approximation model using the index aggregation. Finally, the experimental results and the future research directions are given.

2 Long-term scheduling problem for track tamping

2.1 Outline of the problem

The goal of TTSP is like these; (ⅰ) to minimize the number of track tamping works while meeting all requirements related with gauge irregularity (ⅱ) to minimize the fixed costs from every departure for tamping by confirming continuity of each lot (ⅲ) to disperse works not to be concentrated on (limited) specific days. In this section, mathematical model of TTSP is presented. This paper assumes followings for the model.

2.1.1 Assumptions

ⅰ. The proposed model is applied as a unit of railway maintenance post.

ⅱ. The proposed model uses the concept of 'virtual depot' to take into account multiple maintenance operations during unit period, thus the multiple number of virtual depots may be assigned, if necessary (e.g. 3-5 virtual depots).

ⅲ. One and only one maintenance operation and MTT type are assigned to each virtual depot during unit period.

ⅳ. The various rules and processes concerning TTSP are compliant to processes of track tamping which are carried out by 'High Speed Railroad Railway Control Regulation' of Korean Railroad.

The notations used in the model are defined as follows.

2.1.2 Notations

J The set of virtual depots. The index for indicating the number of maintenance operation(s) in a unit period

L The set of lots. L = {0, 1, 2, ..., L^{max}}. L1 = L - {0}. L2 = L - {L^{max}}. In here, L = 0 indicates depot itself. The indices are sorted in ascending order to outbound direction.

K Types of MTT equipments. K = {new type, old type}.

T T = {1, 2, ..., T^{max}}. T^{max} = 5 years × 365 days.

σ^{max} The standard value of gauge irregularity (standard deviation)

σ_l^+ The increment of gauge irregularity of lot-1 (standard deviation)

σ_k^- The recovering capacity of MTT type-k of train (standard deviation)

D Distance matrix for between lot-pair (km)

NW Prohibition of assigning works.

WT^k The required operation time of MTT type-k (h/lot)

VR^k The moving speed of MTT type-k (km/h)

BT Blocking time (h/day) = working hour + moving time/hour.

UL The length (km) of a unit lot. Basic length = 0.2 km.

F^k The fixed costs occurred assignment of MTT with type-k

δ A very small constant value

2.1.3 Decision variables

$z^{tk} = 0, 1, 2, ...$ The number of maintenance operations with MTT type-k in period-t.

$I^{jtk} = \{0, 1\}$ An indicator function. If a maintenance work is assigned to virtual depot-j during the period-t with a MTT type-k, then it has a value 1, otherwise 0.

$w_l^{jkt} \in \{0, 1\}$ If a maintenance work is assigned to lot-l by a virtual depot-j using MTT type-k during the period-t, then it has a value 1, otherwise 0.

$y_{lm}^{jkt} \in \{0, 1\}$ A variable for a sequence of maintenance works assignments. If $w_l^{jkt} = 1 \Rightarrow w_m^{jkt} = 1$, then it has a value 1, otherwise 0.

$N^{jkt} \equiv \{l \mid w_l^{jkt} = 1, \forall j \in J, \forall k \in K, \forall t \in T\}$ A set of lots where a maintenance work is assigned for virtual depot-j with MTT type-k in period-t, dynamically. $n = |N^{jkt}|$

$\mu^{jkt} = \max\{l | l \in N^{jkt}\}$

$\nu^{jkt} = \min\{l | l \in N^{jkt}\}$

$\sigma_l^t \in R^+$ The levels of gauge irregularity in lot-l at the end of period-t (standard deviation)

$Z_1 \in R^+$ The first term of objective function in TTSP0 (variable cost)

$Z_2^t \in R^+$ The second term of objective function in TTSP0 (fixed cost)

2.2 Objective function

The objective function of TTSP model consists of two terms. The first one is to minimize the frequency of maintenance operations while meeting all requirements. In the case that the amount of gauge irregularity exceeds the criterion, the maintenance operations are indispensable but the number of the operations should be minimized because each operation makes the ballast crushed. Formula (1) is the function to achieve such objective.

$$Z_1 = \sum_{t\in T}\sum_{k\in K}\sum_{j\in J}\sum_{l\in L2}\sum_{m\in L1}\left(w_m^{jkt} - \delta \cdot y_{lm}^{jkt}\right) \tag{1}$$

The second term is to dispose the given works with the minimum number of maintenance works. It is not common that more than maintenance work group is operated in a day at one depot. Therefore, the number of maintenance work groups might be minimized instead of minimizing the number of departure of maintenance works.

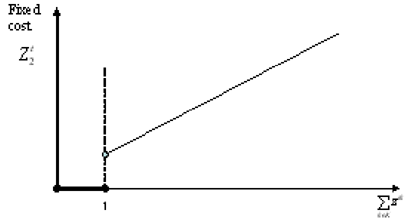


Figure 1: An example of fixed cost.

$$Z_2^t = \begin{cases} 0 & \sum_{k\in K} z^{tk} \le 1 \\ F^k \sum_{k\in K} z^{tk} & \sum_{k\in K} z^{tk} > 1 \end{cases} \tag{2}$$

$$Z = Z_1 + \sum_{t\in T} Z_2^t \tag{3}$$

These objectives can be achieved, as shown in Figure 1, by considering the fixed cost paid whenever maintenance departures more than two are employed during unit period. Thus, the objective function is given, as in formula (3), by the sum of variable cost and fixed cost. To implement the fixed cost in a commercial optimization software package, it needs to modify model. The modification of model will be described in next section.

2.3 Assignments of maintenance operations

It is crucial in TTSP model to keep the amount of gauge irregularity of each lot below the criterion value, which is expressed as the inequality (4).

$$\sigma_l^t \leq \sigma^{\max}, \ \ \forall t \in T, \forall l \in L \tag{4}$$

The maintenance works are avoided being assigned in uniform fashion, since the increasing rates of gauge irregularity (σ_l^+) are considerably different even in the adjacent lots [3]. It is desirable to establish works which let the periodicity of consecutive lots synchronized with similar periods.

Gauge irregularity of each lot increases at every unit period like formula (5-6), and track tamping works (w_l^{jkt}) will be established when the values are reached at regulated level ($\sigma^{\max}$). Meanwhile, formula (6) restricts the feasible space to be limited. Figure 2 depicts the restriction conceptually.

$$\sigma_l^1 = \sigma^{init}, \ \ \forall l \in L \tag{5}$$

$$\sigma_l^t = \sigma_l^{t-1} + \sigma_l^+ - \sum_{k \in K} \sum_{j \in J} \sigma_k^- \cdot w_l^{jkt}, \ \forall t = \{2,3,\ldots,T^{\max}\}, \forall l \in L \tag{6}$$

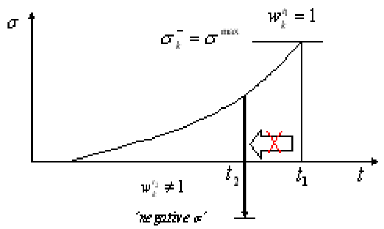


Figure 2: Restriction of feasible space.

When assigning maintenance operations with assumption that corrective effects of gauge irregularity is set exactly same as the regulated level (i.e. $\sigma_k^- = \sigma^{\max}$), assigning operations is possible ($w_l^{t_1} = 1$) only when the amount of gauge irregularity reaches exactly at the standard level ($t = t_1$), and all previous operation assignments are impossible ($w_l^{t_2} \neq 1$). This is because operations are assigned in advance of t_1, $\sigma_l^t \geq 0$ is violated. This paper presents a modified formulation for this point in formula (7).

$$\sigma_l^t = \sigma_l^{t-1} + \sigma_l^+ - (\sigma_l^{t-1} + \sigma_l^+) \cdot \sum_{j \in J} \sum_{k \in K} w_l^{jkt}, \forall t = \{2,3,\ldots,T^{\max}\}, \forall l \in L \tag{7}$$

On contrary to formula (6), if necessary, formula (7) allows operation assignments possible even before the date of reaching to standard level of gauge irregularity. It means that a limitation which was discovered in the state-of-the-art study [1] becomes relaxed. To apply commercial optimization package, further modification of the formula is necessary because decision variables in the right side take the form of multiplication. The modification of model will be described in the next section as that of the previous one.

$$\sum_{k\in K}\sum_{j\in J} w_l^{jkt} \le 1, \quad \forall t \in T, \forall l \in L \tag{8}$$

$$\sum_{k\in K}\sum_{j\in J} w_l^{jkt} = 0, \quad \forall (t,l) \in NW \tag{9}$$

Formula (8) ensures the uniqueness of operations which are assigned in every unit period and lot, and formula (9) is to restrict assignment of operations owing to the operational reasons during cold and hot seasons.

2.4 Restrictions on blocking times

For all lots, the sum of working time and moving time is restricted by blocking time (BT). Figure 3a represents an example that a team of maintenance operation departs the depot, processes/moves lots, the turns back. Such working process can be expressed by a graph of which assignment of operation is node, and movements between assignments are arcs as shown in Figure 3b.

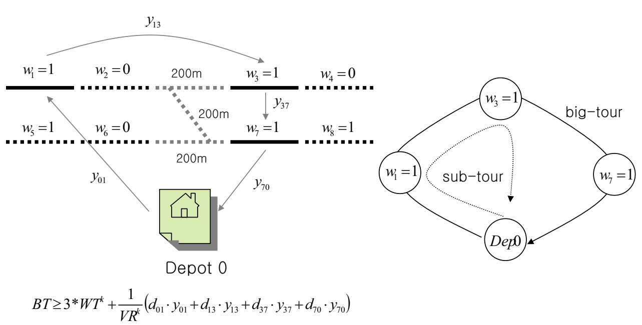


Figure 3: (a) An example of maintenance operation sequence, and (b) its graph representation.

That graph comes under the traveling salesman problem (TSP) which is widely known in the field of combinational optimization. The blocking time of the graph operates only for the big-tour that visits all nodes with a shortest path.

$$\sum_{l\in L_2}\sum_{m\in L_1}\left[(y_{lm}^{jkt}-1)WT^k + \frac{d_{lm}y_{lm}^{jkt}}{VR^k}\right] \le BT,\ \forall j \in J, \forall t \in T, \forall k \in K \tag{10}$$

$$\sum_{l\in S^{kt}}\sum_{m\in S^{kt}} y_{lm}^{jkt} \le \left|S^{jkt}\right| - 1,\ \forall S^{jkt} \subset N^{jkt}, 2 \le \left|S^{jkt}\right| \le n-1 \tag{11}$$

$$\mu^{jkt} \le \nu^{jkt} + 1,\ \forall j \in J, \forall k \in K, \forall t \in T \tag{12}$$

$$w_l^{jkt} + w_m^{jkt} - 2y_{lm}^{jkt} \ge 0,\ \forall j \in J, \forall k \in K, \forall t \in T, \forall l \in L_{2,}\ \forall m \in L_1 \tag{13}$$

Formula (10) represents the constraints of blocking time on big-tours, and formula (11) is sub-tour elimination constraints. These constraints are known as make models very difficult (NP-Complete) to be solved [4]. Formula (12) is a constraint to avoid duplication of operation-assignment-range in each virtual depot. Formula (13) defines the logical connection between operation assignment variables (**w**) and operation sequence variables (**y**), and it works in proper for the minimization form of objective functions.

2.5 Calculating frequency of operations

In the objective functions described in previous sections, to optimize the number of maintenance departures during a unit period, frequency of operations should be calculated.

$$I^{jkt} - w_l^{jkt} \geq 0, \ \forall t \in T, \forall j \in J, \forall k \in K, \forall l \in L \tag{14}$$

$$\sum_{k \in K} I^{jkt} \leq 1, \quad \forall t \in T, \ \forall j \in J \tag{15}$$

$$z^{tk} - \sum_{j \in J} I^{jkt} = 0, \quad \forall t \in T, \forall k \in K \tag{16}$$

Formulas (14-16) is indicator function which is established in accordance with operations in specific lots, and the operation frequency of each MTT type during unit period is calculated by formula (16).

2.6 The range of variables

Formulas (17-21) represents the range of decision variables.

$$z^{tk} = 0,1,2,... \tag{17}$$

$$I^{jtk} = \{0,1\} \tag{18}$$

$$w_l^{jkt} \in \{0,1\} \tag{19}$$

$$y_{lm}^{jkt} \in \{0,1\} \tag{20}$$

$$\sigma_l^t \in R^+ \tag{21}$$

Long-term scheduling model of ballasted track tamping is defined from those defined objective function(s) and constraints as follows.

(TTSP0) - Track Tamping Scheduling Problem
Minimize (3)
Subject to (4-5), (7-21)

3 Approximation of model

3.1 Aggregation of index

Problem size of TTSP0 whose subjects are the newly constructed Korean high speed line (HSL) between Busan and Seoul can be estimated based on the most difficult variables (y_{lm}^{jkt}).

$$y_{lm}^{jkt} \Rightarrow [3\text{-}5] \text{ (departures of maintenance operations)} \times 1825\text{(periods)} \times 2\text{(MTT types)} \times 1105\text{(lots)} \times 1105 = 22{,}283{,}706{,}250 \Rightarrow 2^{22{,}283{,}706{,}250}$$

With the computability of computer systems currently used, this scale of problem is not easy to solve directly. Index aggregation is one of the potential approaches to these large-scaled models. The index aggregation means to reduce scales of the models that can be manipulated using current computer systems, by cutting the number of variables within the range of keeping actual meaning of solutions as far as possible. It seems excessively specific to keep the decision values of long-term scheduling problems for several decades of kilometers by the unit of 0.2 km. In this paper presents a solution approach that integrates those excessively specific decision variables.

As for the operational speeds of MTT equipments which are currently used by Korean Railroad, those of new equipments are about 1.0 km/h (= 5 lots/h, 0.2 km standard), those of old equipments are about 0.8 km/h (= 4 lots/h, 0.2 km standard). According to those operational, on the assumption that average blocking time a day is 4 h (the average processing time a day = the average blocking time a day - the average running time a day = 4 h - 1.5 h = 2.5 h), the average processing lots of new equipments a day is about 12.5 lots/day (= 5 lots × 2.5 h/day), the average processing lots of old equipments is about 10 lots/day (= 4 lots × 2.5 h/day). The number of average lots of new and old equipments is about 11 lots a day. Therefore, in this paper, we propose Aggregated LOT (AL) which integrates lots of 0.2 km (11 lots) which can be operated for a day.

By integrating index of lots, the types of the equipments and the concept of virtual depot of TTSP0 is not necessary any more, and decision parameter associated to lots ($l \in L$) may be re-defined as average value ($s_a^{t'}$) for aggregated lots ($a \in AL$). By aggregating index of lots, the number of variables can be reduced considerably, and relevant formulas (10-13) can be relaxed as well. In the next section, Approximated TTSP (TTSPA) based on the index aggregation is presented.

3.2 Objective function of TTSPA

In the approximated model, the objective function is modified using newly defined variables. First, objective (2) concerning variable cost of TTSP0 is defined as formula (22) in TTSPA.

$$Z_3 = \sum_{t \in T} \sum_{a \in AL} c^t \cdot w_a^t \tag{22}$$

The cost weight (c^t) concerning operation assignment of formula (22) is to solve problem arising from making problem of infinite time horizon smaller to problem of finite time horizon. For example, in the problems of definite time horizon as Figure 4a, operation assignments are possible regardless of the degree(s) of access to the standard (value) at a middle spot, within the range of meeting the standard amount of gauge irregularity in scheduling areas. In the case of infinite time horizon, assigned operations in a cycle are logically right to assign near the standard approximation except for the invincible situations.

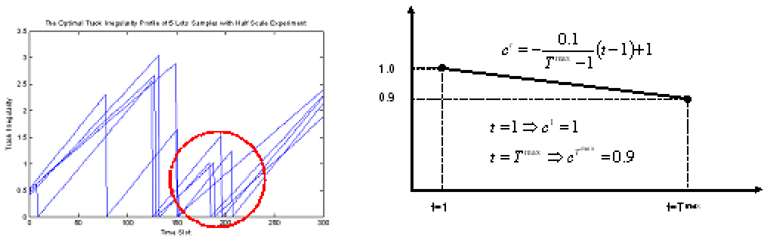


Figure 4: (a) Assignments in infinite time horizon, and (b) Weights to compensate.

The weight concerning assignment of operations compensates such problems arising from the models of finite time horizon, inducing assignment of in the latter half of the period, approaching to the standard. Such weight should be defined in the range not to influence on the original objective function defined by users. Figure 4b depicts an example of methods for establishing weights.

Modification of Formula (2), objective function concerning fixed costs of TTSP0, is for manipulating in commercial optimization software packages. The objective functions which have fixed costs are able to be realized by using Piece-wise Linear (PWL). The PWL is an efficient method which helps realize fixed costs with a small increment number of variables [5].

$$z_{41}^t + z_{42}^t - \sum_{a \in AL} w_a^t = 0, \ \forall t \in T \tag{23}$$

$$0 \le z_{41}^t \le 1, \ \forall t \in T \tag{24}$$

$$0 \le z_{42}^t \le z^{\max} - 1, \ \forall t \in T \tag{25}$$

$$Z_4 = F_{41} \sum_{t \in T} z_{41}^t + F_{42} \sum_{t \in T} z_t^{42} \tag{26}$$

$$Z = Z_3 + Z_4 \tag{27}$$

The objective function is defined as formula (27) by formulas (22) and (26).

3.3 Linearization assignment constraints

Although formula (7) is an alternative to the problem of restrictive feasible space, some problems in the third term of right-hand side arise when two decision variables are expressed as the forms of multiplication, and apply a commercial optimization software packages to the problem. Such a quadratic model can be linearized as follows. First, formula (28) is given by using the variables defined by aggregated index.

$$s_a^t = s_a^{t-1} + s_a^+ - (s_a^{t-1} + s_a^+)\cdot w_a^t, \forall t = 2,...T^{\max}, \forall a \in AL \tag{28}$$

When $\Delta_a^t = s_a^{t-1}\cdot w_a^t$, formula (28) is linearized as formulas (29-33).

$$s_a^t = s_a^{t-1} + s_a^+ - \Delta_a^t - s_a^+ \cdot w_a^t, \forall t = 2,...T^{\max}, \forall a \in AL \tag{29}$$

$$\Delta_a^t \geq 0, \quad \forall t = 2,...T^{\max}, \forall a \in AL \tag{30}$$

$$\Delta_a^t \leq s^- \cdot w_a^t, \forall t = 2,...T^{\max}, \forall a \in AL \tag{31}$$

$$\Delta_a^t \leq s_a^{t-1} + (1-w_a^t)\cdot s^-, \forall t = 2,...T^{\max}, \forall a \in AL \tag{32}$$

$$\Delta_a^t \geq s_a^{t-1} - (1-w_a^t)\cdot s^-, \forall t = 2,...T^{\max}, a \in AL \tag{33}$$

To make understand clear about the model, some formulas are repeated, and TTSPA model is as follows.

(TTSPA)

$$\text{Minimize } Z = Z_3 + Z_4 \tag{27}$$

Subject to

$$Z_3 = \sum_{t\in T}\sum_{a\in AL} c^t \cdot w_a^t \tag{22}$$

$$z_{41}^t + z_{42}^t - \sum_{a\in AL} w_a^t = 0, \ \forall t = T \tag{23}$$

$$0 \leq z_{41}^t \leq 1, \ \forall t \in T \tag{24}$$

$$0 \leq z_{42}^t \leq z^{\max} - 1, \ \forall t \in T \tag{25}$$

$$Z_4 = F_{41}\sum_{t\in T} z_{41}^t + F_{42}\sum_{t\in T} z_t^{42} \tag{26}$$

$$s_a^t = s_a^{t-1} + s_a^+ - \Delta_a^t - s_a^+ \cdot w_a^t, \forall t = 2,...T^{\max}, \forall a \in AL \tag{29}$$

$$\Delta_a^t \geq 0, \quad \forall t = 2,...T^{\max}, \forall a \in AL \tag{30}$$

$$\Delta_a^t \leq s^- \cdot w_a^t, \forall t = 2,...T^{\max}, \forall a \in AL \tag{31}$$

$$\Delta_a^t \leq s_a^{t-1} + (1-w_a^t)\cdot s^-, \forall t = 2,...T^{\max}, \forall a \in AL \tag{32}$$

$$\Delta_a^t \geq s_a^{t-1} - (1-w_a^t)\cdot s^-, \forall t = 2,...T^{\max}, a \in AL \tag{33}$$

$$w_a^t = 0, \ \forall (t,a) \in AWN \tag{34}$$

$$z^t = 0,1,2,..., z^{\max} \tag{35}$$

$$w_a^t \in \{0,1\}, \quad \forall t \in T, \forall a \in AL \tag{36}$$

$$0 \leq s_a^t \leq s^{\max} \tag{37}$$

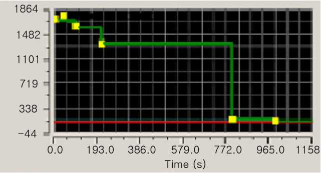
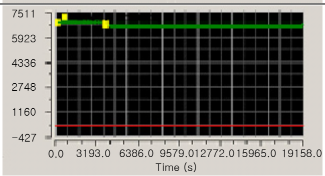
In this section, an approximated model is presented to be handled by using current computers and commercial optimization software packages. In the next section, results of numerical experiments and hereafter direction concerning TTSPA are presented.

4 Experimental results and conclusions

4.1 Experimental results

Two experiments on TTSPA models were performed as shown in Table 1. The one is an experiment about average track tamping problem size (100%) of Korean HSL at each maintenance depot, and the other is a 60%-scaled model of the problem size. They were performed twice for each, and the result of each performance was similar. The result of Table 1 was got from using OPL Studio 4.0 (CPLEX 9.0 is built in) of ILOG Co. in a PC which has 3.4 GHz CPU, 1Gb RAM. Table 1 shows the summary of experiments.

Table 1: Experimental results of TTSPA.

Problem size	Solutions	Experimental conditions
60%		- CPU time = 772 s. - GAP = 15% - Trials = 2 times - Constraints = 60,250 - Int. variables = 45,600
100%		- CPU time = 6 h - GAP = 96.5% - Trials = 2 times - Constraints = 202,399 - Int. variables = 152,500

4.2 Conclusions

TTSPA came to within a scale of commercial optimization software package. Even though the proposed TTSPA model comes into the range where model is manipulated by using current computer systems, further studies are required to improve the convenience of an application program using this model. For the improvement of algorism performance, further studies for user cuts to improve lower bounds of CPLEX Branch & Cut, and for developing heuristic algorithm to improve effective upper bounds.

On the aspect of modeling, some additional studies about effective reduction by using restrictions of assignment of hot and cold seasons, and about extension to the track tamping model related to ballast cleaning are required.

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