On a joint distribution of two successive surf parameters

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Abstract

A joint distribution of two successive surf parameters is provided, and it is represented by a bivariate lognormal distribution. Consequently the joint distribution of two successive breaker indices is represented by a bivariate lognormal distribution. The application of the surf parameter distribution is exemplified by estimating the probability of two successive breakers on slopes by using wave parameters corresponding to typical field conditions.
Keywords: bivariate lognormal distribution, surf zone, surf parameter, breaker index, breaking waves.

1 Introduction

The surf parameter, also often referred to as the surf similarity parameter or the Iribarren number, is used to characterize surf zone processes. It is given by the ratio between the slope of a beach or a structure and the square root of the wave steepness in deep water as introduced by Iribarren and Nogales [1] and used later by Battjes [2]. Shallow water regions where waves break are referred to as the surf zone, and the different breakers on slopes are defined and classified in terms of the surf parameter. It also appears that the surf parameter enters in many empirical and theoretical models for wave-induced phenomena in the surf zone. The breaking of waves is associated with large loss of energy. Within the surf zone along beaches the wave energy flux from offshore is dissipated into turbulence and heat, and consequently the wave height decreases towards the

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shoreline. Wave-breaking also results in strong currents along the shoreline and thereby affects the nearshore circulation. The high intensity of turbulence caused by wave-breaking is also responsible for the intense sediment transport in the surf zone. Wave run-up on beaches and coastal structures such as, e.g., breakwaters, seawalls and artificial reefs are characterized by using the surf parameter. Examples of the relevance and importance of the surf parameter are found in e.g. Herbich [3] and Silvester and Hsu [4].

Tayfun [5] presented a study on the development of approximate theoretical forms of the distributions of wave steepness and surf parameter. The approach is based on assuming the random wave process to be long crested and narrow banded. The results are compared with data from measurements at sea representing two severe storms. Both the wave steepness and the surf parameter are lognormal distributed. The resulting statistics for the surf parameter are applied to breakers at normal incidence on sloping beaches. As stated by Tayfun [5], the joint statistics of wave steepness with wave heights or crest heights, or the wave steepness with wave heights or crest heights above a specified threshold may be appropriate in formulating risks of e.g. capsizing of vessels, overtopping, and slamming forces on seawalls, etc. However, the wave steepness is of interest by itself, particularly in relation with many of the surf zone processes.

In a subsequent discussion Myrhaug and Fouques [6] pointed out that other data sets may result in other distributions. This is exemplified by using the data referred to by Tayfun [5]; the data used by Myrhaug and Kjeldsen [7, 8], Myrhaug and Rue [9], and Myrhaug and Kvålsvold [10]. These papers discuss various aspects of wave steepness statistics using data from a large population of waves obtained by waverider buoys at three different deep water locations on the Norwegian continental shelf. Myrhaug and Fouques [6] found that the wave steepness is Weibull distributed in the right tail and otherwise lognormal distributed, and that the surf parameter is lognormal distributed in the right tail and otherwise Fréchet distributed.

Myrhaug and Rue [9] used the Weibull model to study the statistics of two successive wave steepness parameters with the focus on steep waves, while Myrhaug and Kjeldsen [7], and Myrhaug [11] discussed the joint distribution of wave height and wave steepness. To our knowledge, no studies on the joint distribution of two successive surf parameters, i.e. the values of the surf parameter for two successive waves, are available in the open literature. This is the subject of the present paper, which should represent a useful tool for the assessment of various wave-induced phenomena in the surf zone.

Here the marginal distribution of the surf parameter is taken as the lognormal distribution, as found by Tayfun [5]. The joint distribution of two successive surf parameters is then represented by a two-dimensional lognormal distribution. The application of the results is illustrated by an example; the probability of two successive breaking waves on slopes are given.

2 Background

The surf parameter is defined as $\xi = m/\sqrt{s}$ where $m = \tan\theta$ is the slope with an angle θ with the horizontal, $s = H/\left(g/2\pi\right)T^2$ is the wave steepness in deep water, *H* is the wave height, *T* is the wave period, and *g* is the acceleration of gravity. In the forthcoming the surf parameter is normalized, i.e. $y = \xi/\xi_{rms}$, by defining $\xi_{rms} = m/\sqrt{s_{rms}}$ where s_{rms} is the root-mean-square (*rms*) value of *s*, which will be discussed further in Section 4. The basis for the present approach is that the marginal distribution of the normalized surf parameter follows the lognormal distribution with the probability density function (*pdf*)

$$p\left(y\right) = \frac{1}{\sqrt{2\pi}y\sigma_z}\exp\left[-\frac{\left(\ln y - \mu_z\right)^2}{2\sigma_z^2}\right] \tag{1}$$

where μ_z and σ_z^2 is the expected value and the variance, respectively, of $z = \ln y$. This is in accordance with the results in Tayfun [5]; further details are given in Section 4.

3 Statistics of the joint behaviour of two successive surf parameters

Now the joint distribution of the surf parameter for two successive waves is considered. There are several numbers of possible forms of two-dimensional distributions where marginal distributions are given by the lognormal distribution in Eq. (1). Let $y_1 = \xi_1/\xi_{rms}$ and $y_2 = \xi_2/\xi_{rms}$ denote the variables which are normalized with the same parameter ξ_{rms}. Here y_1 and y_2 are associated with the first wave and the next succeeding wave, respectively. By introducing $z_1 = \ln y_1$ and $z_2 = \ln y_2$ in the two-dimensional Gaussian distribution in Eq. (A1) (see the Appendix), it can be transformed to a two-dimensional distribution with the marginal distributions given by Eq. (1). This change of variables gives the following joint *pdf* of normalized variables (see e.g. Johnson and Kotz [12])

$$p\left(y_1, y_2\right) = \frac{1}{2\pi y_1 y_2 \sigma_z^2\sqrt{1-\rho_{z_1z_2}^2}} \cdot \exp\left[-\frac{\left(\ln y_1 - \mu_z\right)^2 + \left(\ln y_2 - \mu_z\right)^2 - 2\rho_{z_1z_2}\left(\ln y_1 - \mu_z\right)\left(\ln y_2 - \mu_z\right)}{2\sigma_z^2\left(1-\rho_{z_1z_2}^2\right)}\right] \tag{2}$$

where

$$\mu_z = E\left[\ln y_1\right] = E\left[\ln y_2\right] \tag{3}$$

$$\sigma_z^2 = Var\left[\ln y_1\right] = Var\left[\ln y_2\right] \tag{4}$$

The correlation coefficients $\rho_{z_1 z_2}$ and $\rho_{y_1 y_2}$ are related by

$$\rho_{y_1 y_2} = \frac{E[y_1 y_2] - \mu_y^2}{\sigma_y^2} = \frac{e^{\rho_{z_1 z_2}\sigma_z^2} - 1}{e^{\sigma_z^2} - 1} \tag{5}$$

by utilizing that

$$\begin{aligned} E[y_1\, y_2] &= \int_0^\infty \int_0^\infty y_1 y_2 p(y_1, y_2)\, dy_1\, dy_2 \\ &= \exp\left[2\mu_z + \sigma_z^2 \left(1 + \rho_{z_1 z_2}\right)\right] \end{aligned} \tag{6}$$

$$\mu_y = e^{\mu_z + \frac{1}{2}\sigma_z^2} \tag{7}$$

$$\sigma_y^2 = e^{2\mu_z + \sigma_z^2}\left(e^{\sigma_z^2} - 1\right) \tag{8}$$

Or, an alternative to Eq. (5) is

$$\rho_{z_1 z_2} = \frac{1}{\sigma_z^2} \ln\left[1 + \rho_{y_1 y_2}\left(e^{\sigma_z^2} - 1\right)\right] \tag{9}$$

The conditional *pdf* of y_2 given y_1 is also lognormal distributed, given by (Johnson and Kotz [12])

$$p(y_2 \mid y_1) = \frac{p(y_1, y_2)}{p(y_1)} = \frac{1}{\sqrt{2\pi}\, y_2 \hat{\sigma}_z} \exp\left[-\frac{\left(\ln y_2 - \hat{\mu}_z\right)^2}{2\hat{\sigma}_z^{\,2}}\right] \tag{10}$$

where

$$\hat{\mu}_z = \mu_z + \rho_{z_1 z_2}\left(\ln y_1 - \mu_z\right) \tag{11}$$

$$\hat{\sigma}_z^{\,2} = \sigma_z^2\left(1 - \rho_{z_1 z_2}^2\right) \tag{12}$$

The mean (expected) value of y_2 given y_1 is given by (Johnson and Kotz [12])

$$\begin{aligned} E[y_2 \mid y_1] &= \int_0^\infty y_2 p(y_2 \mid y_1)\, dy_2 \\ &= y_1^{\rho_{z_1 z_2}} \exp\left[\frac{1}{2}\sigma_z^2\left(1 - \rho_{z_1 z_2}^2\right) + \mu_z\left(1 - \rho_{z_1 z_2}\right)\right] \end{aligned} \tag{13}$$

A quantity of interest is the probability of the surf parameter of a wave to be in an interval y_l to y_h when the surf parameter of the previous wave has been in the same interval. This probability is given as

$$P = \frac{\int_{y_l}^{y_h}\int_{y_l}^{y_h} p(y_1, y_2)\, dy_1 dy_2}{\int_{y_l}^{y_h} p(y_1)\, dy_1} = \frac{\int_{y_l}^{y_h}\left[\int_{y_l}^{y_h} p(y_2 \mid y_1)\, dy_2\right] p(y_1)\, dy_1}{\Phi(u_h) - \Phi(u_l)} \tag{14}$$

by using the relationship $p(y_1, y_2) = p(y_2 \mid y_1)\, p(y_1)$, and where Φ denotes the standard Gaussian cumulative distribution function (*cdf*), i.e.

$$\Phi(v) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{v} e^{-t^2/2} dt \tag{15}$$

and

$$u_l = \frac{\ln y_l - \mu_z}{\sigma_z} \ , \ u_h = \frac{\ln y_h - \mu_z}{\sigma_z} \tag{16}$$

The evaluation of the inner integral in the nominator of Eq. (14) follows by using Eqs. (10) to (12), giving

$$P = \frac{\frac{1}{\sqrt{2\pi}} \int_{u_l}^{u_h} \left[\Phi(\hat{u}_h) - \Phi(\hat{u}_l) \right] \exp\left(-\frac{1}{2} u_1^2 \right) du_1}{\Phi(u_h) - \Phi(u_l)} \tag{17}$$

where

$$\hat{u}_l = \frac{u_l - \rho_{z_1 z_2} u_1}{\sqrt{1 - \rho_{z_1 z_2}^2}} \ , \ \hat{u}_h = \frac{u_h - \rho_{z_1 z_2} u_1}{\sqrt{1 - \rho_{z_1 z_2}^2}} \tag{18}$$

$$u_1 = \frac{\ln y_1 - \mu_z}{\sigma_z} \tag{19}$$

4 Example of application

In this example the lognormal distribution of the surf parameter proposed by Tayfun [5] is adopted, given by the *pdf*

$$p(\xi) = \frac{1}{\sqrt{2\pi}\xi\sigma_{\ln\xi}} \exp\left[-\frac{\left(\ln\xi - \mu_{\ln\xi}\right)^2}{2\sigma_{\ln\xi}^2} \right] \tag{20}$$

where

$$\mu_{\ln\xi} = \ln\left(\frac{2m}{\sqrt{\alpha}} \right) \ , \ \sigma_{\ln\xi}^2 = \frac{1}{4} \ln\left(\frac{4}{\pi} \right) \tag{21}$$

and α is a parameter related to the wave steepness of the sea state s_{rms}. From Tayfun [5] it appears that his theoretical value of the sea state steepness parameter, $s_{rms} = 0.318\alpha$, is very close to $s_{rms} = 17.6 H_s / 4gT_z^2$ used by Myrhaug and Rue [9]; thus giving $\alpha = 1.41 H_s / T_z^2$. Here H_s is the significant wave height and T_z is the mean zero-crossing wave period.

Now a change of variables from ξ to $y = \xi / \xi_{rms}$ gives the lognormal *pdf* in Eq. (1) with $\mu_z = \mu_{\ln\xi} - \ln\xi_{rms} = \ln\left(2\sqrt{0.318}\right)$ and $\sigma_z^2 = \sigma_{\ln\xi}^2$, giving

$$\mu_z = 0.120 \ , \ \sigma_z = 0.246 \tag{22}$$

Figure 1(a) shows the mean (expected) value of $y_2 = \xi_2 / \xi_{rms}$ given $y_1 = \xi_1 / \xi_{rms}$ versus y_1 according to Eq. (13) for a range of $\rho_{z_1 z_2}$ values from 0 to 0.9. From Fig. 1(a) it appears that $E[y_2 | y_1]$ approaches y_1 as $\rho_{z_1 z_2}$

increases. It should be noted that $E[y_2 \mid y_1] = y_1$ for $\rho_{z_1 z_2} = 1$; see Eq. (13). For zero correlation the mean value of y_2 given y_1 is always constant, i.e. Eq. (13) reduces to Eq. (7).

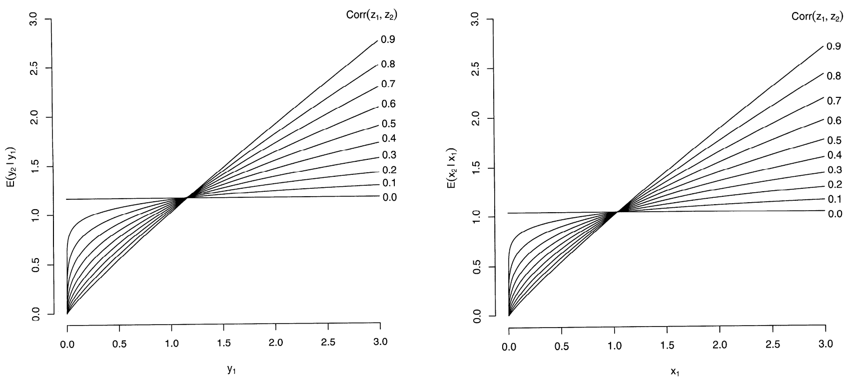

Figure 1: (a) (left) Conditional expected value of $y_2 = \xi_2 / \xi_{rms}$ given $y_1 = \xi_1 / \xi_{rms}$ versus y_1 for different values of $\rho_{z_1 z_2} = \text{Corr}(z_1, z_2)$. (b) (right) Conditional expected value of $x_2 = h_{2b} / h_{brms}$ given $x_1 = h_{1b} / h_{brms}$ versus x_1 for different values of $\rho_{z_1 z_2} = \text{Corr}(z_1, z_2)$.

Furthermore, a sea state specified by $H_s = 7$ metres and $T_z = 7$ seconds is chosen, representing a "steep" sea state which has been measured at a deep water location on the Norwegian continental shelf (Krogstad [13]). Thus, $s_{rms} = 0.064$ and $\alpha = 0.20$, giving $\xi_{rms} = 3.95\,\text{m}$.

Moreover, in this example breaking waves on slopes will be considered. Types of breaking waves are defined in terms of the surf parameter, classified as (see e.g. Tayfun [5])

$$\begin{array}{ll} \text{spilling if} & \xi \le 0.5 \\ \text{plunging for} & 0.5 < \xi \le 3 \\ \text{collapsing for} & 3 < \xi \le 3.5 \\ \text{surging if} & 3.5 < \xi \end{array} \tag{23}$$

Thus, by taking $y_1 = \xi_1 / \xi_{rms}$, $y_2 = \xi_2 / \xi_{rms}$ and μ_z, σ_z from Eq. (22), Eqs. (16) to (19) can be used to calculate the probability of two successive breaking waves on slopes.

Figure 2 shows the probability P of two successive spilling breakers versus the correlation coefficient $\rho_{z_1 z_2}$ for the slopes m = 1/10, 1/5, 1/4, 1/3, 1/2.

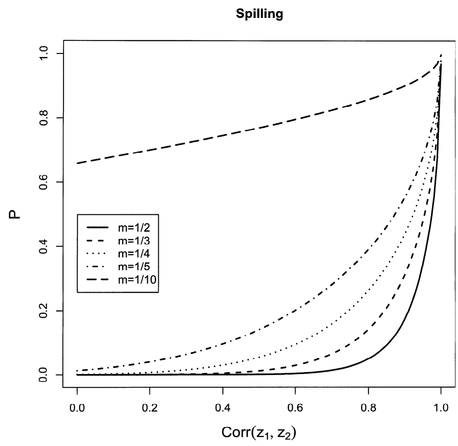

Figure 2: The probability (P) of two successive spilling breakers versus $\rho_{z_1 z_2} = \mathrm{Corr}(z_1, z_2)$ for different slopes m.

Similar results are shown in Figures 3, 4 and 5 for plunging, collapsing and surging breakers, respectively.

From Figs. 2 to 5 it appears that P increases as $\rho_{z_1 z_2}$ increases for a given slope, which is physically sound. Moreover, from Fig. 2 it appears that P decreases as the slope increases for a given value of $\rho_{z_1 z_2}$, which a priori is not quite obvious. However, it can be demonstrated by considering the results for $\rho_{z_1 z_2} = 0$, i.e. when y_1 and y_2 are statistically independent. Then the marginal *pdf* of y_1 (and y_2) is given by Eq. (1) and μ_z, σ_z from Eq. (22), and the *pdf* of $\xi = \xi_1 \left(\text{or}\, \xi_2\right)$ for $\xi_{rms} = 3.95\,\text{m}$ is shown in Fig. 6 for the same slopes m as in Figs. 2 to 5. From Fig. 6 it appears that the probability of a spilling breaker for a given slope (i.e. given by the area under the *pdf* for the slope considered corresponding to $0 < \xi \le 0.5$), is largest for $\text{m} = 1/10$ and decreases as the slope increases. This will also be the case for other values of $\rho_{z_1 z_2}$, and consequently the results are as shown in Fig. 2.

From Fig. 3 it appears that the probability of two successive plunging breakers for a given value of $\rho_{z_1 z_2}$ increases for the slope in the order m=1/10, 1/2, 1/5, 1/3, 1/4; for the three latter values the differences are very small. The understanding of this is supported by the results for $\rho_{z_1 z_2} = 0$ in Fig. 6; for $0.5 < \xi \le 3.0$ it is observed that the area under the *pdf* is smallest for m=1/10 and that it increases for the slope in the order referred to in the discussion of Fig. 3.

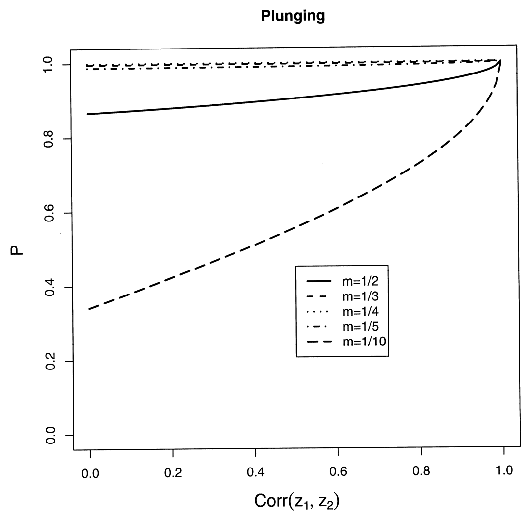

Figure 3: The probability (P) of two successive plunging breakers versus $\rho_{z_1 z_2} = \mathrm{Corr}(z_1, z_2)$ for different slopes m.

The results for the collapsing and surging breakers in Figs. 4 and 5, respectively, are similar; it appears that the probability of two successive collapsing and surging breakers increases as the bed slope increases for a given value of $\rho_{z_1 z_2}$, which is supported by the results in Fig. 6 for $\rho_{z_1 z_2} = 0$.

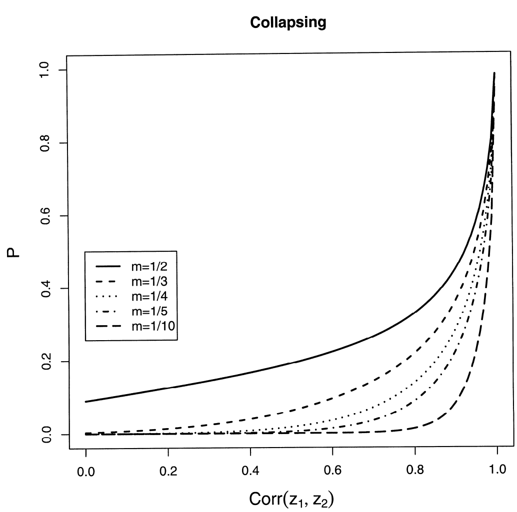

Figure 4: The probability (P) of two successive collapsing breakers versus $\rho_{z_1 z_2} = \mathrm{Corr}(z_1, z_2)$ for different slopes m.

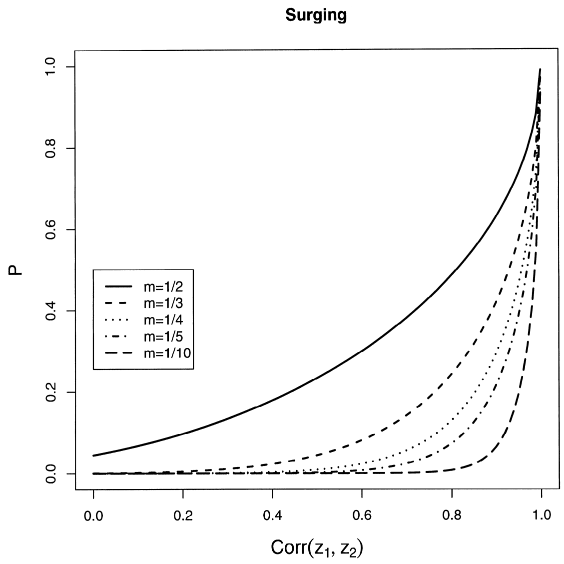

Figure 5: The probability (P) of two successive surging breakers versus $\rho_{z_1 z_2} = \mathrm{Corr}(z_1, z_2)$ for different slopes m.

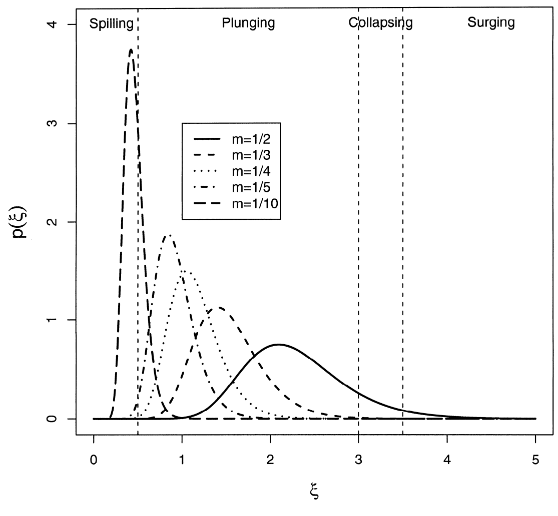

Figure 6: The *pdf* of the surf parameter $\xi = \xi_1$ (or ξ_2) for $\xi_{rms} = 3.95$ m and different slopes m.

Overall, the results given in this example appears to be physically sound, although they are valid for the particular sea state chosen. However, validation with data is required before a conclusion can be drawn on the ability of the present approach to describe measured wave data. Thus the results should be taken as tentative, but in the meantime the present distribution of two successive surf parameters should serve the purpose of being a useful tool for making assessments of wave phenomena in the surf zone, i.e. to obtain an estimate of two extreme successive wave events in the surf zone.

5 Statistics of the joint behaviour of two successive breaker indices

The breaker index h_b is another frequently used quantity in coastal work, which is closely related to the surf parameter. It is defined as the ratio between the wave height H_b and the water depth d_b at breaking. Many empirical relationships exist for h_b; one is related to the surf parameter in the form $h_b \equiv H_b / d_b = a\xi^c$ (Tayfun [5]), where a and c are empirical coefficients. In the forthcoming the breaker index is normalized, i.e. $x = h_b / h_{brms}$, by defining $h_{brms} = a\xi_{rms}^c$ where ξ_{rms} is defined in Section 2, giving $x = \left(\xi / \xi_{rms}\right)^c = y^c$. The *pdf* of x is obtained from Eq. (1) by a change of variable from y to x, taking the form

$$p\left(x\right) = \frac{1}{\sqrt{2\pi}x\sigma} \exp\left[-\frac{(\ln x - \mu)^2}{2\sigma^2} \right] \tag{24}$$

where

$$\mu = c\mu_z \ , \ \sigma^2 = \left(c\sigma_z\right)^2 \tag{25}$$

are the mean value and the variance, respectively, of $\ln x$.

Thus, the statistics of the joint behaviour of two successive breaker indices $x_1 = h_{b1} / h_{brms}$ and $x_2 = h_{b2} / h_{brms}$ normalized with the same parameter h_{brms} follow by utulizing the results in Section 3. More specifically it follows that: $p(x_1, x_2)$ is given in Eq. (2); $p(x_2 \mid x_1)$ in Eqs. (10) to (12); $E\left[x_2 \mid x_1\right]$ in Eq. (13), by replacing $y_1, y_2, \mu_z, \sigma_z$ with $x_1, x_2, c\mu_z, c\sigma_z$. Moreover, $\rho_{x_1 z_2}$ is given in Eq. (5) (or alternatively $\rho_{z_1 z_2}$ in Eq. (9)) by replacing y_1, y_2, σ_z with $x_1, x_2, c\sigma_z$; this is obtained by utilizing the results in Eqs. (6) to (8) by in addition replacing y, μ_z with $x, c\mu_z$. The results in Eqs. (14) to (19) can be re-arranged accordingly to be valid for two successive breaker indices.

Figure 1(b) shows the mean value of $x_2 = h_{b2} / h_{brms}$ given $x_1 = h_{b1} / h_{brms}$ versus x_1 for a range of $\rho_{z_1 z_1}$ values from 0 to 0.9 according to Eq. (13) by replacing $y_1, y_2, \mu_z, \sigma_z$ with $x_1, x_2, c\mu_z, c\sigma_z$, and $(a,c) = (1.20, 0.27)$ from Kaminsky and Kraus [14]. The results are similar to those given in Fig. 1(a) for the surf parameter except for the shift of the values. Results for the breaker index will not be elaborated further here since they will be similar to those presented for the surf parameter.

6 Summary

A joint distribution of two successive surf parameters is provided, and it is represented by a bivariate lognormal distribution. Consequently the joint distribution of two successive breaker indices is represented by a bivariate lognormal distribution. The application of the surf parameter distribution is exemplified to estimate the probability of two successive breakers on slopes; spilling, plunging, collapsing and surging breakers, by using wave parameters corresponding to typical field conditions. Overall, these results appear to be physically sound, although they are valid for the particular sea state chosen. The results should be taken as tentative, because validation with data is required before a conclusion can be drawn on the ability of the present approach to describe measured wave data. However, in the meantime the bivariate lognormal distribution of two successive surf parameters should serve the purpose as a useful tool for making assessments of wave phenomena in the surf zone, i.e. to obtain an estimate of two extreme successive wave events in the surf zone.

Appendix

The joint *pdf* of two Gaussian random variables z_1 and z_2 with the same mean value μ_z and variance σ_z^2, is given by (Bury [15])

$$p(z_1, z_2) = \frac{1}{2\pi\sigma_z^2\sqrt{1-\rho_{z_1 z_2}^2}} \cdot \exp\left[-\frac{(z_1-\mu_z)^2 + (z_2-\mu_z)^2 - 2\rho_{z_1 z_2}(z_1-\mu_z)(z_2-\mu_z)}{2\sigma_z^2\left(1-\rho_{z_1 z_2}^2\right)} \right] \tag{A1}$$

where the correlation coefficient $\rho_{z_1 z_2}$ is given as

$$\rho_{z_1 z_2} \equiv \frac{Cov[z_1, z_2]}{\sigma_z^2} = \frac{E[z_1 z_2] - \mu_z^2}{\sigma_z^2} \tag{A2}$$

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