A methodology for identification of dynamic parameters in assembled aircraft structures

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Abstract

Finite Element Models (FEM) are widely used in order to study and predict the dynamic properties of structures. Comparing dynamic experimental data and analytical results, respectively, of the real and modelled structure, shows that the prediction of the dynamic response can be obtained with much more accuracy in the case of a single component than in the case of assemblies.

Generally speaking, as the number of components in the assembly increases the calculation quality declines because the connection mechanisms among components are not represented sufficiently.

Specifically for aircrafts, it is quite common that Frequency Response Functions (FRF) obtained via Ground Vibration Test (GVT) show a certain degree of discrepancy from the FRF calculated with the FEM, particularly across the sections where joining is discontinued.

When this happens it is necessary to tune up the values of the dynamic parameters of the joints, to allow the numerical FRF to match the results of the experimental FRF. From a modelling and computational point of view, these types of joints can be seen as localized sources of stiffness and damping and can be modelled as lumped spring/damper elements.

In this paper this is done by formulating an optimization problem. The approach has been applied to a FEM that mimics the rear fuselage of a commercial aircraft and the numerical results shows that the procedure is very efficient and promising.
Keywords: aircraft design, assembled structures, ground vibration test, dynamic analysis, optimization methods.

www.witpress.com, ISSN 1743-355X (on-line)
doi:10.2495/CMEM130291

1 Introduction

Modern aircrafts are usually built by connecting a number of structural segments, as fuselage sections, belly fairing, wings, vertical and horizontal stabilizers and so on. Connections between these structures can be of different nature and exhibit dissimilar behaviour. While some of them are fabricated as built-in there are other cases where both parts of the assembly are only connected at a few locations. Figure 1 shows examples of each connection type.

(a) (b)

Figure 1: Examples of assembled structures. (a) Fuselage barrel; (b) VTP and fuselage connection.

A relevant issue of locally connected assembled structure is the evaluation of the mechanical properties of the assembly elements, namely the values of the stiffness, for static purpose and the damping, for dynamic purposes. Proper identification of both characteristics is very important in order to have good enough information or the aircraft expected performance. Also they are necessary for the definition of a finite element model that can be used to carry out structural analysis of several classes of loadings.

2 Ground vibration test of aircraft

Aircrafts are subjected to an ample collection of test to assess their behaviour. Amongst them the one entitled Ground Vibration Test (GVT) is in charge of characterizing the structure dynamically. It means to define the properties of the structure in terms of FRFs, modal base, modal shape and modal damping in the frequency range of interest. The GVT is carried out by applying a set of dynamic loads at specific locations of the aircraft, and installing a number of measurement devices as accelerometers at relevant positions, to obtain information of structural responses from the aircraft. The first are defined as Dynamic Points (DPs) and the latter as Response Points (RPs). Typical output of GVT are the FRFs, whose synthesis, through Experimental Modal Analysis (EMA) technique, allows the definition of modal base, or eigenvalues, and the modal shape, or eigenvector, associate to each mode.

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In figure 2 a CESSNA business jet and a military aircraft are shown, as examples of aircraft subject to GVT.

An important role of the GVT, in addition to provide actual dynamic performance of this full aircraft or the aircraft part, is the possibility to use the experimental data provided by the test to make comparisons with dynamic analysis outputs of the numerical models. It is not rare that in many cases these two types of results do not agree to the expected level, particularly for higher frequency values, where the joining mechanisms became relevant. For long time the lack of reliability in modeling the junctions in complex assemblies has been under estimated, hence its effect on the global dynamic behavior is neglected.

(a) (b)

Figure 2: Aircrafts subject to GVT. (a) CESSNA business jet; (b) military aircraft.

Minimize the degree of discrepancy between EMA and FE Model is very important, because allows the numerical model to be predictive and consequently reliable at time to calculate dynamic response under whatever load condition: i.e. continue and discrete turbulence, wind-milling, dynamic landing. If statically only the stiffness can be considered a key parameter, dynamically speaking, more complex and dissipative mechanisms, like friction at interfaces, play crucial roles. It means that the tuning approach needs to consider both stiffness and damping. Also it is important to point up that these kind of dissipative mechanisms are usually non-linear.

Several models have been proposed to tackle up with this problem. Hurty [1], proposed the Component Mode System (CMS), many years ago. More recently the Harmonic Balance Method has been used for different purposes [2–7]. A research conducted in the University of Coruna in collaboration with AIRBUS has taken a different approach providing good results and will be next described. The procedure explained in the next paragraphs is applied considering a linear formulation of the lumped elements at interfaces but it can be extended, by means of iterative procedures, to non linear lumped elements.

3 Formulation of dynamic analysis of assembled structures

As mentioned before the research deals with the identification of the dynamic parameters of the joints of assembled structures, but the approach is a novel one. It is based upon splitting the stiffness of the complete structure into two parts $\mathbf{k}_{ns}$

and $\mathbf{k}_s$, where $\mathbf{k}_s$ is the stiffness provided by the joints and $\mathbf{k}_{ns}$ the remaining stiffness. Thus the total stiffness $\mathbf{k}$ is

$$\mathbf{k} = \mathbf{k}_{ns} + \mathbf{k}_s \tag{1}$$

therefore the dynamic equilibrium under a vector of forces $\mathbf{f}$ can be written

$$\mathbf{m}\,\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{f} = \mathbf{m}\,\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \left(\mathbf{k}_{ns} + \mathbf{k}_s\right)\mathbf{u} = \mathbf{f} \tag{2}$$

The displacement vector $\mathbf{u}$ is formulated via modal decomposition using the spectral matrix $\mathbf{\Phi}$.

$$\mathbf{u} = \mathbf{\Phi}\mathbf{q} \tag{3}$$

vectors $\mathbf{f}$ and $\mathbf{q}$ can be written as

$$\mathbf{f} = \mathbf{p}e^{i\omega t} \qquad\qquad \mathbf{q} = \mathbf{v}e^{i\omega t} \tag{4}$$

Introducing these expressions into (2) and if the spectral matrix contains the eigenvectors associated to the stiffness $\mathbf{K}_{ns}$ normalized to the total mass it turns out.

$$\mathbf{I} + \ddot{\mathbf{q}} + \mathbf{c}\dot{\mathbf{q}} + \mathbf{k}_{ns}\mathbf{q} + \mathbf{k}_s\mathbf{q} = \mathbf{F} \tag{5}$$

where

$$\mathbf{C} = \mathbf{\Phi}^T c\mathbf{\Phi} \qquad\qquad \mathbf{F} = \mathbf{\Phi}^T\mathbf{p} \tag{6}$$

and

$$\mathbf{K}_{ns} = \mathbf{\Phi}_{ns}^T\mathbf{k}_{ns}\mathbf{\Phi}_{ns} \qquad\qquad \mathbf{K}_s = \mathbf{\Phi}_{ns}^T\mathbf{k}_s\mathbf{\Phi}_{ns} \tag{7}$$

vectors $\mathbf{p}$, $\mathbf{F}$ and $\mathbf{v}$ are complex in general, so

$$\mathbf{p} = \mathbf{p}_r + i\mathbf{p}_i \qquad \mathbf{F} = \mathbf{F}_r + i\mathbf{F}_i \qquad \mathbf{v} = \mathbf{v}_r + i\mathbf{v}_i \tag{8}$$

therefore

$$\begin{pmatrix} \mathbf{R}_{ns} + \mathbf{K}_s & -\omega\mathbf{C} \\ \omega\mathbf{C} & \mathbf{R}_{ns} + \mathbf{K}_s \end{pmatrix} \begin{vmatrix} \mathbf{v}_r \\ \mathbf{v}_i \end{vmatrix} = \begin{vmatrix} \mathbf{F}_r \\ \mathbf{F}_i \end{vmatrix} \tag{9}$$

where

$$\mathbf{R}_{ns} = \mathbf{K}_{ns} - \omega^2\mathbf{I} \tag{10}$$

Solving the system of linear equations of (9) the vector of displacements $\mathbf{u}$ can be obtained for each type of dynamic load.

These expressions can be simplified if the load vector $\mathbf{p}$ is real. In other words

$$\mathbf{p}_r = \mathbf{p} \qquad \mathbf{p}_i = \mathbf{0}; \qquad \mathbf{F}_r = \mathbf{\Phi}^{\mathrm{T}}\mathbf{p}; \qquad \mathbf{F}_i = \mathbf{0} \tag{11}$$

then it turns out

$$\mathbf{u} = \mathbf{\Phi}\mathbf{q} = \mathbf{\Phi}\left[\mathbf{v}_r + i\mathbf{v}_i\right]e^{i\omega t}; \;\; \mathbf{u} = \mathbf{\Phi}\left[\mathbf{v}_r\cos\omega t - \mathbf{v}_i\mathrm{sen}\omega t + i\left(\mathbf{v}_r\mathrm{sen}\,\omega t + \mathbf{v}_i\cos\omega t\right)\right] \tag{12}$$

If the load is $\mathbf{p} = \mathbf{P}\cos\omega t$

$$\mathbf{u} = \mathbf{\Phi}\left[\mathbf{v}_r\cos\omega t - \mathbf{v}_i\mathrm{sen}\omega t\right] \;\; \ddot{\mathbf{u}} = -\omega^2\mathbf{\Phi}\left[\mathbf{v}_r\cos\omega t - \mathbf{v}_i\mathrm{sen}\omega t\right] \tag{13.a}$$

and if the load is $\mathbf{p} = \mathbf{P}\mathrm{sen}\omega t$

$$\mathbf{u} = \mathbf{\Phi}\left[\mathbf{v}_r\mathrm{sen}\omega t + \mathbf{v}_i\cos\omega t\right] \;\; \ddot{\mathbf{u}} = -\omega^2\mathbf{\Phi}\left[\mathbf{v}_r\mathrm{sen}\omega t + \mathbf{v}_i\cos\omega t\right] \tag{13.b}$$

Let consider the load is $\mathbf{p} = \mathbf{P}\mathrm{sen}\omega t$. A component of vector **u,** for instance u_k, is obtained as

$$u_k = \boldsymbol{\psi}_k\left[\mathbf{v}_r\mathrm{sen}\omega t + \mathbf{v}_i\cos\omega t\right] \tag{14}$$

where $\boldsymbol{\psi}_k$ is the k-esime row of matrix $\mathbf{\Phi}$. Parameters of interest of u_k are its amplitude and phase angle. It can be remembered that in a one degree of freedom

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dynamic system under a harmonic load p = Psenωt the displacement can be written as

$$u = u_0 sen(\omega t - \varphi) \tag{15}$$

comparing expressions (14) and (15) is easily obtained that the phase angle φ is

$$\varphi = arctg - \frac{\boldsymbol{\psi}_k \mathbf{v}_i}{\boldsymbol{\psi}_k \mathbf{v}_r} \tag{16}$$

The maximum value of u_k is obtained by differentiating expression (14) and it turns out

$$\frac{du_k}{dt} = \omega \boldsymbol{\psi}_k \left[\mathbf{v}_r \cos \omega t - \mathbf{v}_i sen\omega t \right] \tag{17}$$

when the derivative cancels out the instant linked to the maximum value of u_k is identified. That time is the data required to create the FRF of the dynamic system

$$t = \frac{1}{\omega} arctg \frac{\boldsymbol{\psi}_k \mathbf{v}_r}{\boldsymbol{\psi}_k \mathbf{v}_i} \tag{18}$$

When the load is of the type **p** = **P**cosωt the maximum value of u_k is obtained by differentiating expression (13.a) and finally it turns out

$$t = \frac{1}{\omega} arctg - \frac{\boldsymbol{\psi}_k \mathbf{v}_i}{\boldsymbol{\psi}_k \mathbf{v}_r} \tag{19}$$

This formulation allows us to obtain the displacements, accelerations or velocities of a structure from expression (12) or its simplified version (13) or (21). It has the advantage that it only requires to know the spectral matrix $\mathbf{\Phi}_{ns}$ and the natural vibration frequencies of the structural model excluded the spring elements. That needs to be done only once.

Therefore the evaluation of a FRF in the dynamic system for different values of the spring parameters can be done quite easily carrying out the following steps:

1) Assembly of stiffness matrix of springs $\mathbf{k}_s$ that is a simply task.
2) Calculation of matrix $\mathbf{K}_s$ according to formulae (7)
4) Calculation of matrix **R** according to expression (10).

Afterwards the system of linear equations of (9) can be solved and the FRF obtained for any given value of spring stiffness $\mathbf{k}_s$.

The main idea behind this formulation is to eliminate the inconvenience of carrying out a dynamic analysis of the full structure each time that the mechanical parameters of the points are changed with the objective of matching the experimental and computational results.

In this approach an eigenvalue problem is solved at the beginning considering the structural model after eliminating the contribution of the joints, that in this research have been considered as linear springs. This provides spectral matrix $\mathbf{\Phi}$ and matrices $\mathbf{K}_{ns}$ and **C** that are diagonals.

$$\mathbf{K}_{ns} = \begin{pmatrix} \omega_1^2 & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & \omega_m^2 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 2\zeta_1\omega_1 & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & 2\zeta_m\omega_m \end{pmatrix} \tag{20}$$

Afterwards for any given realization of the set of mechanical parameters of the joints the procedure only requires to solve systems of linear equations to obtain the FRF of a specific degree of freedom.

4 Identification of mechanical parameters by optimization methods

The aim of obtaining a FEM model that can produce FRF that mach those provided by the GVT is done by identifying the values of the mechanical parameters of the degrees of freedom related to the joints of the assembled structure. In this research the mechanical parameters allowed to vary have been the spring stiffness and the set of modal damping coefficients. By doing that and using them as design variables an optimization problem can be formulated using as objective function the summation of the squares of the differences between the values of the natural frequencies in the experimental and the numerical models and also the squares of the differences between the values of the FRF at the natural frequencies. Figure 3 shows graphically the strategy and mathematically the objective function can be written as

$$F = \sum_{i=1}^{n} \left(\omega_{iEX} - \omega_{iNU}\right)^2 + \sum_{i=1}^{n} \left(FRF_{EX}\left(\omega_{iEX}\right) - FRF_{NU}\left(\omega_{iEX}\right)\right)^2 \tag{21}$$

where ω_{iEX} is the i-esime natural frequency and FRF_{EX} (ω_{iEX}) its amplitude in the experimental model. All these values are constant in the optimization process. Also ω_{iNU} is the i-esime natural frequency and FRF_{NU} (ω_{iNU}) its amplitude in the FRF of the numerical model. These values change at each iteration of the procedure. In fact, the first set of natural frequencies is obtained by solving the eigenvalue problem using $\mathbf{K}_{ns}$ matrix. Such values can be entitled ω_{iNUI}. Them at each iteration a natural frequency is identified as the coordinate that provide a local maximum of the frequency response function. This is done searching in an interval located between values $\left(\omega_{iNUI} - \Delta\omega, \omega_{iEX} + \Delta\omega\right)$ as shown in figure 3.

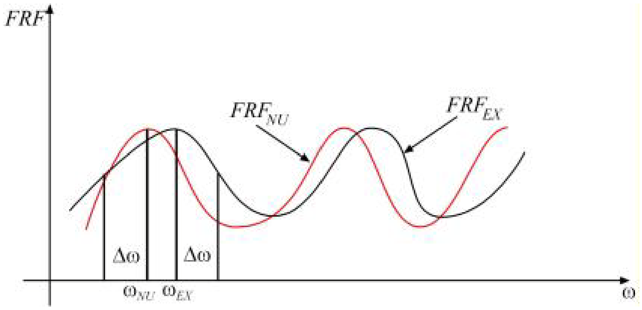


Figure 3: Explanation of optimization procedure.

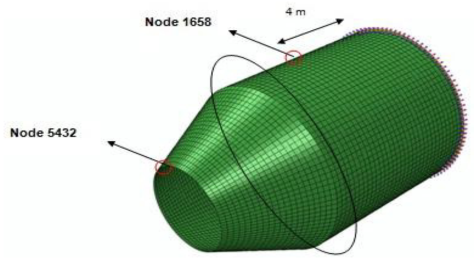
Expression (21) is intended to match the results at a single degree of freedom of an assembled structure but it could be more convenient to match as many degrees of freedom as possible in order enhance the accuracy of the FEM model. In that case the objective of the optimization problem becomes.

$$F = \sum_{j=1}^{N_{FRF}} \left(\sum_{i=1}^{N_w} \left(\omega_{iEX} - \omega_{iNU} \right)^2 + \sum_{i=1}^{N_w} \left(FRF_{EX}\left(\omega_{iEX}\right) - FRF_{NU}\left(\omega_{iEX}\right) \right)^2 \right) \tag{22}$$

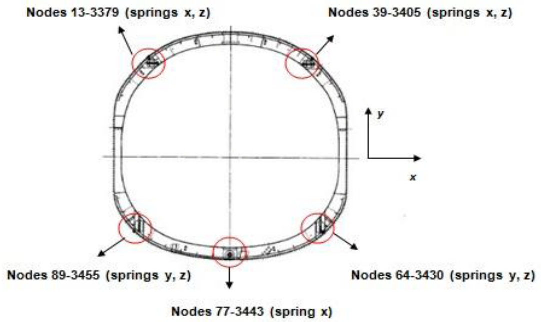
where N_{FRF} is the number of degrees of freedom considered in the study.

5 Application example

The described methodology has been applied to the structural model defined in figure 4 having a shape similar to the connection between the fuselage and the tail cone of a commercial aircraft. Cylinder and cone are connected at the locations shown in figure 4.b. Joints are defined as linear springs, up to a number of nine elements. The structure is undergoing a vertical harmonic load on node 5432 that is the tip upper node of the cone.



(a)



(b)

Figure 4: Structural model of application example. (a) Model geometry; (b) joints location.

A dynamic analysis using ABAQUS code has been made to obtain the FRF response of several degrees of freedom and the FRF provided by this analysis will be the *objective* of the optimization. Afterwards some properties of the FEM will be changed and thus the new FRF will be different and they will be nominated as *initial*. Then, the procedure already described will proceed iteratively until achieving the optimum values of the design variables that will match the *objective* FRF.

5.1 Optimization formulation with a simple FRF

In this case the purpose has been to match the FRF of the vertical displacement of code 5432, where the harmonic load is applied. *Objective* and *initial* values of the spring's stiffness and the modal damping coefficients, that are the design variables, appear in tables 1 and 2. It can be seen in figure 5 that such modification alters substantially the dynamic response of the first and second natural frequencies.

Table 1: Values of spring stiffness.

	1	2	3	4	5	6	7	8	9
Objective	5000	5000	5000	5000	5000	5000	5000	5000	5000
Initial	4500	4500	4500	4500	4500	4500	4500	4500	4500

Table 2: Values of modal damping coefficients (%).

	1	2	3	4	5	6
Objective	2	2	2	2	2	2
Initial	3	3	3	3	3	3

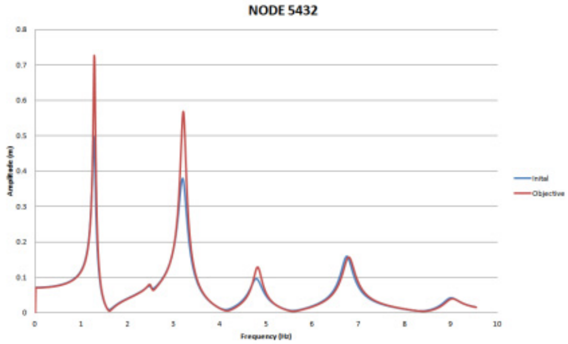


Figure 5: FRF of *initial* and *objective* models.

Carrying out the optimization procedure and using the objective function of expression 21 the initial FRF can be matched as presented in figure 6.

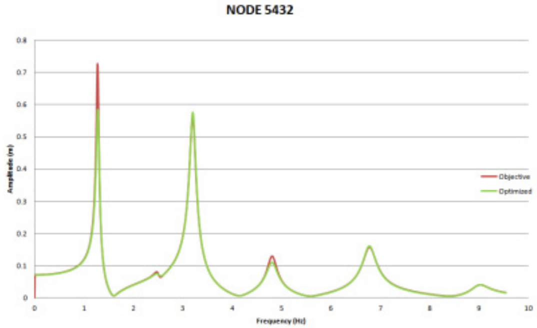


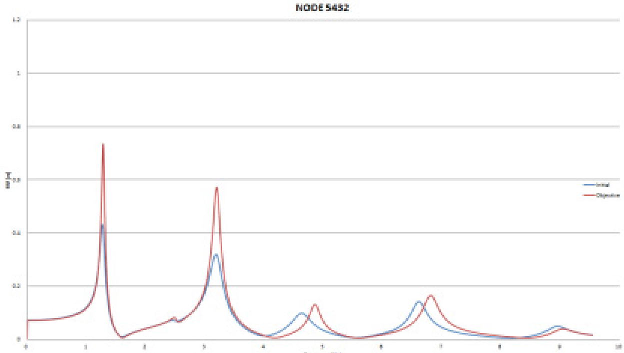
Figure 6: FRF of *objective* and *optimized* models.

5.2 Optimization formulation with multiple FRF

Additionally another case has been solved considering simultaneously the following FRF:

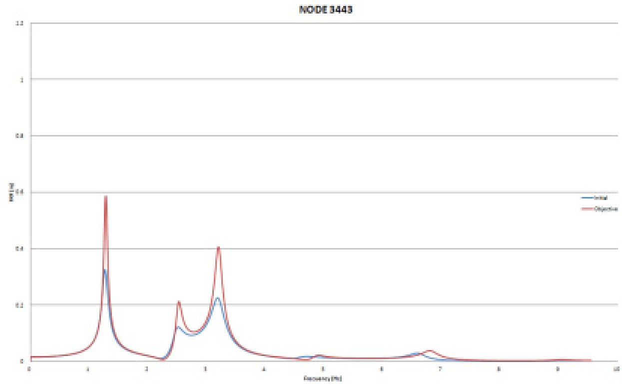
- Vertical displacement of node 5432
- Vertical displacement of node 3443 (bottom spring)
- Vertical displacement of node 3418 (intermediate left spring)
- Vertical displacement of node 3455 (bottom right spring)

Design variables chosen were the stiffness of the nine springs and the modal damping coefficients of six natural frequencies. In figure 7 the FRF of *objective* and *initial* design are shown and the dissimilarities are obvious. On the other hand figure 8 presents the FRF of the *objective* and *optimized* models that are in very good agreement.

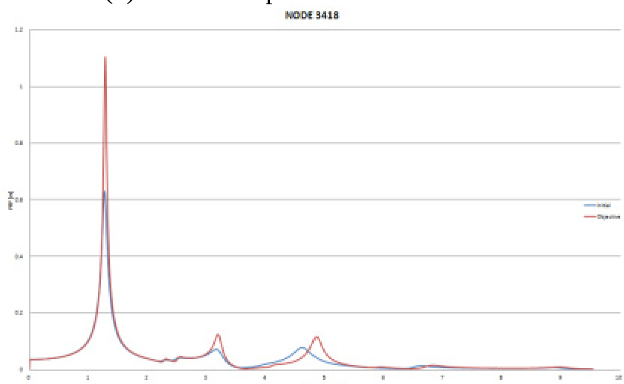


(a) Vertical displacement of node 5432

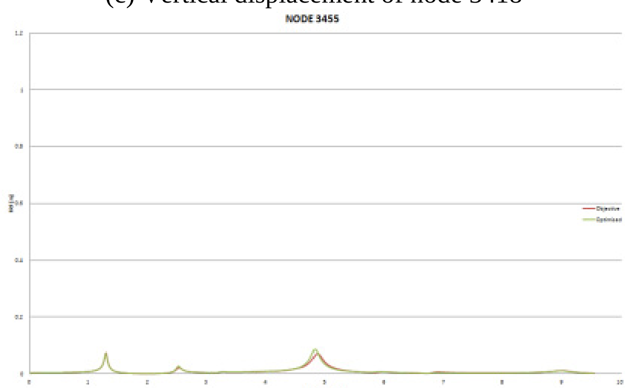
Figure 7: FRF of objective and initial models.



(b) Vertical displacement of node 3443

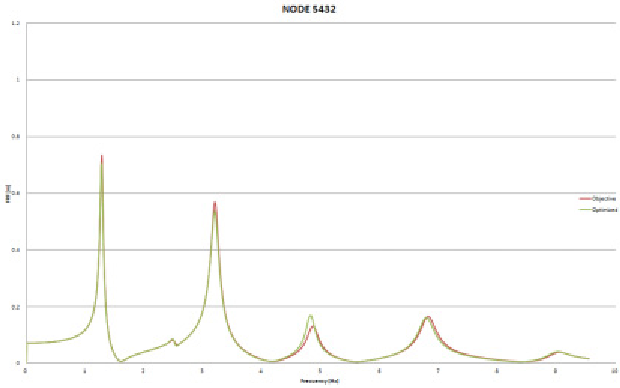


(c) Vertical displacement of node 3418

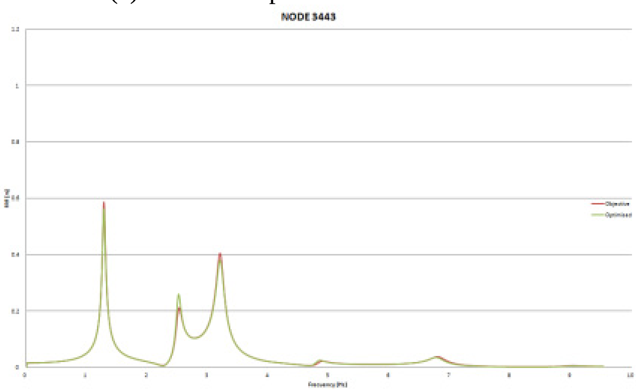


(d) Vertical displacement of node 3455

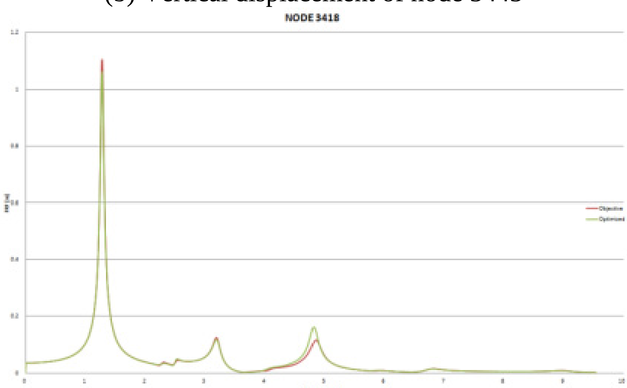
Figure 7: Continued.



(a) Vertical displacement of node 5432

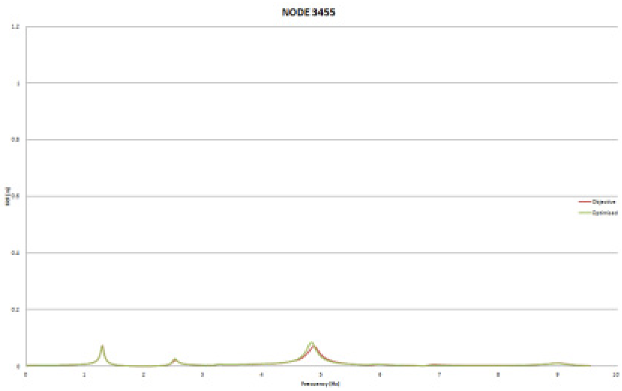


(b) Vertical displacement of node 3443



(c) Vertical displacement of node 3418

Figure 8: FRF of objective and optimized models.



(d) Vertical displacement of node 3455

Figure 8: Continued.

6 Conclusions

The following conclusions can be extracted from the research:

1) Reliable prediction and the studies of the dynamic behavior of assembled structures , as aircrafts, though numerical methods, is a general issue because localized phenomena, involving complex stiffness and damping mechanisms, happen at joint interfaces.
2) A way to increase the accuracy of the FE Models is to set up a robust tuning process able to minimize the error between the experimental and analytical results.
3) Optimization process can be successfully used in order to minimize the degree of discrepancy between these set of data.
4) The optimization process previously described focuses the attention on the use of spring lumped elements at interfaces as local variable and modal damping as global variable, but the methodology is completely general and can be easily adapted at whatever type of element. Also it can be used in order to estimate the dynamic behavior when non-linear lumped elements at interfaces are involved. This approach evidently requires an iterative algorithm for the convergence of the solution.

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