Experimental validation of analytical solutions for a transient heat conduction problem

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Abstract

This paper analyses heat transfer across multilayer systems when boundary conditions are unsteady. The results of analytical simulations and experimental tests were compared in order to validate the analytical formulation. The formulation that is proposed to solve this problem uses Green's functions to handle the conduction phenomena. The Green's functions are established by imposing the continuity of temperatures and heat fluxes at the interfaces of the various layers. The technique used to deal with the unsteady state conditions consists of first computing the solution in the frequency domain (after the application of time and spatial Fourier transforms along the two horizontal directions), and then applying (fast) inverse Fourier transforms into space-time. The thermal properties of the multilayer system materials have been previously defined experimentally.

For the experimental measurements the multilayer system was mounted on a guarded hotplate capable of imposing a controlled heat variation at the top and bottom boundaries of the system. Temperatures were recorded using a thermocouple set connected to a data logger system. Comparison of the results showed that the analytical solutions agree with the experimental ones.
Keywords: experimental validation, transient heat conduction, Green's functions formulation, frequency domain.

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1 Introduction

A dwelling's interior comfort is a fundamental issue in building physics and it depends on the building's envelope. In order to better evaluate the thermal performance of the construction elements used throughout the building envelope, more accurate models must be developed. Thermal behaviour depends largely on unsteady state conditions, and so the formulations for studying those systems should take the transient heat phenomena into consideration.

Most schemes devised to solve transient diffusion heat problems have either been formulated in the time domain (time-marching approach) (e.g. Chang *et al.* [1]) or else use Laplace transforms (e.g. Rizzo and Shippy [2]). An alternative approach is to apply a Fourier transform to deal with the time variable of the diffusion equation, thereby establishing a frequency domain technique, and then obtain time solutions are obtained by using inverse Fourier transforms into space-time (e.g. Tadeu and Simões [3]).

In general, multilayer systems, built by overlapping different layers of materials, are used to ensure that several functional building requirements, such as hygrothermal and acoustic comfort, are met. One of the requirements is to get high thermal performance and thus reduce energy consumption and promote building sustainability. The importance of multilayer solutions has motivated some researchers to try and understand the heat transfer in those systems (e.g. Kaşka and Yumrutaş [4], Chen et al. [5] and Sami A. Al-Sanea [6]).

In this paper is presented an experimental validation of a semi-analytical Green's functions solution that simulates heat conduction through multilayer systems when they are subjected to heat generated by transient sources. The proposed semi-analytical solutions allow the heat field inside a layered medium to be computed, without having to discretize the interior domain. The problem is formulated in the frequency domain using time Fourier transforms. The technique requires knowing the Green's functions for the case of a spatially sinusoidal, harmonic heat line source placed in an unbounded medium. The Green's functions for a layered formation are formulated as the sum of the heat source terms equal to those in the full-space and the surface terms required to satisfy the boundary conditions at the interfaces, i.e. continuity of temperatures and normal fluxes between layers. The total heat field is found by adding the heat source terms equal to those in the unbounded space to the set of surface terms arising within each layer and at each interface (e.g. Tadeu and Simões [3]).

The experimental results were obtained for several systems built by overlapping different materials. These test specimens were subjected to a transient heat flow produced by cooling and heating units which established a heat flow rate that could reach a pre-programmed mean test temperature in the specimen. The temperature changes in the different specimen layers were recorded by a thermocouple data logger system. The thermal properties of the different materials, such as thermal conductivity, mass density and specific heat were obtained experimentally. The temperature variation in the top and bottom surfaces of the multilayer system was used as an input for the semi-analytical model designed using the thermal properties obtained experimentally. This paper first formulates the three-dimensional problem and presents the Green's function in the frequency domain for a heat point source applied to a multilayer formation. A brief description of the mathematical manipulation follows, and the experimental setup is then described. Some final remarks are presented after the experimental measurements have been compared with computational results.

2 Problem formulation

Consider a system built from a set of *m* plane layers of infinite extent bounded by two flat, semi-infinite media, as shown in Figure 1. The top semi-infinite medium is called medium 0, and the bottom semi-infinite medium is assumed to be *m*+1. The thermal material properties and thickness of the various layers may differ. This system is subjected to a point heat source somewhere in the domain.

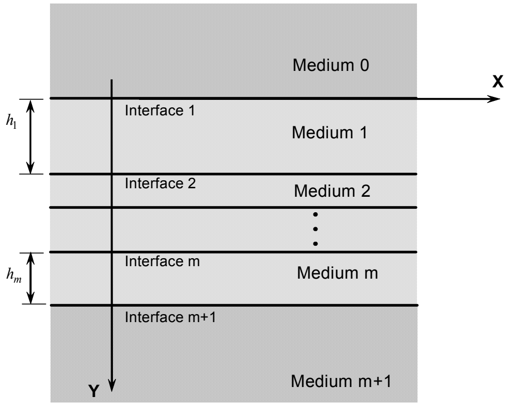

Figure 1: Multilayer system bounded by two semi-infinite media.

The transient heat transfer by conduction in each layer is expressed by the equation

$$k_j \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) T(t, x, y, z) = \rho_j \, c_j \, \frac{\partial T(t, x, y, z)}{\partial t}, \tag{1}$$

in which t is time, $T(t, x, y, z)$ is temperature, j identifies the layer number, k_j is the thermal conductivity, ρ_j is the mass density and c_j is the specific heat.

3 Semi-analytical solutions

The solution is defined in the frequency domain as the superposition of plane heat sources. This is done after applying a Fourier transform in the time domain and a double Fourier transformation in the space domain along the x and z directions.

Applying a Fourier transformation in the time domain to eqn (1) gives equation

$$\left(\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+\left(\sqrt{\frac{-\mathrm{i}\omega}{K_j}}\right)^2\right)\hat{T}(\omega,x,y,z)=0\,, \qquad (2)$$

where $\mathrm{i}=\sqrt{-1}$, $K_j=k_j/\left(\rho_j\, c_j\right)$ is the thermal diffusivity of the layer j, and ω is the frequency. For a heat point source applied at $\left(x_0,y_0,0\right)$ in an unbounded medium, of the form $T_{inc}\left(\omega,x,y,z,t\right)=\delta\left(x-x_0\right)\delta\left(y-y_0\right)\delta\left(z\right)e^{\mathrm{i}(\omega t)}$, where $\delta\left(x-x_0\right)$, $\delta\left(y-y_0\right)$ and $\delta\left(z\right)$ are Dirac-delta functions, the fundamental solution of eqn (2) can be expressed as

$$\hat{T}_{inc}(\omega,x,y,z)=\frac{e^{-\sqrt{\frac{\mathrm{i}\omega}{K_j}}\sqrt{\left(x-x_0\right)^2+\left(y-y_0\right)^2+z^2}}}{2k_j\sqrt{\left(x-x_0\right)^2+\left(y-y_0\right)^2+z^2}}\,. \qquad (3)$$

Applying a Fourier transformation in the z direction leads to the solution

$$\tilde{T}_{inc}(\omega,x,y,k_z)=\frac{-\mathrm{i}}{4k_j}H_0\left(\sqrt{-\frac{\mathrm{i}\omega}{K_j}-k_z{}^2}\; r_0\right), \qquad (4)$$

where $H_0\left(\;\right)$ are Hankel functions of the second kind and order 0, and $r_0=\sqrt{\left(x-x_0\right)^2+\left(y-y_0\right)^2}$.

The full three-dimensional solution is then found by applying an inverse Fourier transform in the k_z domain. This inverse Fourier transformation can be expressed as a discrete summation if we assume the existence of virtual sources, equally spaced at L_z, along z, which enables the solution to be obtained by solving a limited number of two-dimensional problems,

$$\hat{T}_{inc}(\omega,x,y,z)=\frac{2\pi}{L_z}\sum_{m=-M}^{M}H_0\left(\sqrt{-\frac{\mathrm{i}\omega}{K_j}-k_{zm}{}^2}\; r_0\right)e^{-\mathrm{i}k_{zm}z}\,, \qquad (5)$$

with k_{zm} being the axial wavenumber given by $k_{zm}=\frac{2\pi}{L_z}m$. The distance L_z chosen must be big enough to prevent spatial contamination from the virtual sources. Eqn (5) can be further manipulated and written as a continuous superposition of heat plane phenomena,

$$\tilde{T}_{inc}(\omega,x,y,k_z)=\frac{-\mathrm{i}}{4\pi\,k_j}\int_{-\infty}^{+\infty}\left(\frac{e^{-\mathrm{i}\nu_j|y-y_0|}}{\nu_j}\right)e^{-\mathrm{i}k_x(x-x_0)}dk_x \quad , \tag{6}$$

where $\nu_j=\sqrt{-\frac{\mathrm{i}\omega}{K_j}-k_z^{\,2}-k_x^{\,2}}$ and $\mathrm{Im}\left(\nu_j\right)\le 0$, and the integration is performed with respect to the horizontal wave number (k_x) along the x direction.

Assuming the existence of an infinite number of virtual sources, we can discretize these continuous integrals. The integral in the above equation can be transformed into a summation if an infinite number of such sources are distributed along the x direction, spaced at equal intervals L_x. The equation can then be written as

$$\tilde{T}_{inc}(\omega,x,y,k_z)=\frac{-\mathrm{i}}{4k_j}E_{0j}\sum_{n=-\infty}^{n=+\infty}\left(\frac{E_j}{\nu_{nj}}\right)E_d \quad , \tag{7}$$

where $E_{0j}=\frac{-\mathrm{i}}{2k_jL_x}$, $E_j=e^{-\mathrm{i}\nu_{nj}|y|}$, $E_d=e^{-\mathrm{i}k_{xn}(x)}$, $\nu_{nj}=\sqrt{-\frac{\mathrm{i}\omega}{K_j}-k_z^{\,2}-k_{xn}^{\,2}}$

and $\mathrm{Im}\left(\nu_{nj}\right)\le 0$, and $k_{xn}=\frac{2\pi}{L_x}n$, which can in turn be approximated by a finite sum of equations (N). Note that $k_z=0$ is the two-dimensional example.

The total heat field is achieved by adding the heat source terms, equal to those in the unbounded space, to the sets of surface terms arising within each layer and at each interface, that are required to satisfy the boundary conditions at the interfaces, i.e. continuity of temperatures and normal fluxes between layers.

For the layer j, the heat surface terms on the upper and lower interfaces can be expressed as

$$\tilde{T}_{j1}(\omega,x,y,k_z)=E_{0j}\sum_{n=-\infty}^{n=+\infty}\left(\frac{E_{j1}}{\nu_{nj}}A_{nj}^{t}\right)E_d \quad , \tag{8}$$

$$\tilde{T}_{j2}(\omega,x,y,k_z)=E_{0j}\sum_{n=-\infty}^{n=+\infty}\left(\frac{E_{j2}}{\nu_{nj}}A_{nj}^{b}\right)E_d \quad , \tag{9}$$

where $E_{j1}=\mathrm{e}^{-\mathrm{i}\nu_{nj}\left|y-\sum_{l=1}^{j-1}h_l\right|}$, $E_{j2}=\mathrm{e}^{-\mathrm{i}\nu_{nj}\left|y-\sum_{l=1}^{j}h_l\right|}$ and h_l is the thickness of the layer l. The heat surface terms produced at interfaces 1 and m+1, which govern the heat that propagates through the top and bottom semi-infinite media, are respectively expressed by

$$\tilde{T}_{02}(\omega,x,y,k_z)=E_{00}\sum_{n=-\infty}^{n=+\infty}\left(\frac{E_{01}}{\nu_{n0}}A_{n0}^{b}\right)E_d \tag{10}$$

$$\tilde{T}_{(m+1)2}(\omega,x,y,k_z) = E_{0(m+1)} \sum_{n=-\infty}^{n=+\infty} \left(\frac{E_{(m+1)2}}{\nu_{n(m+1)}} A^t_{n(m+1)} \right) E_d \quad (11)$$

A system of 2(m+1) equations is derived, ensuring the continuity of temperatures and heat fluxes along the m+1 interfaces between layers. Each equation takes into account the contribution of the surface terms and the involvement of the incident field. All the terms are organized according to the form $\underline{\underline{F}}\,\underline{a} = \underline{b}$. When the heat source is placed in medium 1, the following system of equations is obtained

$$\begin{bmatrix} -1 & -1 & e^{-i\nu_{n1}h_1} & \cdots & 0 & 0 & 0 \\ \frac{1}{k_0\nu_{n0}} & -\frac{1}{k_1\nu_{n1}} & -\frac{e^{-i\nu_{n1}h_1}}{k_1\nu_{n1}} & \cdots & 0 & 0 & 0 \\ 0 & e^{-i\nu_{n1}h_1} & -1 & \cdots & 0 & 0 & 0 \\ 0 & \frac{e^{-i\nu_{n1}h_1}}{k_1\nu_{n1}} & \frac{1}{k_1\nu_{n1}} & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & -1 & e^{-i\nu_{nm}h_m} & 0 \\ 0 & 0 & 0 & \cdots & -\frac{1}{k_1\nu_{nm}} & -\frac{e^{i\nu_{nm}h_m}}{k_m\nu_{nm}} & 0 \\ 0 & 0 & 0 & \cdots & e^{-i\nu_{nm}h_{m1}} & -1 & -1 \\ 0 & 0 & 0 & \cdots & \frac{e^{-i\nu_{nm}h_m}}{k_m\nu_{nm}} & \frac{1}{k_m\nu_{nm}} & \frac{1}{k_{m+1}\nu_{n(m+1)}} \end{bmatrix} \begin{bmatrix} A^b_{n0} \\ A^t_{n1} \\ A^b_{n1} \\ \cdots \\ A^t_{nm} \\ A^b_{nm} \\ A^t_{n(m+1)} \end{bmatrix} = \begin{bmatrix} -e^{-i\nu_{n1}y_0} \\ \frac{e^{-i\nu_{n1}y_0}}{k_1\nu_{n1}} \\ -e^{-i\nu_{n1}|h_1-y_0|} \\ -\frac{e^{-i\nu_{n1}|h_1-y_0|}}{k_1\nu_{n1}} \\ \cdots \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

The resolution of the system gives the amplitude of the surface terms at each interface. The temperature field for each layer formation is found by adding these surface terms to the contribution of the incident field, which leads to the following equations:

top semi-infinite medium (medium 0)

$$\tilde{T}(\omega,x,y,k_z) = E_{00} \sum_{n=-\infty}^{n=+\infty} \left(\frac{E_{01}}{\nu_{n0}} A^b_{n0} \right) E_d \text{, if } y < 0 \quad (13)$$

layer 1 (source position)

$$\tilde{T}(\omega,x,y,k_z) = \frac{-i}{4k_1} H_0\left(K_{t1}\ r_0\right) + E_{01} \sum_{n=-\infty}^{n=+\infty} \left(\frac{E_{11}}{\nu_{n1}} A^t_{n1} + \frac{E_{12}}{\nu_{n1}} A^b_{n1} \right) E_d \text{, if } 0 < y < h_1 \quad (14)$$

layer j $(j \neq 1)$

$$\tilde{T}(\omega,x,y,k_z) = E_{0j} \sum_{n=-\infty}^{n=+\infty} \left(\frac{E_{j1}}{\nu_{nj}} A^t_{nj} + \frac{E_{j2}}{\nu_{nj}} A^b_{nj} \right) E_d \text{, if } \sum_{l=1}^{j-1} h_l < y < \sum_{l=1}^{j} h_l \quad (15)$$

bottom semi-infinite medium (medium $m+1$)

$$\tilde{T}_{(m+1)2}(\omega,x,y,k_z)=E_{0(m+1)}\sum_{n=-\infty}^{n=+\infty}\left(\frac{E_{(m+1)2}}{\nu_{n(m+1)}}A^t_{n(m+1)}\right)E_d \quad (16)$$

Note that when the position of the heat source is changed, the matrix $\underline{\underline{F}}$ remains the same, while the independent terms of $\underline{b}$ are different. As the equations can be easily manipulated to consider another position for the source, they are not included here.

4 Experimental validation

4.1 Specimen description

The multilayer systems were built by overlapping 500x500 mm^2layers of different current insulating materials: natural cork (NC), molded expanded polystyrene (EPS) and medium-density fibreboard (MDF).

Each multilayer system was composed of 4 layers. One homogeneous (System 1) and two heterogeneous (Systems 2-3) systems were prepared. System 1 is composed of four NC layers (20.63mm, 20.68mm, 20.52mm and 20.60mm thickness). System 2 was prepared with these four layers: EPS (18.84mm), MDF (19.65mm), NC (20.60mm) and EPS (19.86). System 3 had the following four layers: NC (20.52mm), EPS (18.84mm), MDF (19.65mm) and NC (20.60mm). Each material used was tested to determine its thermal conductivity, mass density and specific heat. The thermal conductivity was found by means of the Guarded hot-plate method (ISO 8302:1991 [7]) using the EP-500 Lambda-meter from Lambda-Mebtechnik GmbH Dresden, a single-specimen model. The test procedure defined in EN 12667:2001[8] was used. The mass density was determined using the procedure described in EN 1602:1996[9]. The specific heat was obtained using a Netzsch apparatus, model DSC200F3, following the ratio method.

Table 1 gives the averages of those properties for the three materials used in the experiments.

Table 1: Thermal properties of the materials.

Material	Conductivity, k ($W.m^{-1}.^{o}C^{-1}$)	Mass density, ρ ($kg.m^{-3}$)	Specific heat, c ($J.kg^{-1}.^{o}C^{-1}$)
Natural Cork	0.046	130.0	1638.0
Molded Expanded Polystyrene	0.041	14.3	1430.0
Medium-Density Fiberboard	0.120	712	1550.0

4.2 Experimental procedure

The experiments required imposing an unsteady heat flow rate on each multilayer system using the single-specimen EP-500 Lambda-meter apparatus. Before running any test, the specimens were conditioned in a climatic chamber, Fotoclima 300EC10 from Aralab, in a controlled environment with a set-point temperature of (23±2)ºC and (50±5)% relative humidity, until constant mass was reached.

The tests were carried out in a controlled laboratory environment (temperature (23±2)ºC and relative humidity (50±5)%). The single-specimen Lambda-meter EP-500was first programmed to reach a mean temperature of 23ºC in the test specimen, establishing a 15º temperature difference between the heating and the cooling units. So, during the test, the temperature of the top multilayer surface (in contact with the heating plate unit) increased, while the temperature of the bottom multilayer surface (in contact with the lower plate) decreased. The energy input was maintained until a permanent heat flow rate was reached, that is, when there were no temperature variations at the multilayer interfaces. The system was then allowed to cool until the initial temperatures were reached again.

The temperature variation at each interface layer was measured using type T (copper) thermocouples made of 0.2 mm diameter wire. Three thermocouples were placed at each system interface, including the top and bottom surfaces, which were in contact with the heating and cooling plates (see Figure 2). The data were recorded by a Yokogawa MW 100 data logger, with a time interval of 10 seconds.

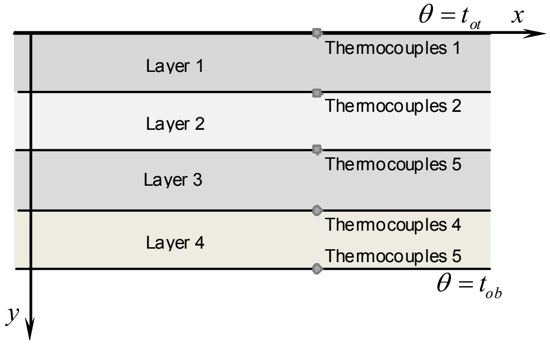

Figure 2: Thermocouple positions.

5 Results and discussion

In this section the experimental measurements are compared with the semi-analytical results. The semi-analytical solutions are obtained using the formulation presented in section 3, after they have been manipulated to simulate the heat transfer through the multilayer systems where the temperature variations are prescribed for the top and bottom surfaces. The materials' thermal properties (see Table 1) were used in these simulations.

5.1 Semi-analytical model

Equations (8) and (9) are manipulated by removing the media 0 and m+1 and by imposing temperatures t_{0t} and t_{0b} on the external top and bottom surfaces (interfaces 1 and m+1). Temperatures t_{0t} and t_{0b} are obtained by applying a Fourier transformation in the time domain to the temperatures recorded at the external multilayer system surfaces during the guarded hot plate test.

The total heat field is achieved by adding together the sets of surface terms arising within each layer at each interface and by imposing continuity of temperatures and normal fluxes at the internal interfaces.

The following system of 2m equations is obtained:

$$\begin{bmatrix} \frac{1}{k_1\nu_{n1}} & \frac{e^{-i\nu_{n1}h_1}}{k_1\nu_{n1}} & \ldots & 0 & 0 \\ e^{-i\nu_{n1}h_1} & -1 & \ldots & 0 & 0 \\ \frac{e^{-i\nu_{n1}h_1}}{k_1\nu_{n1}} & \frac{1}{k_1\nu_{n1}} & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & -1 & e^{-i\nu_{nm}h_m} \\ 0 & 0 & \ldots & -\frac{1}{k_1\nu_{nm}} & -\frac{e^{i\nu_{nm}h_m}}{k_m\nu_{nm}} \\ 0 & 0 & \ldots & \frac{e^{-i\nu_{nm}h_m}}{k_m\nu_{nm}} & \frac{1}{k_m\nu_{nm}} \end{bmatrix} \begin{bmatrix} A_{n1}^t \\ A_{n1}^b \\ \ldots \\ A_{nm}^t \\ A_{nm}^b \end{bmatrix} = \begin{bmatrix} t_{0t} \\ 0 \\ 0 \\ \ldots \\ 0 \\ 0 \\ t_{0b} \end{bmatrix} \tag{17}$$

Given that the temperatures t_{0t} and t_{0b} are uniform along the interfaces, this system is solved by imposing $k_{xn} = 0$ and $k_z = 0$. The resolution of this system gives the amplitude of the surface terms at each interface, leading to the following temperature fields at layer j :

$$\tilde{T}(\omega, x, y, k_z) = E_{0j}\left(\frac{E_{j1}}{\nu_{0j}}A_{0j}^t + \frac{E_{j2}}{\nu_{0j}}A_{0j}^b\right), \text{ if } \sum_{l=1}^{j-1} h_l < y < \sum_{l=1}^{j} h_l \tag{18}$$

5.2 Results

Temperatures t_{0t} and t_{0b} were first defined by applying a direct discrete fast Fourier transform in the time domain to the temperatures recorded by the thermocouples on the external surfaces of the system and subtracting the initial temperature. Analysis of the experimental responses led to an analysis period of 16h being established. This was enough to find the energy equilibrium of the multilayer system with the environment (temperatures at the interfaces were again restored almost to the initial test temperatures). The upper frequency of the analysis was defined such that its contribution to the global response is negligible.

The analytical computations were performed in the frequency domain for frequencies ranging from $0.0\ \text{Hz}$ to $\dfrac{1.0}{32 \times 3600}\ \text{Hz}$, with a frequency increment of $\dfrac{1.0}{32 \times 3600 \times 2048}\ \text{Hz}$, which determined a full analysis window of 16h.

The temperature variation imposed on the top and bottom multilayer surfaces may be of any type. To obtain the temperature in the time domain, a discrete inverse fast Fourier transform was applied in the frequency domain. The aliasing phenomena were dealt by introducing complex frequencies with a small imaginary part, taking the form $\omega_c = \omega - \mathrm{i}\eta$ (where $\omega = 0.7\Delta\omega$, and $\Delta\omega$ is the frequency increment). This shift was subsequently taken into account in the time domain by means of an exponential window, $e^{\eta t}$, applied to the response.

The final temperatures were obtained by adding the initial test temperatures to these responses.

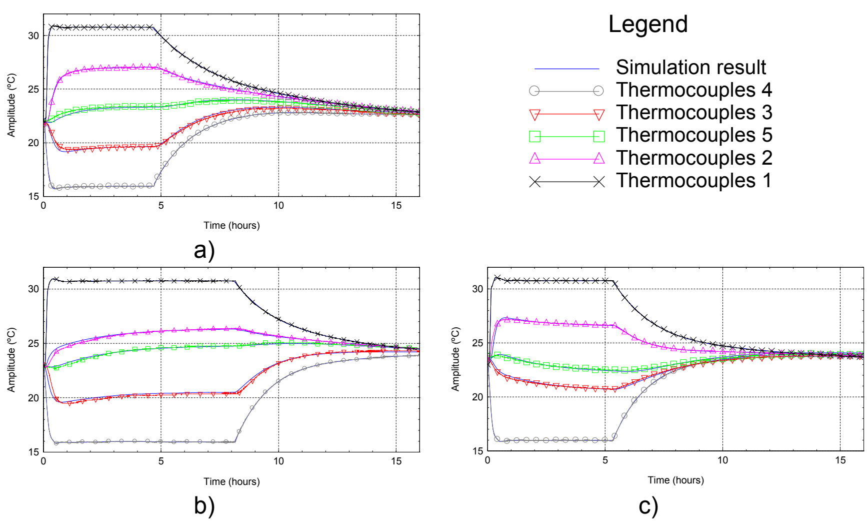

Figure 3: Semi-analytical and experimental results. Temperature change at the layer interfaces of the multilayer systems: a) System 1 – homogeneous multilayer system. b) System 2 – heterogeneous multilayer system. c) System 3 – heterogeneous multilayer system.

The results of the experimental measurements are presented below and compared with those computed analytically. In the figures the solid lines correspond to the semi-analytical responses and the experimental measurements are represented by the lines with marked points. The plotted experimental results at each interface correspond to the arithmetic mean of three thermocouple temperatures.

Figure 3 shows the results obtained for the different multilayer systems. System 1 is the natural cork (NC) homogeneous system, while the other two are heterogeneous (see section 4.1). These responses show a good agreement between the semi-analytical responses and the experimental results. The results are similar over the full time window, i.e. the period in which the heating and cooling units are receiving energy, the period with a constant heat flow rate, and the time when apparatus has no power input. Note that at the beginning of the process all the thermocouples show a temperature similar to the ambient temperature, which conforms with the initial conditions defined for the semi-analytical simulation.

Figure 3b) and c) shows the temperature change at each interface of the heterogeneous systems. System 2 is composed of EPS sandwiching MDF and NC layers, while System 3 has NC layers sandwiching an EPS and MDF layer. Comparing the semi-analytical solutions and experimental measurements for each system, it can be seen that the results are very similar. During the heating and cooling phase, when temperatures are becoming constant, we can see that the lowest temperature gradients occur in the MDF layers, given their higher conductivity and lower thermal diffusivity.

6 Conclusions

Three-dimensional semi-analytical solutions for transient heat conduction in a multilayer system in the frequency domain have been validated experimentally. The results showed a good agreement between experimental measurements and the computed solutions, thus we can conclude that the proposed semi-analytical model formulated in the frequency domain is reliable for studying transient heat conduction in multilayer systems.

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