# Large eddy simulation of a transition process in separated-reattached flows

Z. Yang
*Aeronautical and Automotive Engineering Department,*
*Loughborough University, UK*

## Abstract

Transition from laminar to turbulence in separated-reattached flow occurs frequently and plays a very important role in engineering. Hence, accurately predicting transition is crucial since the transition location has a significant impact on aerodynamics performance and a thorough understanding of the transition process can greatly help to control it, e.g. to delay the turbulent phase where laminar flow characteristics are desirable (low friction drag) or to accelerate it where high mixing of turbulent flow are of interest (in a combustor). However, it is very difficult to predict transition using conventional Reynolds-Averaged-Navier-Stokes (RANS) approach and the transition process is not fully understood. Nevertheless significant progress has been made with the simulation tools such as Large Eddy Simulation (LES) which has shown improved predictive capabilities over RANS and can predict transition process accurately. This paper presents briefly LES formalism and followed by its applications to predict/understand the transition process and unsteady behaviour of the free shear layer in separated-reattached flow.
*Keywords: transition, separated-reattached flow, LES, RANS, shear layer, unsteady, turbulence.*

## 1 Introduction

Separated flows are common and play an important role in many engineering applications from cooling of small electronic devices to airfoil and turbo-machinery design. If a separated flow reattaches downstream a separation bubble is formed and its characteristics are a crucial aspect of the engineering design process. Three types of separation bubble are possible depending on the state of

WIT Transactions on Modelling and Simulation, Vol 51, 
www.witpress.com, ISSN 1743-355X (on-line)
doi:10.2495/CMEM110171

the boundary layer at separation and reattachment: laminar, transitional and turbulent. In a laminar separation bubble the flow at both separation and reattachment is laminar. For a transitional separation bubble it is laminar flow at separation while at reattachment the flow becomes turbulent, and a turbulent separation bubble is formed over an already turbulent boundary layer. Laminar boundary layer separation occurs in many engineering problems such as low Re number flow over aerofoils and turbo-machinery flow. Laminar separated flow has a tendency to become unstable even at relatively low Reynolds numbers and therefore is likely to undergo a transition to turbulence. The location where transition starts and the spatial extent within which transition takes place are of crucial interest in engineering design and performance prediction applications.

Laminar-to-turbulence transition has been under intensive research for many decades. Experimental studies have provided fundamental knowledge of parameters influencing transition, along with indications for related physical mechanisms. However, such data can only provide limited temporal and spatial resolution of flow parameters and hence a thorough description of the transition process is lacking. Theoretical studies on the other hand, suffer from the limitation imposed by nonlinearity of the transition process at later stages.

Conventional RANS approach, based on solving the time- or ensemble-averaged governing equations and hence the effect of all the scales of instantaneous turbulent motion is modelled, is most commonly applied to the solution of engineering turbulent flow problems but is not adequate to predict transition since it only predicts the time- or ensemble-averaged structure and behaviour of transitional bubbles. Other approaches such as the semi-empirical $e^n$ method and correlations are also of limited accuracy and non universal [1].

The alternative approach is LES which was proposed as early as 1963 by Smagorinsky [2]. LES does not adopt the conventional time- or ensemble-averaging RANS approach with additional modelled transport equations being solved to obtain the so called Reynolds stresses resulting from the averaging process. In LES the large scale motions (large eddies) of turbulent flow are computed directly and only small scale (sub-grid scale) motions are modelled.LES can be more accurate than the RANS approach since the larger eddies contain most of the turbulent energy and are responsible for most of the turbulent mixing, and LES captures these eddies in full detail directly whereas they are modelled in the RANS approach. Furthermore the small scales tend to be more isotropic and homogeneous than the large ones, and thus modelling the sub-grid scale motions should be easier than modelling all scales within a single model as in the RANS approach. However, LES has received increased attention in the engineering community only since 1990’s although it was proposed nearly half a century ago, mainly due to the lack of sufficient computational power since LES requires 3D time-dependent calculations with small time-steps and reasonably fine meshes.

The current paper presents briefly LES formalism first followed by its applications to study transitional separated-reattached flows, focusing on the current understanding of physics of the transition process, and concludes with possible future trends in several important areas in LES and transitional bubble study.

## 2 Mathematical formulation

### 2.1 LES governing equations

The governing equations for any fluid flow, called the Navier-Stokes equations, are derived according to the fundamental conservation laws for mass, momentum and energy. In LES only large eddies (large scale motions) are computed directly and hence a low-pass spatial filter is applied to the instantaneous conservation equations to formulate the 3D unsteady governing LES equations. When the finite volume method is employed to solve the LES equations numerically the equations are integrated over control volumes, equivalent to convolution with a top-hat filter, therefore there is no need to apply a filter to the instantaneous equation explicitly and in this case it is called implicit filtering.

The filtered equation expressing conservation of mass and momentum in a Newtonian incompressible flow can be written in conservative form as:

$$\partial_i \overline{u_i} = 0 \tag{1}$$

$$\partial_t (\overline{u}_i) + \partial_j (\rho \overline{u_i u_j}) = -\partial_i \overline{p} + 2\partial_j (\mu \overline{S_{ij}}) - \partial_j (\tau_{ij}) \tag{2}$$

where the bar over the variables denotes the filtered, or resolved scale quantity and:

$$\overline{S_{ij}} = \frac{1}{2}(\partial_i \overline{u_j} + \partial_j \overline{u_i}) \tag{3}$$

$$\tau_{ij} = \rho(\overline{u_i u_j} - \overline{u_i}\,\overline{u_j}) \tag{4}$$

$\overline{S_{ij}}$ is the resolved scale strain rate tensor and $\tau_{ij}$ is the unknown sub-grid scale or residual stress tensor, representing the effects of the sub-grid scale motions on the resolved fields of the LES, which must be modelled or approximated using a so called sub-grid scale model.

### 2.2 Sub-grid scale modelling

Many different kinds of sub-grid scale models have been developed [3–5]and most of them make an eddy-viscosity assumption (Boussinesq's hypothesis) to model the sub-grid scale stress tensor as follows:

$$\tau_{ij} = -2\mu_t \overline{S_{ij}} + \frac{1}{3}\delta_{ij}\tau_{ll} \tag{5}$$

$\mu_t$ is called sub-grid scale eddy viscosity and eqn. (2) then becomes:

$$\partial_t(\rho\bar{u}_i) + \partial_j(\rho\overline{u_i u_j}) = -\partial_i \overline{P} + 2\partial_j[(\mu + \mu_t)\overline{S_{ij}}] \qquad (6)$$

It should be noted that a modified pressure, $\overline{P} = \overline{p} + \frac{1}{3}\tau_{ll}$, has been introduced and hence when the above equation is solved the pressure obtained is not just the static pressure only. The question now is how to determine the sub-grid scale eddy viscosity and the most basic model is the one originally proposed by Smagorinsky [2]:

$$\mu_t = \rho(C_S\overline{\Delta})^2 S \qquad S = (2\overline{S_{ij}}\,\overline{S_{ij}})^{\frac{1}{2}} \qquad \Delta = (\Delta x\Delta y\Delta z)^{\frac{1}{3}} \qquad (7)$$

$C_S$ is the so called Smagorinsky constant and typical value used for it is 0.1.

Despite increasing interest in developing more advanced sub-grid scale models this very simple model has been used widely and proved surprisingly successful although it has clear shortcomings such as that it is too dissipative (not good for transition simulation) and the Smagorinsky constant needs to be adjusted for different flows. An improvement on this simple SGS model was suggested by Germano *et al*. [6] – a dynamic sub-grid scale model, which allows the model constants $C_S$ to be determined locally in space and in time during the simulation.

### 2.3 Numerical methods

The finite volume method is the most popular numerical method used in fluid flow simulation and most of LES studies have been carried out using this method. A brief discussion on many important numerical issues will be presented in this section.

#### 2.3.1 Filtering

When the finite volume method is used there is no need to explicitly filter the instantaneous Navier-Stokes equations since the governing equations can be regarded as implicitly filtered as mentioned in section 2.1.The velocity components at the corresponding grid points are interpreted as the volume average. Any small scale (smaller than the mesh or control volume) motions are averaged out and have to be accounted for by a sub-grid scale model. However, note that it is impossible in this case to discuss the convergence properties (grid independent solution) of the LES equations because with every mesh refinement, more small scale eddies are resolved and strict convergence is only achieved in the limit of the so called Direct Numerical Simulation (DNS).

#### 2.3.2 Spatial and temporal discretization

The most popular spatial discretization scheme used in LES is the second-order central differencing duo to its non-dissipative and conservative properties (not only mass and momentum but also kinetic energy conserving), which are essential for LES. This is the reason why usually first- and second-order upwind schemes or any upwind-biased schemes are not used in LES since they produce too much numerical dissipation. While higher-order numerical schemes, generally speaking, are desirable and can be applied fairly easily in simple geometries, their use in complex configurations is rather difficult. In addition, it is difficult, at least for incompressible flows, to construct high-order energy conserving schemes. Hence it is likely that with increasing applications of LES to flows of engineering interest in complex geometries the second-order central differencing scheme is still the most popular choice.

As for the temporal discretization (time advancement), implicit schemes allow larger time steps to be used. However, they are more expensive because at each time step non-linear equations have to be solved. Furthermore, large time steps are unlikely to be used in LES in order to resolve certain time scales for accurate simulations of turbulence. Hence, explicit schemes seem to be more suitable for LES than implicit schemes and most researchers in LES use explicit schemes such as the second-order Adams–Bashforth scheme. Since the time steps are usually small in LES so that it is not essential to use higher-order schemes either.

### 2.3.3 Inflow boundary conditions

Most boundary conditions used in LES are fairly standard and similar to those used in the RANS approach but specifying inflow boundary conditions accurately for LES proves to be very difficult. This is because in LES of turbulent flow at inflow boundary, unlike the RANS computations where only time-averaged information is required that can be usually specified according to experimental data, three components of instantaneous velocity need to be specified at each time step, which are almost impossible to be obtained from experimental data. Hence normally boundary conditions in LES at inflow boundary have to be generated numerically which usually lack physical flow properties. For example, the simplest way is to specify the mean flow velocity profile (usually obtained experimentally) plus some random perturbations. However, random disturbances are nothing like real turbulence since they have no correlations; neither in space nor in time. Therefore, they decay rapidly and it takes usually a long distance downstream from the inflow boundary for a desired realistic turbulence to develop, and in some cases the use of random noise at the inlet does not develop turbulence at all. On the other hand one can use the so-called precursor simulation technique, which is basically to perform another simulation and store the data as the input for the required simulation. This can generate the most realistic turbulence information at inflow boundary but it is far too expensive. Many efforts have been made to develop a method which can generate numerically three instantaneous inflow velocity components in such a way that they have all the desired turbulence properties. However, so far there are methods developed which can generate inflow turbulence with certain properties but no methods available yet to generate inflow turbulence with all the desired characteristics such as intensity, shear stresses, length scales and power spectrum [7].

### 2.3.4 Near wall modelling

LES has been applied more and more, in recent years, to study practical engineering turbulent flows in complex geometries at higher Reynolds number. However, for high Reynolds number wall-bounded flows the cost of LES that resolves all the important eddies in the near wall region (the wall-layer structures) is far too high. Therefore methods to bypass the near wall region are required to perform high-Reynolds-number LES at a reasonable cost. Several methods have been developed to "model" the near wall region flow rather than resolve it directly using very fine mesh and more details can be found in a review paper by Piomelli and Balaras [8].

# 3 Applications of LES to study transitional bubble

This section presents some LES studies of transition in separated-reattached flows and tries to summarise the current understanding of the transition process, focusing on several important flow phenomena associated with the transition process.

## 3.1 Transition mechanism

Many studies have revealed that in the absence of any finite magnitude environmental disturbances, transition in the separated shear layer of a separation bubble is dominantly initiated through the inviscid Kelvin-Helmholtz (KH) instability mechanism. This mode of instability closely resembles that of the planar free-shear layer in mixing layers and jets [9]. The LES study of Yang and Voke [10]revealed a primary 2D instability of a separated shear layer (induced by a smooth leading edge) via the KH mechanism. A similar mechanism was also observed by Abdalla and Yang [11] in their LES studies of a separation bubble over a sharp leading edge. The LES study by Roberts and Yaras [12] demonstrated that transition of a separated shear layer through the KH instability does not eliminate the existence of a so called Tollmien-Schlichting (TS) instability(a viscous instability typically associated with attached flow boundary layer transition) in the inner part of the flow where the roll up of shear layer into vortical structures occurred at the dominant TS frequency. They emphasized the possibility of an interaction between the TS and KH instability modes. Several other studies have shown that KH instability plays a dominant role in the transition process of separation bubbles. A number of experimental studies have also suggested that the TS instability mechanism plays a significant role in a transitional separation bubble [13–15].

The next stage of the transition process after the dominant primary KH instability is less well understood. In planar free shear layers, the primary spanwise vortices generated by the KH instability are known to undergo an instability leading to the vortex pairing phenomenon [9, 16, 17]. This pairing of vortices is regarded as the governing secondary mechanism associated with growth of planar free shear layers. A similar vortex pairing phenomenon has also been reported in separated shear layer studies [18] but Abdalla and Yang [11] demonstrated that transformation of 2D KH rolls into 3D structures occurs via a slightly different secondary instability known as helical instability associated with helical pairing of vortices. Fig. 1 shows the evolution of 2D KH rolls into 3D Lambda-shaped vortices in a transitional bubble formed on a flat plate with a sharp leading edge [19].

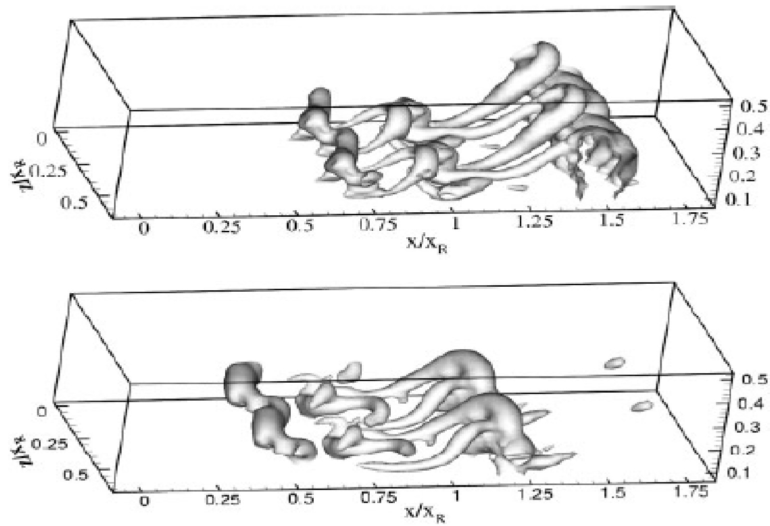

Figure 1: Low-pressure iso-surfaces showing the evolution of 2D KH rolls into 3D Lambda-shaped vortices.

In summary, the transition process in separated-reattached flow generally consists of the following stages:

1) a primary 2D instability (mostly KH instability),
2) a secondary instability leading to significant 3D motions and,
3) a breakdown stage where fully turbulent flow emerges.

Another key parameter influencing the transition process of a separated boundary layer and its following reattachment is free-stream turbulence (FST). Experimental studies have shown that FST increases the shear-layer entrainment rates, decreases the mean reattachment length and results in an earlier transition to turbulence in separated boundary layer. Yang and Abdalla [19, 28] performed LES studies of separated boundary layer transition under 2% FST. They reported a 14% reduction of the mean bubble length and an earlier breakdown of the free shear layer compared with the zero FST case. At 2% FST, 2D KH rolls were not as apparent as in the case with zero FST, but still coherent 2D structures in the early part of the bubble were observable. Lambda-shaped vortices could hardly be identified and streamwise structures were enlarged in the spanwise direction and shortened in the streamwise direction compared with the no FST case. It was concluded that in the presence of 2% FST the primary instability of the free shear layer was still the same as in the zero FST case (KH instability mechanism) but secondary instability was different and needed to be further investigated.

### 3.2 Shedding phenomenon

A key feature of separated-reattached flows is vortex shedding associated with different unsteady flow phenomena of the free shear layer at different frequencies. In a steady laminar separation bubble one can define a reattachment point or line where the skin friction is zero. In transitional and turbulent separation bubbles however, the instantaneous flow field is highly unsteady around the ‘mean’ reattachment point and the notion of a reattachment ‘point’ is misleading as it continuously varies with the time. In this case, it is possible that several small bubbles or vortices are formed and then shed afterwards, leading to a vortex shedding phenomenon.

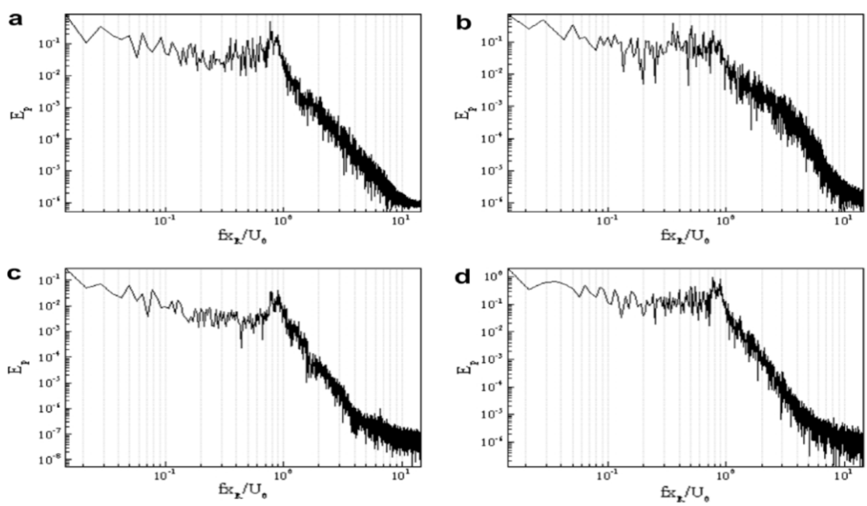


Figure 2: Pressure spectra at x/l=0.75 and four vertical locations: y/l=0.01 (a), y/l=0.05 (b), y/l=0.13 (c), y/l=0.2 (d).

Fig. 2 shows pressure spectra at several different locations in a separated boundary layer transition [19] and a peak frequency band at about 0.8–0.9 $U_0/l$ can be clearly seen ($U_0$ is the free stream velocity and $l$ is the mean bubble length). This peak frequency band was also observed in several experimental studies of separated-reattached flow over a plate with a sharp leading edge at high Reynolds number [20–22]. This peak frequency band was stated to be the characteristic frequency of the large vortices shedding from the free shear layer of the bubble. Furthermore, a low frequency peak (0.12 $U_0/l$) was also reported in those experimental studies near the separation line. This low frequency peak was not clearly understood and was suggested as related to the large scale shrinkage and enlargement of the bubble. A low frequency peak (0.125-0.2 $U_0/l$) was also observed in the LES study by Yang and Voke [10] and they suggested

 
www.witpress.com, ISSN 1743-355X (on-line)

that this was associated with large shrinkage of the bubble caused by a big vortex shedding at a lower frequency as shown in Fig. 3. However, this low frequency peak was not observed in some other separated boundary layer transition studies [11, 19]. Abdalla and Yang [11], in their LES of a transitional bubble over a flat plate with a sharp leading edge, showed a characteristic frequency in the range 0.7-0.875 $U_0/l$ along with some less dominant modes between 0.3-0.6 $U_0/l$. They inferred that this slightly lower frequency content may be related to pairing of vortices as a similar range of frequency had been reported for the pairing phenomenon behind a backward facing step but no low frequency peak as mentioned above was observed. Yang and Abdalla [19] studied the same problem with 2% free-stream turbulence and reported a peak frequency band at about 0.8–0.9 $U_0/l$, in close agreement with the characteristic frequencies already measured in previous studies but again no low frequency peak was observed. Those results indicate that this low frequency mode in separated–reattached flows may only appear in the case of turbulent separation as suggested earlier by Cherry *et al*. [21] but further study is needed to clarify this.

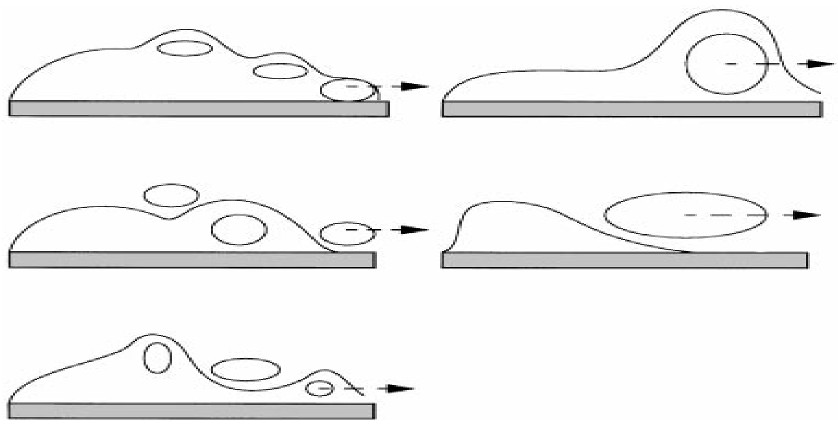

Figure 3: Two different topological structures of a separation bubble associated with the onset of vortex shedding: left, normal shedding; right, low frequency shedding.

### 3.3 Coherent structures

Large-scale structures (large-scale organised motions), usually called coherent structures (CS), have been revealed in many experimental studies to dominate the entrainment and mixing phenomena in free shear flows [23]. It is important to understand the physics of coherent structures so that a better insight into turbulence phenomena can be obtained (such as entrainment and mixing, heat and mass transfer, drag and aerodynamic noise generation etc.). However, despite considerable usage in the literature it seems that an approved definition for coherent structures does not yet exist. Cantwell [24] describes coherent structures as spatially coherent and temporally evolving vortical structures.

*Kelvin-Helmholtz (KH) rolls*, *Streaks*, *Hairpin vortices (also called Lambda-shaped vortices) and Ribs* are some of the common large-scale flow structures which are referred to as coherent structures in the literature and as shown previously in Fig. 1, the evolution of KH rolls into Lambda-shaped vortices in a separated boundary layer transition. Streaky structures are characterized with narrow regions of low velocity fluid stretched in the streamwise direction [25, 26]. Streamwise vortices are vortical structures which are predominantly oriented in the streamwise direction, although they may be bent and make an angle with the streamwise direction. Spanwise vortices are referred to as those primarily oriented in the spanwise direction such as KH rolls. Hairpin vortices (Lambda-shaped vortices) are those with two legs of quasi-streamwise vortex pairs with opposite signs and a tip of spanwise vorticity.

Coherent structures usually depend on flow geometry, flow condition, and location with respect to solid surfaces. Large-scale spanwise vortices in plane mixing layers, Lambda-shaped vortices and low-speed streaks in transitional and turbulent boundary layers and counter-rotating vortices in wakes are the dominant structures controlling the flow dynamics. Vortical structures in separated shear layers grow, merge and shed periodically from the reattachment region. KH rolls and Lambda-shaped vortices have been observed in separated layer transition and the transition process is better understood by studying the evolution of KH rolls into Lambda-shaped vortices [10, 11, 27, 28]. It is believed that reorientation of vorticity in the streamwise direction is a key mechanism for the reattachment process as it provides enhanced momentum exchange in the wall-normal direction. Abdalla *et al*. [29], in a LES study of transitional separated-reattached flow over a surface mounted obstacle and a forward-facing step, demonstrated that the coherent structures such as the Lambda-shaped and rib-like vortices, which are often associated with a flat plate boundary layer and also found in the separated-reattached flow, are not common in the separated-reattached flow over obstacles and forward-facing steps.

## 4 Conclusions

The present paper has presented briefly LES formalism and reviewed some of its applications to study transition process in separated-reattached flow, focusing on the current understanding of physics of the transition process. Several important issues associated with LES have been discussed. Although significant progress has been made towards a better understanding of the transition process in separated-reattached flows our current understanding is far from complete, and there are still many areas where further investigation is needed. According to the author the following issues/areas are particularly important and future research should be focused on:

- numerical methods to generate realistic turbulence at inflow for LES.
- advanced sub-grid scale models for LES of high Reynolds number engineering flow in complex geometry.

- secondary instability and later stage breakdown to turbulence in a transitional bubble.
- effect of high free-stream turbulence on transition in separated-reattached flow.
- transition control, crucial to practical engineering applications.

## References


[1] Langtry, R.B. & Menter, F.R., Transition modelling for general CFD applications in aeronautics. *AIAA 2005-522*, Reno, Nevada, 2005.

[2] Smagorinsky, J., General circulation experiments with the primitive equations: I – the basic experiment. *Monthly Weather Review*, **91**, pp. 99-164, 1963.

[3] Lesieur, M. & Metais, O., New trends in large eddy simulations of turbulence. *Annual Review of Fluid Mechanic,* **28**, pp. 45-82, 1996.

[4] Sagaut, P., *Large Eddy Simulation for Incompressible Flows, an Introduction, Springer*, 2nd edition, 2003.

[5] Kajishima, T. & Nomachi, T., One-equation sub-grid scale model using dynamic procedure for the energy production. *Transaction of ASME*, **73**, pp. 368-373, 2006.

[6] Germano, P., Piomelli, U., Moin, P. & Cabot, W.H., A dynamic sub-grid scale eddy viscosity model. *Physics of Fluids,* **3(7)**, pp. 1760-1765, 1991.

[7] Veloudis, I., Yang, Z., McGuirk, J.J., Page, G.J. & Spencer, A., Novel implementation and assessment of a digital filter based approach for the generation of large-eddy simulation inlet conditions. *Journal of Flow, Turbulence and Combustion*, **79**, pp. 1-24, 2007.

[8] Piomelli, U. & Balaras, E., Wall-layer models for large-eddy simulation. *Annual Review of Fluid Mechanics,* **34**, pp. 349–374, 2002.

[9] Ho, C.M. & Huerre, P., Perturbed free shear layers, *Annual Review of Fluid Mechanics*, **16**, pp. 365-424, 1984.

[10] Yang, Z. & Voke, P.R., Large-eddy simulation of boundary layer separation and transition at a change of surface curvature. *J. Fluid Mech.*, **439**, pp. 305–333, 2001.

[11] Abdalla, I.E. & Yang, Z., Numerical study of the instability mechanism in transitional separating-reattaching flow. *International Journal of Heat and Fluid Flow*, **25**, pp. 593-605, 2004.

[12] Roberts, S.K. & Yaras, M.I., Large-eddy simulation of transition in a separation bubble. *ASME J. Fluids Eng.*, **128**, pp. 232-238, 2006.

[13] Lang, M., Rist, U. & Wagner, S., Investigations on controlled transition development in a laminar separation bubble by means of LDA and PIV. *Exp. Fluids*, **36**, pp. 43–52, 2004.

[14] Roberts, S.K. & Yaras, M.I., Effects of periodic unsteadiness, free-stream turbulence and flow Reynolds number on separation-bubble transition. *ASME-GT2003-38626*, 2003.

[15] R.J. Volino, R.J. & Bohl, D.G., Separated flow transition mechanism and prediction with high and low free stream turbulence under low pressure turbine conditions.*ASME-GT2004-53360*, 2004.
[16] Huang, L.S. & Ho, C.M., Small-scale transition in a plane mixing layer. *J. Fluid Mech*., **210**, pp. 475–500, 1990.
[17] Winant, C.D. & Browand, F.K., Vortex pairing: the mechanism of turbulent mixing-layer growth at moderate Reynolds number. *J. Fluid Mech*., **63**, pp. 237-255, 1974.
[18] Malkiel, E & Mayle, R.E., Transition in a separation bubble. *ASME J. Turbomachinery*, **118**, pp. 752–759, 1996.
[19] Yang, Z & Abdalla, I.E., Effects of free-stream turbulence on a transitional separated-reattached flow over a flat plate with a sharp leading edge. *Int. J. Heat Fluid Flow*, **30**,pp. 1026-1035, 2009.
[20] Kiya, M. and Sasaki, K., Structure of a turbulent separation bubble. *J. Fluid Mech*., **137**, pp. 83-113, 1983.
[21] Cherry, N.J., Hillier, R. & Latour, M.E.M.P., Unsteady measurements in a separating and reattaching flow. *J. Fluid Mech*.,**144**, pp. 13–46, 1984.
[22] Kiya, M. & Sasaki, K., Structure of large-scale vortices and unsteady reverse flow in the reattaching zone of a turbulent separation bubble. *J. Fluid Mech*., **154**, pp. 463-491, 1985.
[23] Hussain, A.K.M.F., Coherent structures and turbulence. *J Fluid Mech*., **173**, pp. 303-356, 1986.
[24] Cantwell, B.J., Organised motion in turbulent flow. *Annual Review of Fluid Mechanics*, **13**, pp. 457–515, 1981.
[25] Kim, H.T., Kline, S.J. & Reynolds, W.C., The production of turbulence near a smooth wall in a turbulent boundary layer. *J. Fluid Mech*., **50**, pp. 133–160, 1971.
[26] Smith, C.R. & Metzler, S.P., The characteristics of low-speed streaks in the near-wall region of a turbulent boundary layer. *J. Fluid Mech*., **129**, pp. 27-54, 1983.
[27] Yang, Z., Large-scale structures at various stages of separated boundary layer transition. *Int. J. Numer. Meth. Fluid*, **40**, pp. 723-733, 2002.
[28] Yang, Z. & Abdalla, I.E., Effects of free-stream turbulence on large-scale coherent structures of separated boundary layer transition. *Int. J. Numer. Meth. Fluid*, **49**, pp. 331-348, 2005.
[29] Abdalla, I.E., Yang, Z. & Cook, M., Computational analysis and flow structure of a transitional separated-reattached flow over a surface mounted obstacle and a forward-facing step. *International Journal of Computational Fluid Dynamics*, **23**, pp. 25-57, 2009.