An improved algorithm for thermal dynamic simulation of walls using Z-transform coefficients

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Abstract

The Transfer Function Method (TFM), recommended by American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE), is one of the most modern tools available to solve heat transfer problems in building envelopes and environments. TFM utilises Z-transform to solve the equations system that describes the heat transfer in a multi-layered wall. Due to an analogy with an electric circuit, it is possible to write the equations system in a matrix suitable to be solved by computer. Authors carried out an analysis on an historical building placed in the south of Italy to test the reliability and the quality of the thermal dynamic simulation using TFM. The analysis is performed using some control systems tools like Bode plots, step response, root and poles location. Results show clearly that, for very massive building, a simply application of TFM fails but if the poles and the residuals are re-ordered in a different position, it will be possible to solve the numerical problem. Furthermore, the analysis shows that the choice of the sampling period is a significant factor to obtain a reliable simulation.

1 Introduction

Many of the most used tools for thermal building simulation, like TRNSYS [1], are based on the use of TFM and Z-Transform set. In previous papers [2] the authors showed that these software, developed contemplating USA building typologies, have some trouble when applied to the Mediterranean buildings, characterised by high value of thermal inertia. Furthermore, most of these software do not permit to change and to fit the basic parameters of calculus.

Transactions on Modelling and Simulation vol 33, © 2003 WIT Press, www.witpress.com, ISSN 1743-355X

Authors developed the software THELDA 2000 (Thermal Elements Dynamic Analysis) [3], based on a new TFM algorithm, to determine the Z-transform coefficients able to perform a thermal building simulation. THELDA 2000 is capable to re-order the position of the poles and residuals, to change the sampling period, and to fix the preferred number of poles and coefficient used for the simulation.

2 The non–steady state heat problems in multilayered walls

The non–steady–state heat problems in multilayered walls are described by a set of differential equations which can be solved by mathematic operators reaching the matrix [4]:

$$\begin{vmatrix} T_e \\ Q_e \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \times \begin{vmatrix} T_i \\ Q_i \end{vmatrix} \tag{1}$$

in which T and Q are, in this case, Laplace Transform (LT) of the temperatures t_i and t_e and of the heat fluxes q_i and q_e in correspondence of the inside and outside surfaces of the wall, while A, B, C and D are the coefficients of the wall transmission matrix reached through the product of transmission matrixes, of each n layers forming the wall:

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \prod_{m=1}^{n} \begin{vmatrix} a_m & b_m \\ c_m & d_m \end{vmatrix} \tag{2}$$

a_m b_m c_m d_m are respectively:

$$a_m = \cosh\left(L\sqrt{\frac{s}{\beta}}\right); b_m = \frac{\sinh\left(L\sqrt{\frac{s}{\beta}}\right)}{\lambda\sqrt{\frac{s}{\beta}}} \tag{3}$$

$$c_m = \lambda\sqrt{\frac{s}{\beta}} \cdot \sinh\left(L\sqrt{\frac{s}{\beta}}\right) \tag{4}$$

$$d_m = a_m \ ; \ \beta = \frac{\lambda}{\rho C_p} \tag{5}$$

and the used symbols have, for each m layer, the following meaning:

λ thermal conductivity $\left[\frac{W}{mK}\right]$

ρ density $\left[\frac{kg}{m^3}\right]$

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C_P specific heat $\left[\dfrac{kJ}{kgK}\right]$

L thickness $[m]$

[5].

The system can be described using the Zeta Transform (ZT) at the LT place. For a time-continuous $f(t)$, with sampling period T, LT is:

$$f(0)+f(T)e^{-sT}+f(2T)e^{-2sT}+\dots \tag{6}$$

Supposing $e^{sT}=z$ we obtain:

$$f(0)z^{0}+f(\Delta)z^{-1}+f(2\Delta)z^{-2z}+\dots, \tag{7}$$

that is the ZT of the function $f(t)$.

If a system is solicited by an input signal whose ZT is $U(z)$ and the output is $Y(z)$, the link we have to determine is:

$$\frac{Y(z)}{U(z)} \tag{8}$$

We briefly introduce some dynamical systems basilar concepts that are useful in the sequel.

2.1 Transfer function

Let $u(t)$ be the input signal and $y(t)$ be the output signal. Let $U(z)=Z[u(t)]$ and $Y(z)=Z[y(t)]$ be the corresponding z-transformed signals. Then, the transfer function of the linear model for the system is given by $G(z)=\dfrac{Z[y(t)]}{Z[u(t)]}=\dfrac{n(z)}{d(z)}$, where $n(z)$ is the a polynomial called *numerator* and $d(z)$ is a polynomial called *denominator.*

2.2 Poles, zeros and residuals

The roots of the numerator (z_i such that $n(z_i)=0$) are called *zeros* and the roots of the denominator (p_i such that $d(p_i)=0$) are called *poles.* Clearly the number of poles n_p denotes the *order* of the polynomial $d(z)=(z-p_1)(z-p_2)\dots(z-p_{np})$. Residuals are directly

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linked to the poles that, when calculated, are not ordered respect the absolute value [6].

2.3 Sampling period

The sampling period T is related to the hours of data-collection. If the temperatures are collected every hour then the sampling time is $T = 1$ hour, and so on. For different sampling period we get different transfer function in the *z*-domain. A signal that is sampled every T period is said a discrete-time signal.

2.4 Transfer function coefficient

From a generic transfer function $\dfrac{Y(z)}{U(z)}$ with *n* poles and zeros we can evaluate up to *n* transfer function coefficients. These coefficients permit to utilise a set of recursive formulas to get the solution of the equation system.

3 The software THELDA 2000

It is possible to use the above relations to write the thermal balance of a room composed by a group of outward walls, inward walls and indoor air.

Authors have developed a software called TH.EL.D.A 2000 (Thermal Elements Dynamic Analysis) that is able to simulate the thermal behaviour of a single thermal zone using the Z–Transform and the TFM. This tool presents characteristics of flexibility and innovation with respect to the software programs mostly diffused for the analysis of the dynamic thermal behaviour of building multilayered structures, because it allows one to modify all of the input parameters of the calculus as number of poles, number of coefficients, sampling period. The user can introduce different levels of precision in the individualisation of the transfer coefficients, adapting these parameters to the analysed room.

Furthermore, we have implemented an innovative algorithm that permits to re-order the residuals for every transfer function calculated by THELDA 2000. Re-location of residuals implies a re-location of poles. We tidy up the residuals by a hierarchy founded on their absolute value. We have observed that is possible to eliminate the residuals and the correspondent poles when residuals present values that have an order of magnitude lower than 10^{-10}. Results show clearly as the possibility to change the calculus parameters has a strong influence in the carried out simulations.

Authors have utilised TH.EL.D.A. 2000 to perform a simulation on a single thermal zone belonging to a very massive ancient building in the city of Marsala, in the south of Italy. The features of this building give some troubles carrying out a thermal simulation using other software [7]; consequently so we have decided to perform a deep analysis over the transfer functions of this building.

The room used for the analysis is composed by six walls, one bordering with the external environment. Every wall is composed by a tufe layer in the middle and inside-outside plaster. The thickness of the walls is 90 cm. Such features are typical of the Mediterranean building historical heritage and they are not an exception. Concerning the response of the system, it represents a matrix in which the first six columns represents the hourly behaviour of the temperature on the inner surface of each wall and the last column represents the hourly temperature of the air (free floating simulations).

The input signal is the air–sol temperature on the external surface of the first wall calculated for the city of Marsala in July (orientation south-east). Air-sol temperature is a fictitious temperature used to join the effects of conductive and radiative thermal phenomena [8]. Moreover, other input signals are radiative heat flux due to a window and the air ventilation. The input signals are periodic and repeat themselves every 24 hours.

In order to evaluate the accuracy of the calculations carried out by using a given set of ZT coefficients, we have to compare them with a reference response coming from a procedure having a different mathematical background and even able to give the time continuous response of the system. So, the calculated temperatures have been compared to those ones obtained by Fourier's analysis in periodic steady state conditions. Naturally, to operate this comparison, in THELDA 2000 simulations we have repeated the input signal 200 times. Under such conditions after transient the room has reached the stabilized thermal behaviour.

Table 1. Input signals of the systems.

	Radiative flux (W/m^3)	External air temperature (°C)	Air-sol temperature (°C)
Hour			
1	0	23.74	23.74
2	0	23.45	23.45
3	0	23.22	23.22
4	0	23.05	23.05
5	2.01	22.99	23.04
6	313.88	23.11	30.02
7	564.69	23.39	35.81
8	715.68	23.91	39.56
9	777.60	24.65	41.50
10	761.75	25.51	41.76
11	677.00	26.49	40.56
12	551.22	27.40	38.24
13	356.27	28.09	34.47
14	311.92	28.55	34.00
15	275.32	28.72	33.51
16	230.55	28.55	32.56
17	172.35	28.15	31.15

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20	94.01	27.52	29.15
21	1.22	26.77	26.79
22	0	26.03	26.03
23	0	25.40	25.40
24	0	24.82	24.82

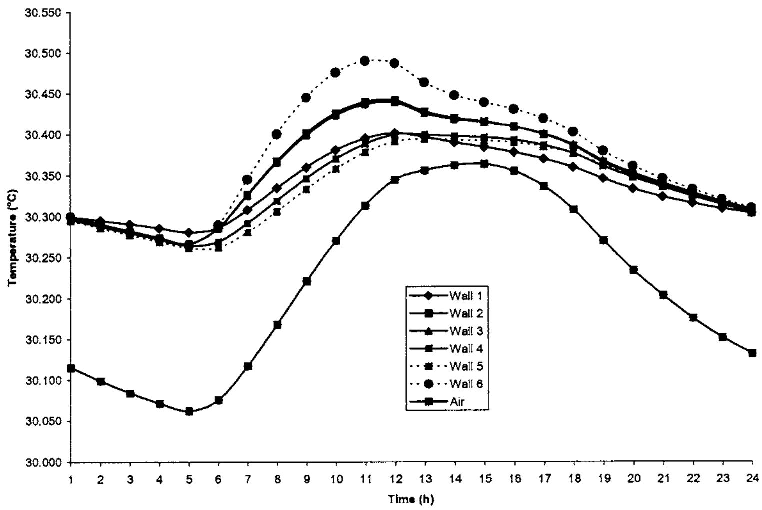

Figure 1: plots of Fourier period steady-state response

4 Model analysis

In order to select the *best model* representing the real system we carried out a comparison between the simulation data obtained from Fourier steady state algorithm and those one obtained from TFM. Referring to the first method, we have interpolated the discrete input data using into the generic sampling interval t_n, t_{n+1} the following rule:

$$x'(t) = at^3 + bt^2 + ct + d \tag{9}$$

for each interval the coefficients a, b, c, d have to be evaluated according to the following constraints:

- the $x'(t)$ assumes the values $x(t_n)$ to $x(t_{n+1})$
- the tangent to the curve at the point $[t_n, x(t_n)]$ forms equal angles with the segments joining $x(t_n)$ to $x(t_{n-1})$ and $x(t_n)$ to $x(t_{n+1})$.

This kind of spline is able to smooth out any sharpness and even to follow a linear time variation. We used the time continuous input functions to apply the Fourier analysis and solve the system for 200 harmonic components. Finally we recombine all of systems sinusoidal outputs in a non–sinusoidal output obtaining the solution showed in Figure 1 and Table 2.

5 Comments

The model obtained using sampling period T = 1, 10 poles, 8 coefficients and without re–ordering of the residuals is completely wrong with order of magnitude of output temperatures 10^5. Clearly showed from the profiles of temperatures plotted in Figure 2, the model obtained re–ordering residuals is not correct. In particular, medium values are close to the right value but the dynamics of the system is not well represented.

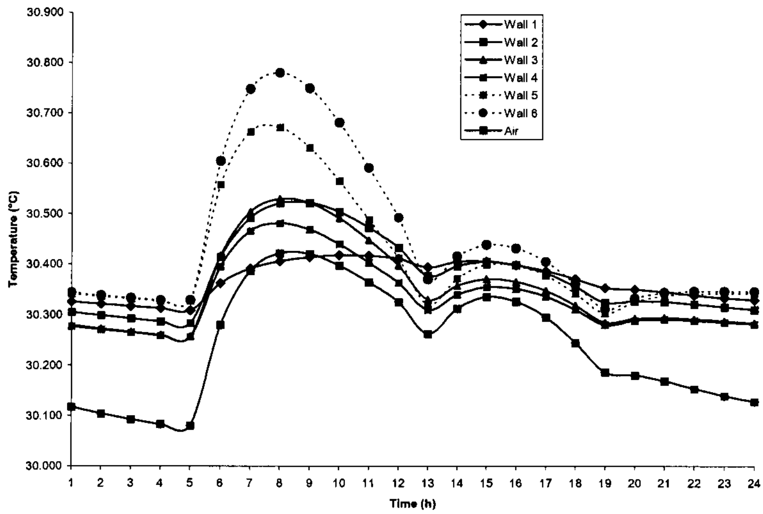

Figure 2: output temperatures obtained with transfer functions using 10 poles, 8 coefficients, sampling period T = 1 hour and re-ordered residuals.

Increasing the sampling period, T = 2 hours, without re–ordering of the residuals, output temperatures have order of magnitude 10^4, and as we can see in Figure 3 the simulation using 10 poles, 8 coefficients and with re–ordered residuals shows a better steady state representation, even if still is not correct.

Again, with sampling period T = 3 hours, without re–ordering of the residuals, using 10 poles and 8 coefficients there are some numerical problems that does not permit to evaluate as acceptable the results. Re–ordering the residuals we can have results very similar to those one obtained with T = 2 hours, plotted in Figure 4.

With sampling period T = 1 hour, increasing the number of poles up to 20 and coefficients up to 16 the behaviour is getting worse: without re–ordering residuals results have order of magnitude 10^5 and with re–ordering model is not capable to give the correct medium value too.

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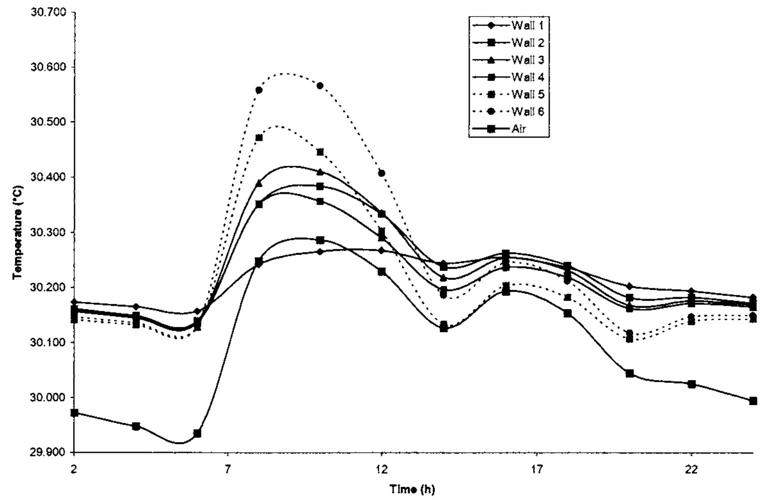

Figure 3: output temperatures obtained with transfer functions using 10 poles, 8 coefficients, sampling period $T = 2$ hour and re-ordered residuals.

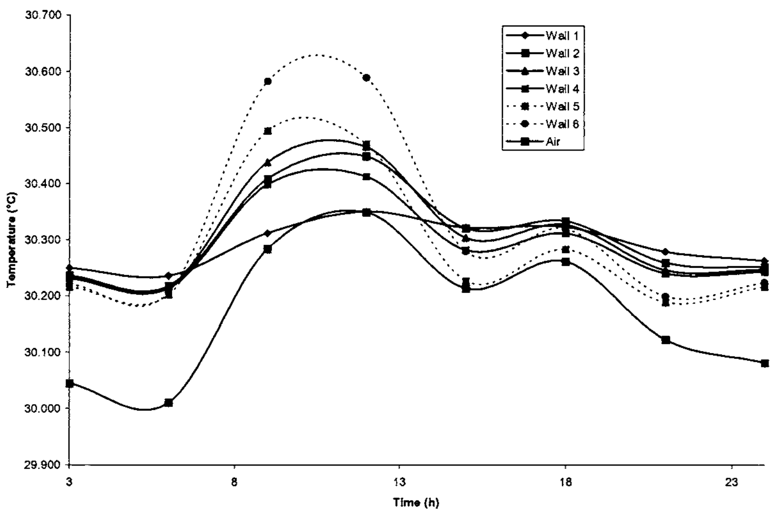

Figure 4: output temperatures obtained with transfer functions using 10 poles, 8 coefficients, sampling period $T = 3$ hour and re-ordered residuals.

Simulation carried out with sampling period $T = 2$ hours, with re–ordering of the residuals, using 20 poles and 16 coefficients gives us a good results referring to medium values and dynamics behaviour too, as drowned in Figure 5. The same above simulation but without re–ordering of the residuals fails some medium values and it is not correct. Finally, increasing the sampling period to $T = 3$ hours and using 20 poles and 16 coefficients, with or without re–ordering of residuals, the simulation gives us a very good results in terms of medium values and dynamics behaviour.

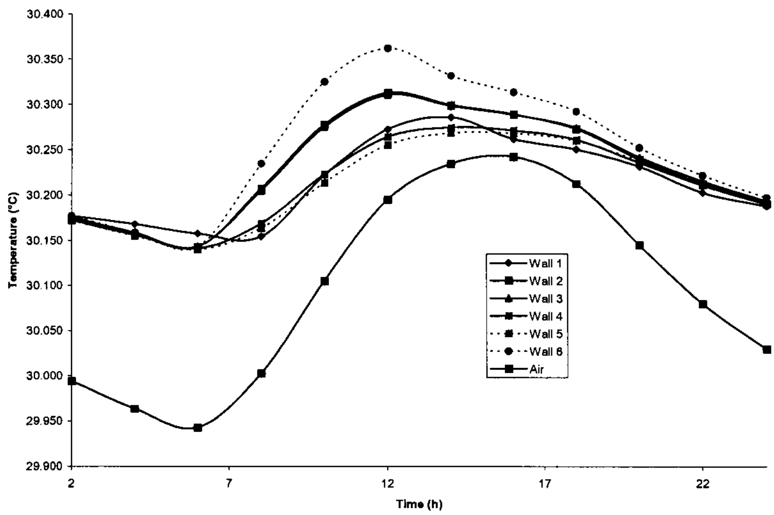

Figure 5: output temperatures obtained with transfer functions using 20 poles, 16 coefficients, sampling period $T = 2$ hour and re-ordered residuals.

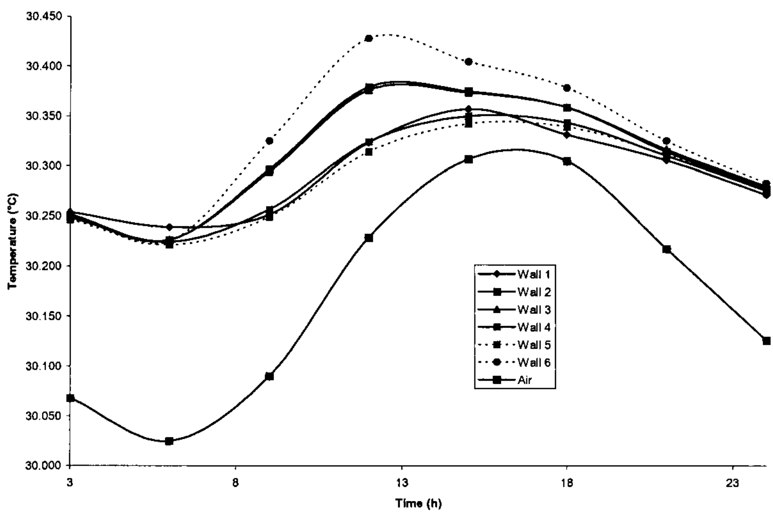

Figure 6: output temperatures obtained with transfer functions using 20 poles, 16 coefficients, sampling period $T = 2$ hour and residuals not re–ordered.

Conclusions

Thermal mass in buildings can be used to avoid dealing with instantaneous high cooling loads, to reduce by up to 20% energetic consumptions in commercial buildings ([9], [10], [11]) and to attenuate indoor temperature swings. In climates where cooling is of primary concern, thermal mass can reduce energy consumption, provided that the building is unused in the evening hours and the stored heat can be dissipated during this idle period. The results from the analysis developed on building typologies characterised by high values of thermal inertia show that optimal choice of initial parameters of calculation is very important. Moreover, the above mentioned typologies need particular attention in choosing number of poles and sampling period to be used in the Z-transform method. The case study developed showed that in massive buildings available software presents numerical problems. The presented analysis showed effectiveness of the changing in sampling period, the choice of number of used poles and the re-ordering of the residuals to get a suitable model.

The objective of current research and future works concerns on optimal selection of the residuals and poles by reordering the residuals and the selection of the appropriate sampling period.

References

[1] Trnsys 15 user guide sez. 5.1-2

[2] C. Giaconia, A. Orioli; "*On the reliability of conduction transfer function coefficients of walls*"; Applied Thermal Engineering 20, 2000 Pergamon Press, Cambridge, U.K

[3] M. Beccali, M. Cellura, C. Giaconia, V. Lo Brano, A. Orioli; "*La trasmissione del calore in regime vario nelle pareti multistrato: il software TH.EL.D.A.*"; Condizionamento dell'Aria 5/2000–Sezione Ricerche

[4] Clark J. A. *"Energy simulation in building design"*- Butterworth-Heinemann, 1985

[5] Carslaw H. S., Jager J.C. "*Conduction of heat in solid",* xford University Press, 1959

[6] J. D'Azzo and C. Houpis, *'Linear Control System Analysis & Design : Conventional & Modern',* Mc-Graw Hill, (Burr Ridge:1995).

[7] C. Giaconia, A. Orioli; "*TRANSFER FUNCTION METHOD - Analisi dei metodi di calcolo dei coefficienti*"; La Termotecnica - Anno LIII n 2, Editrice BIAS, 1999

[8] Yunus A. Cengel, *"termodinamica e trasmissione del calore"* McGraw-Hill 1997

[9] M. Brown, *'Optimization of thermal mass in commercial building applications",* ASME Journal of Solar Energy Engineering, Vol. 112, pp. 273-279.

[10] Lara Pasqualetto, Ramu Zmeureanu, Paul Fazio; "*A Case Study Of Validation Of An Energy Analysis Program: Micro-Doe2.1 E*"; Building and Environment; 1998 Pergamon Press.

[11] N. K. Bansal, M. S. Bhandari; "*Comparison Of The Periodic Solution Method With Trnsys And Suncode For Thermal Building Simulation*"; Solar Energy; 1996 Elsevier Science.