An empirical investigation of the short-term relationship between interest rate risk and credit risk

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Abstract

Empirical results from several studies indicate that changes in interest rates and changes in credit spreads are negatively related in the short run. These findings are further investigated by examining the dependence structure between interest rate and credit risk factor changes that are computed from sovereign and corporate bond indices. Several copulas (Gaussian, Student t, BB1, and Frank copula) are calibrated and their goodness-of-fit is compared. No clear pattern of the dependence structure can be observed as it varies substantially with the duration and – concerning the credit risk factor changes – the rating of the obligors. The Student t copula's fit in terms of the AIC goodness-of-fit measure is superior to that of all other copulas. The null hypothesis of a specific copula being the true copula can be rejected for the Student t copula in the least cases. Additionally employing a likelihood-ratio test, the null hypothesis of a Gaussian copula can be rejected in favour of a Student t copula. The Gaussian copula seems to underestimate the probability of joint strong risk factor changes, while the Student t copula seems to overestimate it.
Keywords: copula, dependence, short-term, interest rate risk, credit risk, risk measurement.

1 Introduction

Credit spreads, the difference in yield between corporate and government bonds of similar maturity, reflect the credit risk associated with corporate bonds. The dependence of interest rate risk, i.e. the risk associated with changes of the interest rate term structure, and credit risk, i.e. the risk associated with rating-migrations or defaults of obligors, is an interesting topic to explore in the context

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of risk measurement, as it enables an improvement of the risk aggregation process.

Several empirical studies report that short-term *changes* in the level of interest rates are negatively related with *changes* of credit spreads: when interest rates rise, credit spreads tend to narrow and vice versa (Longstaff and Schwartz [10], Duffee [5], Neal et al. [12], Collin-Dufresne et al. [3], and Papageorgiou and Skinner [13] find a negative relationship for monthly changes, Leake [9] and Pape and Schlecker [14] for weekly changes and Leake [9] and Wagner et al. [18] for daily changes. However, Neal et al. [12] report a positive relationship for a long-term horizon (36 months) which is not supported by the theoretical literature) This is sometimes interpreted as a high level of interest rates being related to a low level of credit spreads. Such a relationship is in accordance with the widely used structural credit risk models based on the contingent claim approach by Merton [11] if one assumes that an increase in interest rates does not trigger a drop in the obligors' asset values. It is also consistent with business cycle developments, where in times of contraction the interest rates and the credit quality tend to decrease jointly, while in times of expansions they tend to increase jointly. This is associated with central banks' cyclical adaptation of the interest rates.

This paper focuses on the explicit dependence structure, or copula, for one-day interest rate and credit risk factor changes that are computed from sovereign and corporate bond indices. Copulas are rather a new instrument in risk management that have attracted a lot of attention recently. One advantage of copula-based approaches is that they allow for a separate modelling or calibration of the marginal distributions and the dependence structure (the copula). Copulas allow to combine arbitrary marginal distributions to a joint distribution.

The paper is structured as follows. Section 1 explains the computation of the risk factors. Section 2 gives a short introduction to copula-based approaches and section 3 contains the empirical investigation.

2 Data and computation of the risk factors

The data base includes daily sovereign and corporate bond indices from the iboxx € index family for Euro-denominated fixed-coupon bonds for the period from January 31st, 2000 to September 15th, 2006. For both sovereign and corporate bond indices, subindices with specific maturity bands are considered. These maturity bands are: (i) all maturities, (ii) 1Y to 3Y maturities, (iii) 3Y to 5Y maturities, (iv) 5Y to 7Y maturities, and (v) 7Y to 10Y maturities.

The corporate bond indices' constituents are further grouped according to their ratings. These ratings are: (i) all ratings, (ii) AAA-rated, (iii) AA-rated, (iv) A-rated, and (v) BBB-rated.

We are interested in the dependence structure of joint risk factor changes, specifically of joint interest rate and credit risk factor changes. We define the interest rate risk factor changes at day t as the log-returns of the sovereign bond indices, $i_{j,t}$, with maturity bands j

$$i_{j,t} = \ln P_{sov,j,t} - \ln P_{sov,j,t-1} \qquad \forall j \in \{1,...,5\}. \tag{1}$$

If there is an upward shift in the yield curve, we will observe negative interest rate risk factor changes, while the contrary holds for a downward shift of the yield curve. The credit risk factor changes at day t, $c_{j,k,t}$, are defined as the excess returns of corporate bonds over sovereign bonds for a given maturity band and a given rating

$$c_{j,k,t} = \ln P_{j,k,t} - \ln P_{j,k,t-1} - i_{j,t} \qquad \forall j \in \{1,...,5\},\ k \in \{1,...,5\}. \tag{2}$$

The corporate bond indices' log-returns are adjusted by subtracting the sovereign bond indices' log-returns such that the resulting risk factor changes $c_{j,k,t}$ only display the change in the value of the corporate bond indices that is due to a change in the credit quality of the constituents, assuming a similar composition of the sovereign and corporate bond indices with identical maturity band concerning duration, and assuming a constant risk-appetite of the market participants and constant premia for the corporate bonds' liquidity risk.

From these riskless returns and excess returns, we construct data pairs that consist of ($i_{j,t}$, $c_{j,k,t}$) for maturity bands j and rating classes k, resulting in 25 bivariate empirical sample pairs.

A closer look at the risk factor changes shows that they all are non-Gaussian. Using a Jarque-Bera test (Jarque and Bera [8]), the null hypothesis of normally distributed marginal distributions can be rejected at the 1% significance level in all cases (in fact they can be rejected even at the 0.012% significance level). Hence, a copula-based approach clearly seems preferable to the assumption of a multivariate Gaussian distribution for the data sample at hand.

For many of the risk factor changes, autocorrelation is detected. To adjust the data for autocorrelation, an AR(2)-model is fitted where the observed risk factor changes are modelled as

$$r_t = \beta_1 + \beta_2 r_{t-1} + \beta_3 r_{t-2} + \varepsilon_t \tag{3}$$

where r_t is either $i_{j,t}$ or $c_{j,k,t}$. The coefficients and their statistical significance are estimated using the standard OLS-estimates of a classical normal linear regression.(generally, in the context of estimating AR models, alternative estimation methods are preferred to the OLS-method, as the estimated standard errors of the coefficients are downward biased. However, the estimates of the coefficients are consistent and the bias reduces as the sample size increases) If the estimates of either β_2 or β_3 turn out not to be statistically significantly different from 0 at the 5% significance level, models of the type $r_t = \beta_1 + \beta_2 r_{t-1} + \varepsilon_t$ and $r_t = \beta_1 + \beta_3 r_{t-2} + \varepsilon_t$, respectively are estimated. If both β_2 and β_3 turn out not to be statistically significantly different from 0, $r_t = \beta_1 + \varepsilon_t$ is modelled, where $\widehat{\beta}_1 \equiv \bar{r}$. For 21 out of the 25 credit risk factor changes, AR(2) models are fitted. For only one of the interest rate risk factor changes, autocorrelation is detected. Here, an AR(1)-model is fitted.

Being interested only in the innovations that cannot be explained by lagged values of the observed returns, the autoregressions' residuals are used as the empirical return observations in what follows. (A part of the empirical analysis was also conducted for the unadjusted data. The results do not differ strongly from the results obtained from the autocorrelation-adjusted observations.) The 25 autocorrelation-adjusted data pairs include 1,727 observations each.

3 Copulas

This paper focuses on the investigation of the goodness-of-fit of selected copulas. The term copula was introduced by Sklar [17] in 1959 (a similar concept for modelling dependence structures of joint distributions was independently proposed by Höffding [7] some twenty years earlier).

Copulas are functions that combine or couple (univariate) marginal distributions to a multivariate joint distribution. Sklar's theorem (using a slightly different notation in the original article) states that a n-dimensional joint distribution function $F(x)$ evaluated at $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ may be expressed in terms of the joint distribution's copula C and its marginal distributions $F_1, F_2, \ldots, F_n$ as

$$F(\mathbf{x}) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)), \qquad \mathbf{x} \in \mathbf{R}^n . \tag{4}$$

The copula function C is itself a multivariate distribution with uniform marginal distributions on the interval $\mathbf{U}_1 = [0,1], C : \mathbf{U}_1^n \to \mathbf{U}_1$.

As far as the calibration of joint distribution functions from empirical data is concerned, copula-based approaches allow for a separate modelling of (i) the marginal distributions (in the present case the univariate distributions of the interest and credit risk factor changes) and (ii) the dependence structure (the copula).

We shall restrict our empirical investigation to some selected copulas from the family of elliptical and Archimedean copulas. These are: the Gaussian and Student t copula (elliptical copulas), the BB1 copula and its two special cases, the Clayton and the Gumbel copula, and the Frank copula (Archimedean copulas). Arbitrary marginal distributions that are combined by a copula to joint distributions are referred to as meta-distributions (meta-Gauss, meta-Student t, meta-BB1, etc.).

The above-mentioned copulas assign different probabilities to joint extreme observations. For example, a Student t copula assigns a higher probability to joint extreme observations than does a Gaussian copula. This is referred to as *positive tail dependence*.(formally, lower and upper tail dependence are defined as $\lambda_L = \lim_{\alpha\to 0} \mathrm{P}(u_1 \leq \alpha \mid u_2 \leq \alpha)$ and $\lambda_U = \lim_{\alpha\to 1} \mathrm{P}(u_1 > \alpha \mid u_2 > \alpha)$. Positive tail dependence is prevalent if λ_L and/or λ_U is positive) Some copulas assign different probabilities to joint extreme positive deviations from the median than to joint extreme negative deviations (*asymmetric tail dependence*).

Figure 1 displays contour plots of bivariate meta-Student t, meta-Clayton, meta-Gumbel, and meta-Frank distributions with standard normal marginal distributions with a Spearman's rho of 0.4 (top row) and 0.8 (bottom row). Additionally, for reasons of comparison, contours of a Gaussian distribution with identical marginal distributions and identical correlation are displayed. It can be seen that the Student t copula assigns a higher probability to joint extreme co-movements than does the Gaussian copula. Also the symmetric nature of the tail dependence can be identified visually. The Clayton copula's lower tail dependence and the Gumbel copula's upper tail dependence can also be identified well. The Clayton copula's lower tail dependence exceeds the Gumbel copula's upper tail dependence. The Frank copula assigns a lower probability to joint extreme co-movements than a Gaussian copula.

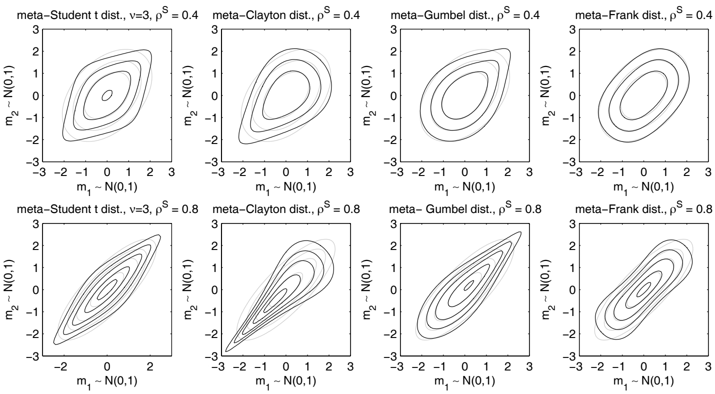

Figure 1: Contour plots of bivariate meta-Student t (parameter $\nu = 3$), meta-Clayton, meta-Gumbel, and meta-Frank distributions with standard normal margins an a Sperman's rho of 0.4 (top row) and 0.8 (bottom row).

Some copulas allow to model both positive and negative dependence in their 'standard' versions by assigning appropriate copula-parameters. Amongst these copulas are e.g. the Gaussian, Student t and Frank copula. Other bivariate copulas like e.g. the BB1 copula and its two special cases, the Clayton and Gumbel copula in their 'standard' version allow to model positive dependence only.(in fact, the Clayton copula may also be used in its standard version to model negative dependence if the copula parameter $\theta \in [-1, 0)$. Such a parameterisation is not further considered in this paper) *Copula rotation* allows to transform copulas such that they may be used to model negative dependence also. Further, copula rotation allows to transform (bivariate) copulas depending on whether and/or where the empirical data at hand requires the copula to display positive tail dependence.(we use rotated copulas C^{--}, C^{+-} and C^{-+} with densities $c^{--}(u_1, u_2) = c(1\text{-}u_1, 1\text{-}u_2)$, $c^{+-}(u_1, u_2) = c(u_1, 1\text{-}u_2)$, and$c^{-+}(u_1, u_2) = c(1\text{-}u_1, u_2)$. C^{--} is also referred to as 'survival copula'.)

To calibrate the copula parameters the *pseudo-log-likelihood method* (also referred to as CML – canonical maximum likelihood – or semiparametric method) is employed in this paper. Here, no assumptions have to be made on specific parametric marginal distribution. Scaillet and Fermanian [16] who conduct a Monte Carlo study to assess the impact of misspecified marginal distributions suggest that 'if one has any doubt about the correct modeling of the margins, there is probably little to loose but lots to gain from shifting towards a semiparametric approach'.

To measure the goodness-of-fit of the calibrated copulas, we use the Akaike information criterion *AIC* (Akaike [1]). This measure takes into account that the likelihood increases with the number of copula parameters by adjusting the measure accordingly.

Concerning statistical tests that explicitly test whether a parameterised copula at hand is indeed the true copula, one standard test does not yet exist. A widely used test is based on Rosenblatt's [15] *probability integral transform*.(this test is presented e.g. in Dias [4], pp. 27f) Statistical tests based on the probability integral transform suffer from the fact that they test for the whole joint distribution (i.e. the copula and the marginal distributions) while the focus should indeed be on the copula. An alternative test that, roughly speaking, examines the null hypothesis of the Gaussian copula being the true copula against the Student t copula can be conducted with a likelihood ratio test, where the Gaussian copula is regarded as a limiting case of the Student t copula with $\nu \to \infty$. Research on statistical tests examining the goodness-of-fit of copulas is still ongoing. Other tests than the ones presented above have been proposed by e.g. Fermanian [6] and Berg and Bakken [2].

The *computing time* for the copula-parameter estimation procedure and for the simulation of copulas varies considerably, depending on the copula.(the computations were done on a 'standard' personal computer (3.5 GHz processor, 1 GB RAM), using the software 'Matlab', version 7.2, in a MS-Windows environment) The parameter estimation for a Student t copula takes about 150 times as long as for the Gaussian copula. Simulations of a Gumbel and BB1 copula take much (roughly 1,000 times) longer than those for the other copulas. This is due to the necessity of using numerical optimisation for the simulation of these two copulas.

4 Empirical investigation

This section contains the empirical investigation of the dependence structure of daily joint interest rate and credit risk factor changes described in section 1. Table 1 shows sample estimates of Spearman's rho for the 25 bivariate empirical samples and reports whether they are statistically significantly different from 0. In all but 3 samples, the Spearman's rho correlation measure is negative, which is in line with the reported empirical evidence on the short-term relationship between interest and credit risk factor changes.

Bivariate Gaussian, Student t, BB1, Clayton, Gumbel, and Frank copulas are calibrated to the 25 data-pairs, using the pseudo-log-likelihood method. For the BB1, Clayton and Gumbel copulas also the rotated versions are fitted. Results are reported only for the rotated version with the best goodness-of-fit in terms of the AIC. The average AIC that is obtained by the copulas is displayed in table 2. One can see that the Student t copula on average yields the best goodness-of-fit, followed by the BB1 copula. Both copulas yield a better goodness-of-fit than their restricted versions, the Gaussian and the Clayton and Gumbel copulas, respectively. The Frank copula yields a substantially worse goodness-of-fit than the Student t and BB1 copulas.

Table 1: Spearman's rho for the 25 bivariate observation pairs.

	maturity bands					
rating	all	1Y-3Y	3Y-5Y	5Y-7Y	1Y-10Y	*mean*
all	-0.53***	-0.04*	-0.25***	-0.28***	-0.35***	*-0.29*
AAA	-0.78***	-0.08***	-0.28***	0.00	-0.08***	*-0.24*
AA	-0.26***	0.10***	-0.05**	-0.13***	-0.29***	*-0.12*
A	-0.43***	0.09***	-0.20***	-0.22***	-0.29***	*-0.21*
BBB	-0.56***	-0.21***	-0.23***	-0.28***	-0.30***	*-0.32*
mean	*-0.51*	*-0.03*	*-0.20*	*-0.18*	*-0.26*	

Statistically significantly different from 0 at the * 10%, ** 5%, *** 1% significance level.

Table 2: Mean AIC and number of times that H_0: 'copula at hand is the true copula' can be rejected for the 25 data samples.

		No. of times that H_0 can be rejected at significance level		
	mean AIC	10%	5%	1%
Gaussian	-204	22	19	14
Student t	-266	3	1	0
BB1	-237	12	11	6
Clayton	-186	19	17	6
Gumbel	-225	15	13	7
Frank	-207	17	11	10

The Student t copula is also the copula for which the null hypothesis of the copula at hand being the true copula can be rejected in least of the cases (see table 2). The null hypothesis of the BB1 copula being the true copula is rejected more often than the analogous null hypothesis for the Student t copula. The null hypothesis of the Gaussian copula being the true copula is rejected in most of the cases.

Additionally conducting a likelihood ratio test, the null hypothesis of the true copula being the Gaussian copula can be rejected in all of the cases in favour of the Student t copula at the 1% (and even at the 0.2%) significance level. This result is not surprising when the parameter distribution of the estimate of Student t copula parameter ν is regarded (as the Gaussian copula corresponds to a Student t copula with $\nu \rightarrow \infty$): The highest value is 12.09, the lowest 2.83. The mean (median) value is 5.68 (5.17).

The results suggest that 'positive tail dependence'(as the risk factor changes for the data sample at hand are generally negatively correlated, we define lower and upper 'positive tail dependence' for negatively correlated sample pairs in this paper as $\lambda_L = \lim_{\alpha\rightarrow 0} P(u_1 > 1-\alpha \mid u_2 \leq \alpha)$ and $\lambda_U = \lim_{\alpha\rightarrow 0} P(u_1 \leq \alpha \mid u_2 > 1-\alpha)$. This definition differes from the 'official' definition of lower and upper tail dependence $\lambda_L = \lim_{\alpha\rightarrow 0} P(u_1 \leq \alpha \mid u_2 \leq \alpha)$ and $\lambda_U = \lim_{\alpha\rightarrow 1} P(u_1 > \alpha \mid u_2 > \alpha)$) could be prevalent as the Student t and BB1 copulas that display positive tail dependence in all cases have a better goodness-of-fit than the Gaussian and Frank copula, which do not. The data sample at hand is large enough so that we may examine the potential existence of tail dependence in more detail. To do so, we introduce the concept of corner dependence, which is an empirical counterpart to the measure of positive tail dependence.

For positively correlated pairs in terms of Spearman's rho, we define the empirical lower and upper corner dependence as

$$\hat{\lambda}_{L,\alpha}^{empirical} = P(\hat{u}_1 \le \alpha \mid \hat{u}_2 \le \alpha),\ \hat{\lambda}_{U,\alpha}^{empirical} = P(\hat{u}_1 > 1-\alpha \mid \hat{u}_2 > 1-\alpha) \quad (5)$$

where û1 and û2 are empirical estimates of the quantiles of the interest rate and credit risk factor changes.

For negatively correlated pairs in terms of Spearman's rho, we define the empirical lower and upper corner dependence as

$$\hat{\lambda}_{L,\alpha}^{empirical} = P(\hat{u}_1 > 1-\alpha \mid \hat{u}_2 \le \alpha), \hat{\lambda}_{U,\alpha}^{empirical} = P(\hat{u}_1 \le \alpha \mid \hat{u}_2 > 1-\alpha) \quad (6)$$

To compute e.g. the empirical lower corner dependence for positively correlated pairs, we first identify the observation pairs for which $\hat{u}_2 \le \alpha$. The empirical lower corner dependence is the fraction of these pairs for which $\hat{u}_1 \le \alpha$.

Furthermore, for the Gaussian, Student t and Frank copula (symmetric copulas), we define the corner dependence that is implied by a specific copula with parameters θ for positive or negative Spearman's rho respectively as

$$\hat{\lambda}_{L,\alpha}^{implied} = \hat{\lambda}_{U,\alpha}^{implied} = \frac{C(\alpha,\alpha;\hat{\boldsymbol{\theta}})}{\alpha}, \hat{\lambda}_{L,\alpha}^{implied} = \hat{\lambda}_{U,\alpha}^{implied} = \frac{\alpha - C(1-\alpha,\alpha;\hat{\boldsymbol{\theta}})}{\alpha} \quad (7)$$

For the BB1 copula (asymmetric copula) and its rotated versions we define

$$\hat{\lambda}_{L,\alpha}^{implied} = \frac{C(\alpha,\alpha;\hat{\boldsymbol{\theta}})}{\alpha} \text{ and } \hat{\lambda}_{U,\alpha}^{implied} = \frac{2\alpha - 1 + C(1-\alpha,1-\alpha;\hat{\boldsymbol{\theta}})}{\alpha} \quad (8)$$

for C and C^{-+} copulas, and

$$\hat{\lambda}_{L,\alpha}^{implied} = \frac{2\alpha - 1 + C(1-\alpha,1-\alpha;\hat{\boldsymbol{\theta}})}{\alpha} \text{ and } \hat{\lambda}_{U,\alpha}^{implied} = \frac{C(\alpha,\alpha;\hat{\boldsymbol{\theta}})}{\alpha} \quad (9)$$

for C^-- and C^+- copulas.

For symmetric copulas (Gaussian, Student t and Frank) table 3 summarises in how many of the 25 cases the implied corner dependence exceeds both the lower and upper empirical corner dependence and in how many cases it is below for $\alpha \in \{0.05, 0.1\}$. The most evident observation is that the Frank and the Gaussian copula tend to underestimate the empirical corner dependence in all/most of the cases. The Student t copula overestimates the corner dependence in a lot more cases for $\alpha = 0.05$ than it does for $\alpha = 0.1$.

The corner dependence that is implied by the Student t and BB1 copulas is more closely examined in table 4, where the number of cases in which the lower and upper empirical corner dependence is over- or underestimated is reported. The Student t copula overestimates both lower and upper cornerdependence more frequently than it underestimates these measures for α = 0.05. No such behaviour can be observed for the BB1 copula.

Table 3: Number of cases where the implied corner dependence are below/exceed the empirical corner dependence for symmetric copulas.

$a = 0.1$	$\hat{\lambda}^{implied} < \min\{\hat{\lambda}_L^{emp.}, \hat{\lambda}_U^{emp.}\}$	$\hat{\lambda}^{implied} > \max\{\hat{\lambda}_L^{emp.}, \hat{\lambda}_U^{emp.}\}$
Gaussian	22	0
Student t	4	5
Frank	25	0

$a = 0.05$	$\hat{\lambda}^{implied} < \min\{\hat{\lambda}_L^{emp.}, \hat{\lambda}_U^{emp.}\}$	$\hat{\lambda}^{implied} > \max\{\hat{\lambda}_L^{emp.}, \hat{\lambda}_U^{emp.}\}$
Gaussian	20	0
Student t	1	12
Frank	24	0

Table 4: Number of cases where the implied upper and lower corner dependence of the Student t and BB1 copula are below/exceed the empirical corner dependence.

$a = 0.1$	$\hat{\lambda}^{implied} < \hat{\lambda}_L^{emp.}$	$\hat{\lambda}^{implied} > \hat{\lambda}_L^{emp.}$	$\hat{\lambda}^{implied} < \hat{\lambda}_U^{emp.}$	$\hat{\lambda}^{implied} > \hat{\lambda}_U^{emp.}$
Student t	10	15	14	11
BB1	14	11	15	10

$a = 0.05$	$\hat{\lambda}^{implied} < \hat{\lambda}_L^{emp.}$	$\hat{\lambda}^{implied} > \hat{\lambda}_L^{emp.}$	$\hat{\lambda}^{implied} < \hat{\lambda}_U^{emp.}$	$\hat{\lambda}^{implied} > \hat{\lambda}_U^{emp.}$
Student t	6	19	8	17
BB1	14	11	10	15

Focussing on the deviation of the implied corner dependence from the empirical corner dependence, the values in table 5 reported for 'diff. Ga λL', 'diff. Ga λU', 'diff. t λL', 'diff. t λU', 'diff. B λL', and 'diff. B λU' are the mean, median, and standard deviation of the log-differences of the implied and the empirical lower and upper corner dependence measures, $\ln \hat{\lambda}_{L,\alpha}^{implied} - \ln \hat{\lambda}_{L,\alpha}^{empirical}$ and $\ln \hat{\lambda}_{U,\alpha}^{implied} - \ln \hat{\lambda}_{U,\alpha}^{empirical}$, for the Gaussian, Student t and BB1 copula, respectively. Negative values indicate that the implied corner dependence is lower than the empirical corner dependence, which means that the parameterised copula underestimates the probability of joint excessive observations. The values presented in table 5 show that the Gaussian copula seems to systematically underestimate the probability of joint excessive events while the Student t copula overestimates them; in addition, these deviations are in absolute terms higher for α = 0.05 than they are for α = 0.1. However, the corner dependence implied by the Student t copula does not deviate as much from the empirical corner dependence as does the corner dependence implied by the Gaussian copula. For the BB1 copula, the deviations for $\alpha = 0.05$ are less pronounced than they are for the other two copulas, so one might infer that the BB1 copula's fit in that region is better than that of the Student t copula. Still, remember that the Student t copula's goodness-of-fit (indeed for the whole unit cube region) is superior in all cases to that of the BB1 copula, which allows for a flexible modelling of asymmetric corner dependence. This suggests that either this asymmetry is of minor importance as far as copula parameterisation for the data sample at hand is considered or that other asymmetric copulas than the BB1 copula should be employed.

Table 5: Log-Differences between implied and empirical corner dependence.

$a = 0.1$	Diff. Ga λ_L	Diff. Ga λ_U	Diff. t λ_L	Diff. t λ_U	Diff. B λ_L	Diff. B λ_U
mean	-0.22	-0.23	0.04	0.03	-0.04	-0.08
median	-0.17	-0.23	0.05	-0.03	-0.04	-0.04
std.dev.	0.18	0.13	0.10	0.13	0.12	0.13

$a = 0.05$	Diff. Ga λ_L	Diff. Ga λ_U	Diff. t λ_L	Diff. t λ_U	Diff. B λ_L	Diff. B λ_U
mean	-0.36	-0.39	0.14	0.12	0.00	-0.07
median	-0.42	-0.40	0.13	0.09	-0.03	0.02
std.dev.	0.42	0.23	0.24	0.24	0.31	0.27

5 Conclusion

Several studies have reported that changes in interest rates and changes in credit spreads are negatively related in the short run. These results are more closely examined in the present paper, which focuses on the dependence structure, i.e. the copula, between daily interest rate and credit risk factor changes that are computed from Euro-denominated sovereign and corporate bond indices for the time period from January 31st, 2000 to September 15th, 2006. The time series were adjusted for autocorrelation.

The empirical investigation shows that the daily risk factor changes are negatively correlated, which is in line with the empirical literature on this subject. However, the dependence structure seems to be very heterogeneous in an unsystematic way, depending on the rating of the obligors (credit risk) and the time until maturity of the financial instruments. As none of the marginal distributions of the risk factor changes are normally distributed, a multivariate Gaussian distribution should not be assumed for risk measurement purposes. Copula-based approaches are a promising and easily implementable alternative.

Gaussian, Student t, BB1, Clayton, Gumbel and Frank copulas were calibrated to the data sample, employing the pseudo-log-likelihood method. The best fit in terms of the AIC goodness-of-fit measure is achieved by the Student t copula. A likelihood ratio test rejects the null hypothesis of a Gaussian copula in favour of a Student t copula in all cases considered. The BB1 copula also yields good results (in terms of the AIC) and the goodness-of-fit is considerably better than for its two special cases, the Clayton and the Gumbel copula. However, the null hypothesis of the BB1 copula being the true copula is rejected quite often compared to the Student t copula. The Frank copula yields inferior results for the data sample at hand. Concerning the computing time needed for the estimation and simulation of copulas it is found that for Student t copulas parameter estimation takes very long while simulation is comparably fast and that the contrary can be said about the BB1 copula. One advantage of the Student t copula over the BB1 copula is that the former easily allows for a modelling of multidimensional copulas with a reasonable computational effort as far as simulation time is concerned – the simulation of a 5-dimensional copula takes only about 3 times as long as that of a bivariate copula.

Finally, the implied probability of joint strong risk factor movements is compared to the empirical probabilities. The Gaussian copula seems to systematically underestimate the probability of joint strong risk factor changes while the Student t copula seems to overestimate it. No such pattern can be found for the BB1 copula.

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