The relation between stroke work and end-diastolic volume in the ventricles

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Abstract

Several experimental studies have been published in the literature about the apparent linearity of the relation between stroke work (*SW*) and end-diastolic volume (V_{ed}), which is usually referred to as the preload recruitable stroke work (*PRSW*). Experimental results seem to suggest that in the study of the performance of the ventricles the *PRSW* relation gives more consistent results compared to the variability of the parameters based on the end-systolic pressure-volume relation (*ESPVR*). In this theoretical study it is shown that the relation between *SW* and V_{ed} is non-linear. Moreover it is shown that this relation can be described by the same parameters used to describe the *ESPVR*. Applications to experimental data published in the literature are discussed.
Keywords: preload recruitable stroke work, end-systolic pressure-volume relation, preload of left/right ventricular ejection, pressure-volume relation in the ventricles, active force of the myocardium, peak isovolumic pressure.

1 Introduction

The relation between stroke work (*SW*) and end-diastolic volume (V_{ed}) is also called the preload recruitable stroke work (*PRSW*) (Glower et al [1], Little and Cheng [2]). The *PRSW* relation shows an apparent linearity and is considered as a way to express the Frank-Starling mechanism; the larger the initial stretch of the cardiac muscle, the stronger is the contraction of the myocardium. It can be used to describe the contractility of the cardiac muscle and appears to give more consistent results than the parameters derived from the end-systolic pressure-volume relation (*ESPVR*) (Suzuki et al [3], Karunanithi et al [4]).

In a previous study (Shoucri [5]) it was shown that the *PRSW* relation is mathematically non-linear. It was also suggested that by using the same parameters used to describe the *ESPVR*, one can derive the relation between the stroke work *SW* and V_{ed}. This approach is further investigated in this study. Experimental data are presented to suggest new approaches for the study of the relation between *SW* and V_{ed}.

2 Mathematical model

The mathematical model used in this study is based on results obtained in previous studies by the author and has been consistently applied to a wide variety of experimental data related to the study of the performance of the left and right ventricles (Shoucri [5]-[11]). The left (or right) ventricle is represented in a simplified way as a thick-walled cylinder contracting symmetrically, with transverse isotropy with respect to the axis of the cylinder. Because of the assumed symmetry, a helical fibre in the myocardium will generate a radial force *D(r)* per unit volume of the myocardium as shown in fig. 1. At a given instant of the contraction cycle of the myocardium, *D(r)* generates a force per unit area on the inner surface of the myocardium given by $\int_a^b D(r)\,dr = \underline{D}h$, $r = a$ is the inner radius of the myocardium, $r = b$ is the outer radius of the myocardium, $h = b - a$ is the thickness of the myocardium, $\underline{D}$ is a value of *D(r)* calculated by the mean value theorem. In a quasi-static approximation (inertia and viscous forces neglected) the equilibrium of forces in the radial direction on the inner surface of the myocardium is expressed as

$$\underline{D}h - P = E\,(V_{ed} - V) \tag{1a}$$

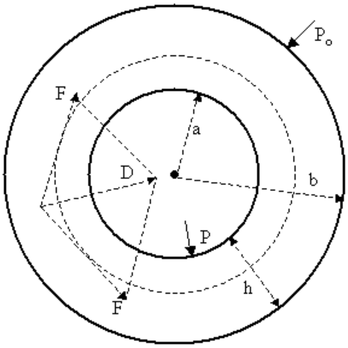

Figure 1: Cross-section of a thick-walled, cylinder representing the left or right ventricle. A helical fibre in the myocardium is projected on the cross-section as a dotted line. *D(r)* is the radial active force/unit volume of the myocardium. *P* is the ventricular pressure, P_o is the outer pressure on the pericardium (neglected), *a* = inner radius, *b* = outer radius, *h* = *b* – *a* = thickness of the myocardium.

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Eqn. (1a) can be split into two equations

$$P = E \ (V - V_d) \tag{1b}$$

$$\underline{D}h = E \ (V_{ed} - V_d) \tag{1c}$$

with the following definitions

V = ventricular volume.
V_{ed} = end-diastolic ventricular volume (when $dV/dt = 0$).
P = ventricular pressure.
E = ventricular elastance, slope of the pressure-volume line in fig. 2.
V_d = volume axis intercept of the pressure-volume line in fig. 2

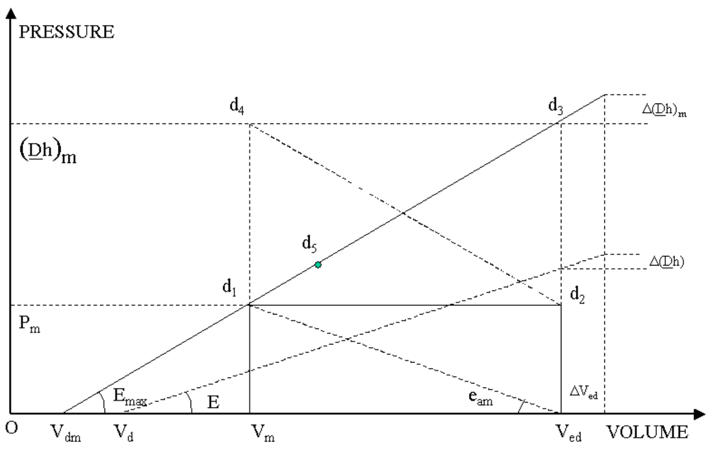

Figure 2: Simplified drawing showing the pressure volume loop in a normal ejection cycle $V_{ed}d_2d_1V_m$. The *ESPVR* is represented by the line d_3V_{dm} with slope E_{max}, the point d_5 is the middle of the segment d_3V_{dm}. $(\underline{D}h)_m$ is the peak radial active force/unit area generated by the muscular fibres on the inner surface of the myocardium (corresponding to the peak isovolumic pressure). It is assumed that $V_m \approx V_{es}$ (end-systolic volume). The changes $\Delta(\underline{D}h)$ and $\Delta(\underline{D}h)_m$ correspond to the change ΔV_{ed} in the end-diastolic volume according to the Frank-Starling mechanism.

A suffix m is added to the variables when E reaches its maximum value E_{max} near end-systole as shown in fig. 2. We get the following equations near end-systole

$$(\underline{D}h)_m - P_m = E_{max} \ (V_{ed} - V_m) \tag{2a}$$

$$P_m = E_{max} (V_m - V_{dm}) \tag{2b}$$

$$(\underline{D}h)_m = E_{max} (V_{ed} - V_{dm}) \tag{2c}$$

The approximation $V_m \approx V_{es}$ (end-systolic volume when $dV/dt = 0$) will be used in our computation. In the simplified diagram of fig. 2, d_3V_{dm} represents the *ESPVR* and $V_{ed}d_2d_1V_m$ represents the pressure volume loop in a normal ejection cycle, its area being equal to the stroke work *SW*. According to the simplified diagram given in fig. 2, the area *SW* is maximum when the point d_1 coincides with d_5 (the middle point of the *ESPVR* represented by the line d_3V_{dm}), in which case the ratio of ventricular elastance E_{max} to arterial elastance e_{am} is given by $E_{max}/e_{am} \approx 1$ and $P_m/(\underline{D}h)_m \approx 1/2$. The normal physiological state for the ventricles seem to correspond to $E_{max}/e_{am} \approx 2$ and $P_m/(\underline{D}h)_m \approx 1/3$, which corresponds to a state of maximum efficiency for oxygen consumption in the myocardium (Burkhoff and Sagawa [12], Asanoi et al [13], De Tombe et al [14]), $E_{max}/e_{am} < 1$ corresponds to a severely depressed state of the myocardium.

From fig. 2 we have the following expression for *SW*

$$SW \approx P_m (V_{ed} - V_m) \tag{3}$$

$SV = V_{ed} - V_{es} \approx V_{ed} - V_m$ is the stroke volume. The *PRSW* relation is usually written in the form

$$SW \approx M (V_{ed} - V_I) \tag{4}$$

where the slopes *M* and the intercept with the volume axis V_I are supposed to be constant (Glower [1], Little and Cheng [2], Karunanithi [4]). By comparing eqns (2b), (3) and (4) we see that we can establish the following relations

$$M \approx E_{max} (V_m - V_{dm}) \tag{5a}$$

$$V_I \approx V_m \tag{5b}$$

From eqns. (3), (4) and (5) we see that the relation between *SW* and V_{ed} is a complex and non-linear relation. It can be described by the same parameters derived from the *ESPVR*. The study of this relation remains linked to the unsolved problem of the way to determine the volume axis intercept V_{dm} of the line d_3V_{dm} representing the *ESPVR* (see fig. 2). In the following, we give some experimental evidence for this result.

3 Experimental applications

Fig. 3 is based on data for the left ventricle taken from table 1 of Little and Cheng [2] and obtained from experiments carried out on mongrel dogs. We have used these data to verify eqn. (5a) (left hand side) and eqn. (4) (right hand side) with $V_m \approx V_{es}$ substituted for V_I according to eqn. (5b). We have included the end-diastolic pressure P_{ed} in the calculation of the right hand side relation of fig. 3, but neglecting this quantity in the calculation gives a similar result.

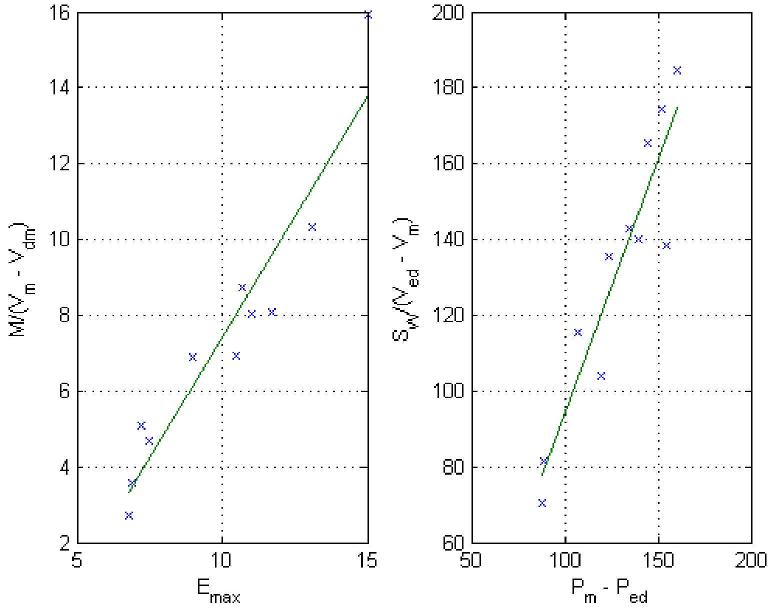

Figure 3: Verification of eqns. (5), least-squares relation for the left hand side is y = 1.28 x – 5.36 with correlation factor r = 0.977; least-squares relation for the right hand side is y = 1.34 x – 40.07 with correlation factor r = 0.994. Experimental data are taken from Little and Cheng [2].

Fig. 4 is based on experimental data for the left ventricle taken from table 2 of the work of Suzuki et al [3], the experiments were carried out on burned dogs. We have focused on the second group of data published in this table; the first group of data was discussed in Shoucri [5]. In the study by Suzuki et al [3] least-squares fit was used to calculate M and V_I from the relation between $SW = \int PdV$ and V_{ed} (eqn. 4), and E_{max} and V_{dm} were calculated by least-squares fit from the relation between P_m and V_m (eqn. 2b). In order to show how fig 4 was calculated, we show in table 1 some of the experimental data taken from Suzuki et al [3] and indicate how the calculation was carried out. A factor of 1.333 x 10^3 dyne/cm^2 is used to convert the values of M to mmHg in table 1. The procedure is as follows:

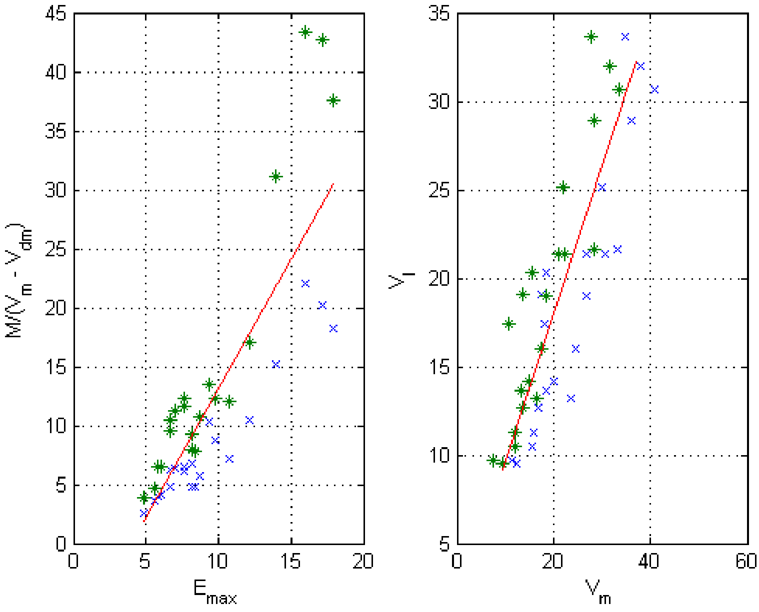

Figure 4: Verification of eqns. (5), least-squares relation for the left hand side is y = 2.19 x – 8.83 with correlation factor r = 0.958; least-squares relation for the right hand side is y = 0.84 x + 1.21 with correlation factor r = 0.989. Experimental data are taken from Suzuki et al [3].

Table 1: Verification of eqns (5); some of the experimental data used to draw fig. 4 and taken from Suzuki et al [3].

$P_m \approx PSP/1.2$ (mmHg)	E_{max} (mmHg/ml)	V_{dm} (ml)	M (mmHg)	V_I (ml)	V_m (ml)	$M/(V_m - V_{dm}) \approx E_{max}$ (mmHg/ml)
119.2	5.6	11.8	77.56	21.6	33.1	3.64
93.33					28.5	4.65
109.2	6.0	19.9	75.91	32.0	38.1	4.17
70.0					31.6	6.51
111.7	6.7	13.4	80.78	25.1	30.1	4.85
56.67					29.9	9.55
131.7	10.8	6.20	87.23	13.6	15.6	7.16
78.30					13.5	12.0
120.0	7.0	1.10	110.0	17.4	15.9	6.42
68.33					10.9	11.3

Table 2: Verification of the relation M/MEP ≈ (Ved – Vm)/(Ved – VI). Experimental data taken from Karunanithi et al [4].

V_{ed} (ml)	SV (ml)	M (mmHg)	V_I (ml)	V_{dm} (ml)	MEP (mmHg)	M/MEP	$SV/(V_{ed} – V_I)$
60	20	57.16	24.0	0.1	93.0	0.615	0.556
72	19	52.06	21.2	-15.5	116.0	0.449	0.374
61	24	61.59	24.4	2.5	92.0	0.669	0.656
74	20	49.96	22.4	-39.5	123.0	0.406	0.388

1) The peak systolic pressure PSP in table 2 of Suzuki et al [3] is divided by a factor of 1.2 to obtain an approximate value for P_m, the left ventricular pressure when E_{max} is reached. One should notice that for each value of E_{max} and V_{dm} in table 2 of Suzuki et al [3] there are two PSP values, as shown in table 1 of this study.
2) The values of $V_m = P_m/E_{max} + V_{dm}$, $(V_m \approx V_{es})$ were calculated (eqn. (2b)). Notice that the calculated values of V_m are approximately equal to V_I, graphical representation of the relation between V_m and V_I is shown in fig. 4 where the average of the two calculated values of V_m is used in the least-squares fitting.
3) The values of M are taken from table 2 of Suzuki et al [3], graphical representation of the relation between $M/(V_m – V_{dm})$ and E_{max} (eqn. (5a)) is shown in fig. 4. Notice in table 1 that these two values are of the same order of magnitude.

Finally table 2 is based on data taken from Karunanithi et al [4]. When two representations $M\ (V_{ed} – V_I) \approx MEP\ (V_{ed} – V_m)$ are used to calculate the same quantity SW ($MEP = SW/SV$ mean ejection pressure used when P_m is not assumed constant), one cannot expect exact verification of eqns (5) because different statistics give different results. But one can verify that the relation $M/MEP \approx (V_{ed} – V_m)/(V_{ed} – V_I)$ is reasonably verified as shown in table 2.

4 Suggestion for future work

Eqn (3) shows that the relation between SW and V_{ed} depends on P_m, and eqn. (2b) shows that P_m depends on the volume axis intercept V_{dm} of the $ESPVR$ represented by the line d_3V_{dm} in fig. 2. Consequently the (SW, V_{ed}) relation depends on finding a way to determine the quantity V_{dm} (or equivalently E_{max}). This problem is still unsolved, and is directly related to the study of the different areas under the $ESPVR$ (Shoucri [7,8]). We point out to possible approaches that can help future work in this field. Since one has to look for possible relations involving V_{dm}, we have shown in fig. 5 relations between $V_m - V_{dm}$ and $V_{ed} - V_{dm}$ on the left hand side, and $V_m - V_{dm} + V_w$ and $V_{ed} - V_{dm} + V_w$ on the right hand side where V_w is the volume of the myocardium (we assume $V_m \approx V_{es}$). The possibility to investigate similar relations can lead to a solution of the problem of the calculation of V_{dm}. The results of fig 5 are based on experimental data taken from Burns et al [15], and calculated in table 1 of Shoucri [11].

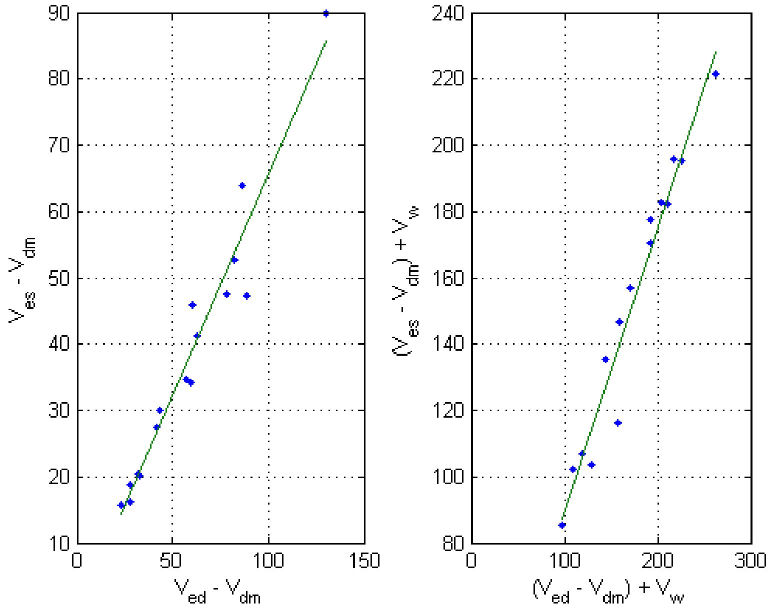

Figure 5: Least squares relation for the left hand side is y = 0.67 x –1.16 with correlation factor r = 0.976; least square relation for the right hand side y = 0.86 x + 3.93 with correlation factor r = 0.981. Experimental data are taken from Burns et al [15] and discussed in Shoucri [11].

5 Conclusion

The results of the present study tend to show that the relation between stroke volume *SW* and end-diastolic volume V_{ed} should be based on the parameters describing the *ESPVR*. The problem of the calculation of V_{dm} (or equivalently E_{max}) is still unsolved; a suggestion for a possible approach to this problem has been presented.

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